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Cyclic soil degradation/hardening models: A critique

Dégradation/durcissement de sol modèles cycliques: Une critique

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ABSTRACT
The response of soils to applied loading is affected by the current state of stress and previous loading history. To account for this when evaluating their response to cyclic or dynamic loading, it is necessary to track the state of stress at each given time. Cyclic degradation or hardening of soils results in an increase or decrease in soil stiffness and strength. This is as a result of changes in the effective confining stress due to void ratio or pore pressure changes. Therefore, all cyclic soil response models account for this effect in an implicit or an explicit manner by tracking a specified memory parameter. For most empirical models, the state parameter is usually related to the number of uniform or equivalent uniform loading cycles experienced by the soil. This paper discusses various implementations of such models in the literature from a general fatigue framework point of view. Various unique features of the models are highlighted to show their capabilities, underlying assumptions and limitations.

RÉSUMÉ
La réponse des sols au chargement appliqué est affectée par l'état actuel des contraintes et de l'histoire précédente de chargement. Pour évaluant leur réponse au chargement cyclique ou dynamique, il est nécessaire de moniteur l'état des contraintes à chaque temp. La dégradation ou le durcissement cyclique des sols est résultant une augmentation ou une diminution de rigidité et de résistance de sol. C'est en résultant des changements des contraintes confinez effective parce que des changements du rapport de vide ou de pression interstitielle. Donc, tous les modèles cycliques de réponse de sol compte cet effet d'une façon implicite ou explicite, par moniteur un paramètre spécifique de mémoire. Pour la plupart des modèles empiriques, le paramètre d'état est habituellement lié au nombre de cycles uniformes ou uniformes équivalents de chargement éprouvés sur le sol. Cet article discute de diverses réalisations de tels modèles dans la littérature d'un point de vue général de théorie de fatigue. De divers dispositifs uniques des modèles sont accentués pour montrer leurs possibilités, prétentions fondamentales et limitations.

1 INTRODUCTION

The accurate prediction of soil behaviour under cyclic or dynamic loading is of prime importance in many geotechnical engineering applications. However, this task can be challenging due to the complex interaction of many deformation mechanisms. The factors that influence the response include: void ratio and pore pressure changes, which affect the confining stress; stress and strain history; strain level; rate effects; frequency of loading; and the natural variability of soil deposits. Generally, for strains below a certain threshold shear strain (γ_{tl}), soil deformations are mainly elastic and recoverable. For strains above a certain volumetric threshold shear strain (γ_{tv}), pore pressure and volumetric changes occur, resulting in permanent degrading or hardening changes in the soil properties. Between γ_{tl} and γ_{tv}, the response is nonlinear, but does not result in any significant changes in the soil's characteristics.

The soil degradation or hardening occurs mainly due to plastic volumetric strains that lead to changes in void ratio or pore pressure. An increase in the pore pressure results in soil degradation, while a decrease in void ratio (volumetric compression) results in soil hardening. In advanced constitutive cyclic soil models, the void ratio and pore pressure changes are directly evaluated from the analysis. Some of these models are: multi-surface hardening models (Mroz et al., 1978); bounding surface models (Dafalias & Hermann, 1982); endochronic theory models (Valanis & Read, 1982); modified Cam Clay models (Carter et al., 1982); and more recently, the hierarchical single surface (HiSS) plasticity model version of the Disturbed State Concept (DSC) (Desai, 2001). These models utilize different approaches to estimate plastic volumetric and shear strains, and use them to establish memory parameters; in most models a single parameter (Van Eekelen, 1980), representing the past loading history at any point in time.

Cyclic nonlinear soil models account for degradation or hardening by using unload and reload curves that are similar to the original backbone curve but have been degraded or hardened. Two approaches are used to introduce the degradation/hardening into a model. One approach is to directly track pore pressure and/or volumetric strains. This is done by either using empirical incremental volumetric strain equations with a knowledge of the soil's rebound characteristics (Martin et al. 1975); or by evaluating them as a function of the number of cycles based on the results of uniform strain or stress-controlled cyclic triaxial or simple shear experiments (Seed et al. 1976). Another approach is to directly correlate the degradation/hardening of stiffness or strength with the number of cycles. Idriss et al. (1978) used this approach within the context of a total stress formulation to characterize the cyclic degradation of soft clays. Regardless of the approach used, fatigue is modeled by tracking the evolution of the chosen variable from an initial state to a final state and is referred to as damage accumulation.

