# Relative density effects on drained and undrained strengths of sand at high pressures

Les effets de la densité rélative sur la résistance du sable drainé et non drainé aux hautes pressions

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# ABSTRACT

Drained and undrained triaxial tests were performed to study effects of sand density on behavior at high pressures in compression and extension. Initial relative densities of 30, 60, and 90% and confining pressures from 0.25 to 60 MPa were used. Isotropic compression curves merge at high pressures resulting in a single compression curve and similar sand response during shearing, independent of initial relative density. Failure envelopes merge at high pressures, and shear strength continues to increase linearly with confining pressure. Total stress interpretation shows that sand at high pressures behaves as normally consolidated clay.

#### RÉSUMÉ

Des tests sur sol drainé et non drainé ont été effectués afin d'étudier l'influence de la densité de sable sur son comportement aux hautes pressions lors des phases de compression et d'extension. Des densités rélatives initiales de 30, 60, et de 90% et des pressions de confinement de 0,25 à 60 MPa ont été utilisées. Les courbes de compression isotropique convergent aux hautes pressions, donnant ainsi une courbe de compression unique et une réponse du sable semblable lors des glissements, indépendamment de la densité rélative initiale. L'interprétation de la tension totale montre que le sable aux hautes pressions se comporte comme de la glaise normalement consolidée.

#### 1 INTRODUCTION

High pressure testing of granular materials under drained conditions has traditionally been associated with construction of large-scale rockfill dams, deep wells and tunnels, and driven pile foundations, where soils under high pressures are found. Interest in the behavior of granular materials under undrained conditions originates from the fact that these may be loaded at relatively high rates during e.g. earthquakes, blasting, and pile driving. For such conditions the granular material may become undrained and develop substantial pore pressures, especially if the material has low relative density and high compressibility. This, in turn, may lead to instability, a condition that can occur inside the failure surface. Instability is the prerequisite for soil liquefaction, and this may lead to catastrophic events of manmade and natural earth structures. Significant particle crushing occurs at high pressures, and this affects the stress-strain, volume change or pore pressure, and strength characteristics, thus making the soil behavior at high pressures hard to extrapolate from low pressure experiments.

Granular materials with different densities respond in different ways to applied shear stress. At low confining pressures, loose sand will compress and dense sand will dilate during shear (Casagrande 1940, Lee and Seed 1967). The behavior of sand at high confining pressures has been studied in detail in connection with a larger investigation of instability of granular materials at high stresses. In this investigation it was found that material that dilates exhibits stable behavior until the failure surface has been reached, whereas materials that compress may become unstable inside the failure surface (Lade et al. 1988, Bopp and Lade 1997, Yamamuro and Lade 1997). The dilative nature of dense sand decreases with increasing confining pressure, and only contractive behavior is observed at higher confining pressure. The magnitude of pressure at which the dilatant behavior is suppressed is dependent upon the initial relative density. Besides, the effect of initial sand fabric, which may play an important role at lower confining pressures, is not significant at high

pressures where the fabric is controlled by the contraction and crushing of the grain structure.

Presented here are the results of a detailed study of the behavior of Cambria sand at three different relative densities (30, 60 and 90%) at high confining pressures (up to 60 MPa) under drained and undrained conditions in triaxial compression and extension. The experimental results show consistent patterns of behavior from low to high confining pressures in both compression and extension.

# 2 TESTING EQUIPMENT, SAND, AND PROCEDURES

The primary attributes of the high pressure testing equipment employed to perform the experiments presented here are similar to those of a standard triaxial setup. The majority of the components were oversized in relation to their low pressure counterparts so that confining pressures up to 100 MPa could be achieved during testing. The high pressure equipment was briefly described by Yamamuro and Lade (1993).

The granular material used in the experimental program was Cambria sand. This coarse, uniform sand consisted of subangular to well-rounded grains with diameters between 0.83 and 2.00 mm. The maximum void ratio was determined to be 0.792, the minimum void ratio was 0.503, and the specific gravity was 2.69.

All triaxial compression tests in the experimental program were performed on cylindrical specimens 7.1 cm (2.8 in.) in diameter and 17.8 cm (7.0 in.) in height, for a height-to-diameter ratio of 2.5. Triaxial extension test specimens were also cylindrical in shape with equal height and diameter of 8.9 cm (3.5 in.) for a height-to-diameter ratio of 1.0. Lubricated ends were employed in all tests. The specimens were prepared by dry pluviation of the sand from different fall heights to produce initial relative densities of 30% (zero fall height), 60%, and 90%. To prevent leaks at high pressures due to puncture of the membrane by the sand grains, a total of five 0.64 mm (0.025 in.) thick latex

membranes were placed around the specimen. The load taken by the membranes was automatically subtracted by the control program calculating the stresses. After preparation of a specimen, saturation was achieved using the  $CO_2$ -procedure, after which a back pressure of 0.52 MPa (75 psi) was applied to ensure full saturation. Further details regarding the experimental techniques employed for the high pressure testing was discussed by Yamamuro and Lade (1996).



Fig. 1. Void Ratio Related to Isotropic Confining Pressure for Loose, Medium Dense, and Dense Cambria Sand.