To extrapolate the results from constant amplitude loading experiments to typical dynamic loading cases characterized by variable amplitude loads, extrapolation procedures are needed. In earthquake geotechnics and soil dynamics, these extrapolation procedures were developed empirically and their mathematical basis and underlying assumption were not presented. Since many geotechnical researchers and practitioners are not familiar with fatigue analysis, empirically based fatigue models can easily be misused and incorrectly implemented in geotechnical applications. Therefore, the main objective of this paper is to provide a detailed assessment and review of different soil degradation models in order to highlight their important features and assumptions.

2 GENERAL SOIL FATIGUE FORMULATION

Assuming uniform stress-controlled two-way cyclic loading for soils under the same initial conditions and negligible variability, the cyclic strain can be expressed by:

$$\gamma_c = f(D, \tau) \tag{1}$$

where τ represents a general stress variable, and D is a fatigue variable, which represents the effect of the past loading history. A similar equation exists for a uniform strain-controlled loading, however, the discussion is limited here to the stress-controlled loading but the results can also be applied to the strain-controlled loading case. It is important to note that a fatigue model developed for stress-controlled tests cannot be directly applied to strain-controlled tests, and vice-versa (Idriss et al., 1978; Van Eekelen, 1977). Eq. (1) can also be expressed in terms of the cyclic stress ratio, s, i.e.,

$$\gamma_c = f(D, s) \tag{2}$$

where $s = \tau/\sigma_m'$, and σ_m' is the mean effective confining stress. For a uniform stress-controlled loading, the fatigue damage function, D, is assumed to be a single-valued deterministic non-dimensional increasing (non-decreasing) function of the stress ratio and number of fatigue cycles, and is given as:

$$D = D(n, s) \tag{3}$$

with properties $D(0, N_f(s)) = 0$; $\quad D(N_f, N_f(s)) = 1$ (4a, b)

where N is the current number of cycles and $N_f(s)$ is the number of cycles at a constant stress ratio s to achieve a full damage condition (failure). The condition $D = 1$ refers to a damage curve termed either the *S-N* curve for the stress-controlled loading, or the ε_f-*N* curve for strain-controlled loading. The failure in this case is defined as the attainment of a specific value of a stated physical property. Since the damage function is non-decreasing,

$$\frac{dD}{dN} = q_1(N, s) > 0 \tag{5a}$$

and

$$\frac{d^2 D}{dN^2} = q_2(N, s) \tag{5b}$$

Combining Eq. (2) with Eq. (3), the fatigue stress-strain behaviour for a uniform stress-controlled loading can be defined fully. However, since we seek the response for a variable-amplitude loading history, the damage function D has to be defined for the general case of varying-amplitude loading. Fig. 1 shows a typical strain and damage contour from a uniform stress-controlled test (s is represented as the relative stress ratio $s_r = s/s_f$ in Fig. 1). The principle of "damage equivalence" dictates that, for a two-staged loading process (i.e. from a stress ratio of s_1 to s_2):

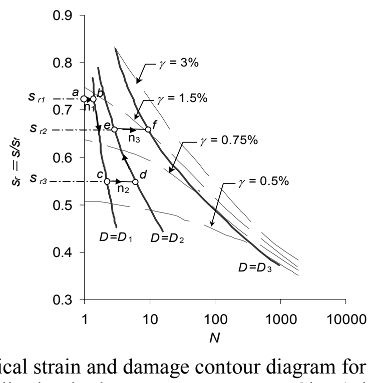

Figure 1. Typical strain and damage contour diagram for uniform stress-controlled cyclic simple shear test on Drammen Clay (adapted from Van Eekelen, 1977)

$$D(N_{11}, s_1) = D(N_{21}, s_2) = D_1 \tag{6}$$

where N_{11} is the number of cycles at s_1 to achieve a damage D_1 and N_{21} is the number of cycles at s_2 to achieve a damage D_1. Since the damage function in Eq. (3) is single-valued over the domain of interest, the number of uniform cycles at a given stress ratio to attain a specified damage, D, can be given by:

$$N = N(D, s) \tag{7}$$

Substituting Eq. (7) into Eq. (6), the number of equivalent cycles N_{21} can then be evaluated as:

$$N_{21} = N(D_1, s_2) \tag{8}$$

Referring to Fig. 1, continuing the loading with n_2 cycles at a stress ratio s_2 gives the new damage D_2 as:

$$D_2 = D((N_{21} + n_2), s_2) \tag{9}$$

This can be extended for a multi-stage loading sequence by applying Eq. (3) and Eq. (7) recursively.

Assuming that $q_1(N, s)$ in Eq. (5) is separable, this may be rewritten in the form (Van Eekelen, 1977):

$$q_1(N, s) = h_1(N) g_1(s) \tag{10}$$

and redefining the fatigue variable such that only the numerical values of fatigue change, and substituting Eq. (10) into Eq. (5a), the following equation is obtained

$$\frac{dD}{dN} = g_1(s) \tag{11a}$$

which yields the damage function:

$$D = N\, g_1(s) \tag{11b}$$

For the general loading process shown in Fig. 1, Eq. (11b) gives:

$$D_1 = n_1\, g_1(s_1) \qquad N_{12} = n_1 \frac{g_1(s_1)}{g_1(s_2)} \tag{12a}$$

$$D_2 = n_1\, g_1(s_1) + n_2\, g_1(s_2) \qquad N_{23} = \ldots$$

$$D_k = \sum_{i=1}^{k} n_i g_1(s_i) = \sum_{i=1}^{k} \frac{n_i}{N_f(s_i)} \tag{12b}$$

For the final damage condition, which is the case when the *S-N* curve is reached, $D = 1$, therefore:

$$\sum_{i=1}^{k} n_i g_1(s_i) = \sum_{i=1}^{k} \frac{n_i}{N_f(s_i)} = 1 \tag{12c}$$

For a stress ratio, s_o, in uniform stress-controlled loading, $D_k = N_{ok}\, g_1(s_o)$ and the number of equivalent cycles, NEC, estimated at s_o that gives the same damage D_k is given by:

$$N_{ok} = \sum_{i=1}^{k} \frac{n_i g_1(s_i)}{g_1(s_o)} = \sum_{i=1}^{k} \frac{n_i N_f(s_i)}{N_f(s_o)} \tag{12d}$$

Equation (12b) implies that the cumulative effect of damage caused by a number of stress cycles of different intensities can be obtained as the linear superposition of their various incremental damage effects, and is independent of the order in which the cycles occur. This also implies that the rate of fatigue damage evolution is the same throughout the process. Further, it can be shown that the curves of constant fatigue damage are parallel and equally spaced on a log (N) axis. This is the classical Palmgren-Miner (P-M) superposition rule (Palmgren, 1924; Miner, 1945) and is also referred to as linear damage accumulation. Thus, a necessary condition for the P-M rule to apply to a damage function is that its first derivative, $q_1(N, s)$ must be separable (Van Eekelen, 1977). Equation (12d) allows estimating the number of equivalent cycles at a given stress ratio when the P-M rule holds.

The damage is expressed in Eq. (11b) as a linear function of N, and Eq. (5b) stipulates that $q_2(N, s)$ in this case is equal to zero. This leads to the P-M rule (Eq. 12b) with linear damage accumulation. It may then be argued that the P-M rule is applicable only for the case where damage evolution is a linear function of N, a result of the separability assumption.

Equations (6)-(9) show the general process of damage accumulation for a variable loading history, and show that the damage function is continuous. It is noted that Eqs. (6) and (7) are used recursively to calculate the NEC and the amount of damage, D. Since for the P-M rule damage accumulation is a linear function of N, substituting Eq. (7) into Eq. (6) (i.e., forming the composition function) should yield a linear function of N, for the rule to hold. Mathematically speaking, the composition of these two functions can only result in a linear function of N, when Eqs. (7) and (6) are inverse functions, a characteristic of the damage function given in Eq. (11b). This implies that separability of $q_1(N, s)$ is not the only necessary condition for the P-M rule to apply. The requirement for the P-M rule to apply can thus be restated to be, "the P-M applies when over the domain of the damage function, the inverse of the damage function exists, and is a function corresponding to the number of equivalent cycles". A corollary of this statement is that the P-M rule can apply for cases in which damage accumulation is nonlinear, i.e., for cases where $q_2(N, s) \neq 0$.

As previously implied, the main features of the P-M rule are load-level independence and linear damage accumulation. With the revised statement of the P-M rule, two possible scenarios of nonlinear damage accumulation are examined:

Scenario 1: The load-level independence feature is maintained but damage evolution is nonlinear, implying that $q_2(N, s) \neq 0$. A nonlinear damage function D means that the damage rate changes with the number of cycles. Modifying the linear damage function given in Eq. (11b), an example of such a nonlinear damage function is:

$$D(N,s) = \left(N\, g_1(s)\right)^c \tag{13}$$

were c is a stress-independent variable and thus D is related to s only by way of g_1. For such cases the P-M rule is applicable in terms of the summation of incremental damage over different stress ratios. Since the function is nonlinear, damage evolution is nonlinear. However, the damage rate changes in the same manner at each stress level. For time-history analysis, this leads to a nonlinear evolution of damage with time, which in light of Eq. (1) results in different strain evolution trajectories for uniform stress-controlled loading. This would still be the case even if $f(s)$ was represented by a linear stress-strain relationship. Nonetheless, the complete damage occurs at $D = 1$ and the P-M rule can still be used to compute the fatigue life of the process, regardless of the pattern of damage evolution.

Scenario 2: This is the case where the damage evolution is load-level dependent and the expression for fatigue life given by the P-M rule (Eq. 12c) does not hold. Fatigue life for this case can be less than or greater than 1 depending on the load sequence (Van Paepegem & Degrieck, 2002). For this case also, $q_2(N, s) \neq 0$ and the change of the damage rate with the number of cycles is different at each stress level. Using Eq. (13) as an example, this occurs when c is a stress-dependent variable. For this case, the P-M rule does not apply under any circumstance, and its use in computing fatigue life is incorrect.

Different soil fatigue models available in the literature are now examined within the above general framework. This helps to assess the underlying assumptions of a particular fatigue evolution procedure, and shows how it is correctly applied in damage estimation.

3 SOIL FATIGUE MODELS

3.1 *Fatigue Contour Diagram Model*

Anderson et al. (1993) discussed an approach used by researchers at the Norwegian Geotechnical Institute (NGI) to evaluate soil fatigue necessary for the cyclic/dynamic response analysis of offshore gravity foundations founded on different types of soil. This approach involves the construction of a contour diagram of the desired fatigue variable (e.g. diagram shown in Fig. 1), which is established from experimental results. It encompasses three different fatigue accumulation procedures: pore pressure accumulation (PPA); cyclic shear strain accumulation (CSSA); and permanent shear strain accumulation (PSSA). The choice of a particular procedure to be used in the analysis is based on which variable is observed to be more prominent during the cyclic loading process, which in turn depends on the soil type and loading conditions. Since the contour diagram is established from experimental results, the mode of fatigue evolution is inherently captured. The choice of a particular approach implies defining that variable as the fatigue damage parameter, and the other variables are consequently evaluated. Anderson et al. (1993) proposed this approach as a pseudo-static methodology to determine the cyclic soil shear strength and the cyclic soil shear moduli for maximum load conditions, and to determine the soil stiffness for dynamic analysis.

Since the wave-loading regime is modeled using a chosen variable-load spectrum, it is known prior to the analysis. The damage accumulation can then be computed using the contour diagram and employing a procedure similar to the general fatigue formulation discussed previously. Apart from the choice of a particular damage parameter, no assumptions are inherent in the method. The issues of load-sequence and load level independence are irrelevant in this case as the mode of fatigue evolution is inherently captured within the experimentally derived fatigue contour. However, the CSSA procedure assumes that an increment in shear stress results in a corresponding increment in shear strain. From empirical evidence, the incremental strain is estimated using the virgin cyclic stress-strain curve implying that, contrary to the general fatigue formulation, cyclic weakening does not affect the stress-strain response.

Generally, fatigue contour diagrams established from experiments are more accurate than analytical models in representing the fatigue evolution. This is especially true because most analytical models correctly predict either the small-strain or large-strain response, but not both (Yoshida et al. 2004). This is an interesting approach, which holds a lot more promise for the future.

3.2 *Degradation Index Model*

3.2.1 *Background*

This model was first developed by Idriss et al. (1978) to represent the cyclic degradation of undrained clay under uniform strain-controlled loading in a total stress cyclic nonlinear analysis. It is the most popular phenomenological soil fatigue model and has been widely used in many applications.

For uniform strain-controlled cyclic shear strain loading and defining G_{s1} and G_{sN} as the secant shear moduli at cycles 1 and N, respectively, it can be shown that:

$$\delta_N = \frac{G_{sN}}{G_{s1}} = \frac{\left(\dfrac{\tau_{cN}}{\gamma_c}\right)}{\left(\dfrac{\tau_{c1}}{\gamma_c}\right)} = \frac{\tau_{cN}}{\tau_{c1}} \tag{14}$$

This yields

$$\delta_N = N^{-t} \tag{15}$$

Vucetic & Dobry (1988) have shown that the degradation parameter, t, depends on the plasticity index (PI) and the overconsolidation ratio (OCR). Low plasticity normally consolidated soils have higher t values; while high plasticity overconsolidated soils have lower t values. Pyke & Beikae (1993) have proposed a general power law expression for t:

$$t = s_t\left(\gamma_c\right)^{r_t} \tag{16}$$

where s_t and r_t are curve-fitting constants.

The degradation index approach has been shown to yield reasonable results for cohesive soils, but poor results for cohesionless soils (Matasovic & Vucetic, 1995). It is interesting to note this varying performance albeit the excess pore pressure is the main cause of degradation in both cases. Pore pressure models for clay show that a maximum pore pressure is associated with each cyclic strain level. On the other hand, the degradation index model as given by Eq. (15) is an ever-decreasing function that is asymptotic to the *N*-axis, which implies that full degradation is possible at all strain levels. This requires that, for any strain level, after pore pressure stabilizes, there are other mechanisms that lead to the complete degradation of the soil stiffness. Even though this is theoretically possible, physically this mechanism does not exist.

This underscores the importance of a defined failure condition representing full damage, such as the *S*-*N* (ε_f -*N*) curve. None of the implementations of the degradation index model, however, have been presented in this form. This may be attributed, at least partly, to two factors: the model was empirically developed; and that no significant event occurs that defines onset of failure. In this approach, failure can only be defined as the attainment of a specified amount of degradation. The following development of the fatigue evolution for the degradation index model is therefore based on the assumption of a failure condition.

3.2.2 *Mathematical formulation*

Assuming a minimum level of degradation (δ_{Nf}), and equating slopes, it can be shown that:

$$\frac{\log(\delta_N)}{\log(\delta_{Nf})} = \frac{\log(N)}{\log(N_f(\gamma_c))} \tag{17a}$$

where $$N_f(\gamma_c) = N_f(\gamma_c, \delta_{Nf}) = 10^{\frac{\log(\delta_{Nf})}{-t(\gamma_c)}} \tag{17b}$$

which gives $$\delta_N = \delta_{Nf}^{\frac{\log(N)}{\log(N_f(\gamma_c))}} \tag{17c}$$

Inspecting Eq. (17a), δ_N is represented as a function of δ_{Nf}, and N_f is a function of γ_c and δ_{Nf}, which imply that for the degradation index model, N_f is not an independent quantity. This means that the definition of the *S*-*N* (ε_f -*N*) curve can fully define the fatigue evolution model.

Equation (4) requires that the fatigue damage function satisfies the condition $0 \le D \le 1$. The damage function can then be defined using Eq. (17a) as:

$$\frac{\log(\delta_N)}{\log(\delta_{Nf})} = D(N, \gamma_c) = \frac{\log(N)}{\log(N_f(\gamma_c))} \tag{18}$$

Equation (18) describes a damage function that also satisfies Eq. (4) as shown in Fig. 2. This damage function has a $q_2(N, s) < 0$ (concave) trend, with an initial slope of $1/\log(N_f(\gamma_c))$, which decreases rapidly at the beginning, at $D = 0$, and later slows down to a slope of $1/(N_f(\gamma_c)\log(N_f(\gamma_c)))$ at $D = 1$.

Assuming that the graph in Fig. (1) is in terms of cyclic strain (γ_c) instead of stress ratio, and considering Eq. (18), the equation for damage in recursive form can be derived as:

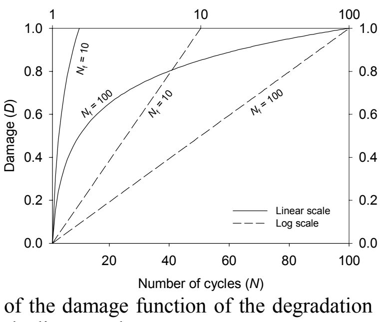

Figure 2: Plot of the damage function of the degradation index model on both a log and a linear scale

$$D_k = \frac{\log\left(\left(N_f(\gamma_{ck})\right)^{D_{k-1}} + n_k\right)}{\log(N_f(\gamma_{ck}))} \tag{19a}$$

Using Eq. (17a), it can then be shown that

$$\delta_{N_k} = \left(\left(\delta_f^{\,D_{k-1}}\right)^{\frac{1}{t(\gamma_{c_k})}} + n_k\right)^{t(\gamma_{c_k})} \tag{19b}$$

From Eqs. (17b) and Eq. (15), it can be inferred that:

$$\delta_{N_k} = \left(\delta_{N_{k-1}}^{\frac{1}{t(\gamma_{c_k})}} + n_k\right)^{t(\gamma_{c_k})} \quad \text{where} \quad \delta_{N_{k-1}} = \delta_f^{\,D_{k-1}} \tag{19c}$$

Equation (19b) was initially derived by Idriss et al. (1978) in the context of the degradation index approach for transient nonlinear analysis applications. Equation (19c) also shows the relationship between the damage function and the degradation function. This formulation therefore sets the degradation index model within the correct fatigue context.

3.2.3 *Comments*

The derivation of Eq. (19b) indicates clearly that the P-M rule does not apply to the degradation index model. Fig. 3 shows degradation contours for normally-consolidated VNP marine clay, a high plasticity stiff clay with $s_t = 0.05$ and $r_t = 0.5$ (Matasovic & Vucetic, 1995); and normally consolidated Alaska GAL clay, a low-plasticity marine clay with $s_t = 0.23$ and $r_t = 0.52$ (Idriss et al., 1977). As expected, the contours are not parallel and GAL clay experiences more degradation. The unique form of the damage function in Eq. (18) is similar to the form $D = h(N)g(s)$, which is separable but $q_1(N, s)$ is not separable. The function can be classified as a stress-dependent model because of its load-level dependency.

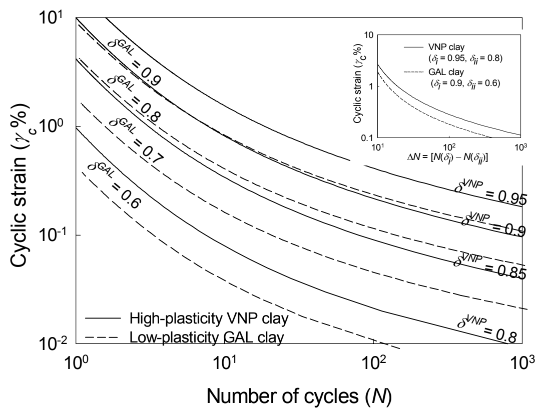

Figure 3: Degradation contour plot for high plasticity VNP clay (Matasovic and Vucetic, 1995) and low plasticity Alaska GAL clay (Idriss et al., 1977) for uniform strain-controlled test

The ε_f -*N* curve defines the model fully and thus it is not possible to define the ε_f -*N* curve and the fatigue evolution expression separately. This is because the function can only be represented by a straight line in a semi-log plot as can be noted from Fig. 2. This reduced "degree of freedom" decreases the adaptability of the model and diminishes its utility to ascertain the applicability of the P-M rule.

The degradation index model can only describe concave shape fatigue evolution, i.e., $q_2(N, s) < 0$ trends, and this is an attribute of the model that could cause it to be incorrectly applied. For example, Poulos (1982) and Sawant & Dewaikar (1993) have used it to model the degradation of the limiting pile-soil interaction force for piles in clay, due to degradation of the undrained strength. However, undrained strength of clays degrade with a fatigue evolution function characterized by a convex form, i.e., $q_2(N, s) > 0$. Therefore, they rather model degradation of the limiting pile-soil interaction force due to degradation of the soil modulus, since that has a trend $q_2(N, s) < 0$.

3.3 *Seed's Liquefaction Model*

3.3.1 *Background*

Liquefaction is a soil degradation phenomenon that occurs in saturated sands, for which total and effective stress models have been developed. For effective stress analysis, the generation of excess pore pressure needs to be tracked. Seed et al. (1976) developed a model based on the results of cyclic simple shear tests (De Alba et al., 1975). The results obtained by De Alba et al. (1975) fell within a fairly narrow band when plotted in a normalized form. Based on their experimental results, De Alba et al. (1975) proposed a model to predict pore pressure evolution, i.e.,

$$r_u = \frac{2}{\pi}\arcsin\left(\frac{N}{N_f(s)}\right)^{1/2\theta_p} \tag{20}$$

where r_u is the pore pressure ratio and θ_p is a curve-fitting parameter. The data from De Alba et al. (1975) fell within the range of $\theta_p = 0.5 - 1.3$, and they recommended the use of an average curve with $\theta_p = 0.7$. The effective stress pore pressure model can then be defined fully by combining Eq. (20) and the liquefaction initiation curve, which represents the number of uniform stress cycles at a given stress ratio to initiate liquefaction. In fatigue terms, the liquefaction initiation curve is the *S-N* curve and Eq. (20) is related to the fatigue evolution function. This model can be seen to have an extra "degree of freedom" when compared to the degradation index model.

Seed et al. (1982) and Liyanapathirana & Poulos (2002) used this model to predict pore pressure generation. Seed et al. (1976) used the NEC approach (Eq. (12d)) and evaluated the NEC at a specified stress ratio of $0.65\tau_{max}$, where τ_{max} is obtained from a total stress analysis. A drawback of this method is that the maximum shear stress (τ_{max}) must be known prior to performing the nonlinear effective stress analysis. Liyanapathirana & Poulos (2002) adapted this approach using pore pressure equivalency, which is essentially based on a damage equivalence principle. Their results including: pore pressure ratio; shear stress versus time; and shear stress versus shear strain compared well with results of laboratory experiments and field case studies. This proved this model to be superior to the method by Seed et al. (1976). In addition, it does not require prior knowledge of the maximum shear stress.

3.3.2 *Mathematical formulation*

A suitable fatigue damage function must satisfy the conditions given in Eq. (4). It is evident that Eq. (20) satisfies this criterion and can be taken as the fatigue damage function, i.e., $D = r_u$. The damage function is observed to be not separable, and depends on s only through $N_f(s)$. It is assumed that pore pressure dissipation effects are minimal (Seed et al., 1982). The damage function has the following characteristics:

- $\theta_p > 0.5$ Double curvature – starts with a $q_2(N, s) > 0$ trend and a slope of infinity, and ends with the $q_2(N, s) < 0$ trend and a slope of infinity
- $\theta_p < 0.5$ Single curvature – starts with a $q_2(N, s) < 0$ trend and a zero slope and ends with a slope of infinity.
- $\theta_p = 0.5$ Single curvature – starts with a $q_2(N, s) < 0$ trend and a slope of 0.637 and ends with a slope of infinity

From Fig. 1, starting from b onwards and assuming $\theta_p = \theta_p(s)$ it can be shown that:

$$D_k = \frac{2}{\pi}\arcsin\left(\left(\left(\frac{n_1}{N_f(s_1)}\right)^{\theta_p(s_2)/\theta_p(s_1)} + \frac{n_2}{N_f(s_2)}\right)^{\theta_p(s_3)/\theta_p(s_2)} \cdots + \frac{n_k}{N_f(s_k)}\right)^{1/2\theta_p(s_k)} \tag{21a}$$

which for constant θ_p, gives

$$D_k = r_u = \frac{2}{\pi}\arcsin\left(\sum_{i=1}^{k}\frac{n_i}{N_f(s_i)}\right)^{1/2\theta_p} \tag{21b}$$

3.3.3 *Comments*

The model developed by Seed et al. (1976) is a nonlinear damage accumulation model, with Eq. (21a) representing its stress-dependent form, while Eq. (12a) represents the stress-independent form. For the stress-independent form, it is noted that the argument of the arcsin function is the P-M damage function given in Eq. (12b). This means that the contribution to fatigue damage from different stress ratios is equal; however, the pattern of damage evolution is nonlinear.

Fig. 4 shows the pore pressure contours (damage contours) based on the liquefaction-triggering curve for Monterey sand of relative density 68% (De Alba et al., 1976). The contours are parallel because the damage is load-level independent, but the contours are unevenly spaced due to the nonlinear nature of damage evolution.

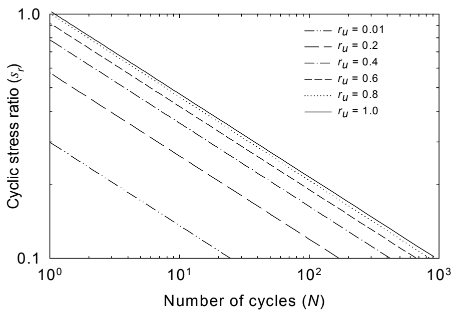

Figure 4: Typical pore pressure (damage) contours for Monterey sand of relative density 68%

In developing their model, Seed et al. (1976) assumed that the P-M rule applies, and hence, computed the NEC at $0.65\tau_{max}$, before estimating r_u. This approach requires the prior knowledge of τ_{max}, which may necessitate an initial total stress analysis before conducting the effective stress analysis. However, observing Eq. (12b) it can be noted that if the damage accumulation is calculated using this equation instead of the NEC, this would not be necessary. This underscores the benefit of clearly outlining the fundamental concepts of the fatigue phenomenon and their implementation in the model.

From Eq. (21), the assumption of constant θ_p is a necessary condition for the P-M rule to apply. It implies that the pattern of evolution of pore pressure is the same at all stress levels. This observation is fundamental and critical to the Seed approach, since constant θ_p implies that the P-M rule applies. For the practical cases studied by Seed et al. (1976), the differences in the evolution patterns at different stress levels were slight. However, for situations when this is not the case, the application of the model to estimate the pore pressure would be incorrect.

The Liyanapathirana & Poulos (2002) approach is based on damage equivalence and thus, the condition of constant θ_p is not required. However, they used the stress-independent form (Eq. (21b)) of the pore pressure damage function. Since the model can account for the case of $\theta_p = \theta_p(s)$, it would be beneficial to extend it to cases where there is a significant difference in the pore pressure evolution with stress.

4 CONCLUSIONS

Various cyclic soil degradation formulations used in earthquake geotechnics and soil dynamics are assessed within the fatigue framework, based on the general mathematical fatigue formulation for multi-stress or strain loading, based on the principle of damage equivalence. A summary of the assessment is given below.

i. The main requirement for the P-M rule to apply to a fatigue damage function is that the change of subject transformation to obtain the number of cycles function yields its inverse function.
ii. The P-M rule applies to stress-independent nonlinear damage accumulation models, but not to stress-dependent models.
iii. The degradation index model developed by Idriss et al. (1978) is stress-dependent with a damage function that couples the *S-N* (ε_f -*N*) curve and fatigue evolution function. This limits its ability to model different fatigue evolution patterns and can only model a fatigue process with $q_2(N, s) < 0$.
iv. The pore pressure model proposed by Seed et al. (1976) is stress-independent, and assumes that θ_p is constant, and thus, the P-M rule applies. If the damage expression given by Eq. (12b) is used instead of the expression of the equivalent number of cycles (Eq. (12d)), τ_{max} need not be known apriori.

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