# 3 EFFECTS OF RELATIVE DENSITY IN ISOTROPIC COMPRESSION

The effect of initial relative density on the isotropic compression curves shown in Fig. 1 indicates that after application of confining pressures higher than 15 MPa, sand specimens with different initial void ratios have approximately the same void ratio for a given isotropic confining pressure. However, the compression curves merge in sequence with increasing pressures and initial densities. Similar conclusions have been made by other investigators for isotropic and K<sub>0</sub>-compression of sands (e.g. Pestana and Whittle 1995; Yamamuro et al. 1996). This also conforms closely to results produced by Vesic and Clough (1968), who concluded that the initial void ratio does not seem to have any effect on behavior of specimens consolidated to pressures above 20 MPa for Chattahoochee River sand. Similarly, Miura and Yamanouchi (1975) found that the effect of initial density on the stress-strain relationship decreases with increasing confining pressure. However, they concluded that even at high confining pressures such as 50 MPa, the initial density continued to have some influence on the stress-strain relations.

With increasing confining pressure a greater degree of particle crushing and grain rearrangement occurs during the isotropic compression phase, and the effect of initial relative density on the behavior during shear is progressively reduced. For tests with confining pressures greater than 15 MPa, the stressstrain, volume change (drained tests) or pore pressure curves (undrained tests) for specimens at the same consolidation pressure are very similar and the effect of initial relative density appears to be negligible.

# 4 EFFECTIVE FRICTION ANGLES IN COMPRESSION

Fig. 2 shows a comparison of the effective friction angles from the drained and undrained triaxial compression tests on specimens with all three initial relative densities. The effect of initial relative density appears to be minimal at high pressures.

The friction angles from the *drained* tests decrease initially as the confining pressure and effective mean normal stress at failure increases. This trend is caused by a progressive reduction in the dilatant behavior of the material with increasing confining pressures. The friction angles reach low values at 3.5 MPa for the experiments with relative densities of 30 and 60%, while the lowest friction angle occurs near 7 MPa for the dense sand. The friction angles reach very similar values around 7 MPa, and above this value there appears to be little effect of initial relative density on the Mohr-Coulomb secant friction angles.

The effective friction angles from the *undrained* tests are also shown in Fig. 2, and they merge together and stabilize at a constant value beyond an effective mean normal stress at failure between 6 and 7 MPa where the friction angles from the drained compression tests also merge together. At the lower mean normal stresses, the friction angles from the undrained tests are similar in magnitude to the drained friction angles. Specimens consolidated to higher confining pressures, however, exhibit measurably larger friction angles under undrained conditions than their drained counterparts when compared at the same mean normal stress at failure.

# 5 EFFECTIVE FRICTION ANGLES IN EXTENSION

The *drained* Mohr-Coulomb secant friction angles in extension for the Cambria sand with initial relative densities of  $D_r = 30\%$ , 60%, and 90% are plotted against the effective mean normal stress at failure in Fig. 3. The highest friction angles are obtained at low stress magnitudes, and they decrease with increasing mean normal stress at failure. At approximately 11 MPa the friction angles reach a minimum, and they remain almost constant or decrease very little for the remainder of the high pressure range.

The effect of initial density on the friction angle is apparent over the entire pressure range in triaxial extension. At low pressures the dense specimens exhibit dilative behavior and achieve high friction angles. With decreasing relative density, dilative behavior is reduced leading to lower friction angles. In contrast with the compression friction angles shown in Fig. 2, the friction angles in extension never merge at high pressures. The pattern of higher friction angles with higher initial relative density is maintained even at the highest stress magnitudes. The effect of particle crushing and densification that reduces and even eliminates most of the effect of initial relative density in compression tests is not nearly as severe in extension tests. Thus, initial relative density continues to play a role over the entire range of high pressure testing.

The Mohr-Coulomb secant friction angles determined from the maximum effective stress ratios in the *undrained* extension tests are also shown in Fig. 3. The friction angles for the three relative densities vary only little in the range from  $32^0$  to  $35^0$ over the entire pressure range, and they are slightly higher than the drained extension friction angles.



Fig. 2. Comparison of Effective Friction Angles for Drained and Undrained Triaxial Compression Tests on Loose, Medium Dense, and Dense Cambria Sand.



Fig. 3 Comparison of Effective Friction Angles for Drained and Undrained Triaxial Extension Tests on Loose, Medium Dense, and Dense Cambria Sand.

### 6 STRENGTH RESULTS IN TERMS OF TOTAL STRESSES

Numerous attempts have been made in the past to develop an encompassing set of principles applicable to all soil behavior. One of the more widely acknowledged concepts include the critical state theories proposed by Casagrande (1940), Rutledge (1947), Roscoe et al. (1958), and Seed and Lee (1967). Lade and Yamamuro (1996) examined these theories and they concluded that the Rutledge Hypothesis, originally applied to normally consolidated clays, provides a unifying behavior pattern for sands at high pressures.

Rutledge (1947) found that (1) the consolidation water content (or void ratio) for fully saturated, normally consolidated clays depends only on the major principal stress, and that (2) the compressive strengths from triaxial compression tests on saturated, normally consolidated, homogeneous clays depends only on the water content at failure. When plotted on a diagram of water content vs. log(stress), the consolidation curve, represented by the major principal stress on the log(stress)-axis, and the strength curve, represented by the compressive strength, would form parallel curves. The importance of this outcome is that strength predictions, whether for drained or undrained conditions, can in principle be made from a conventional onedimensional consolidation test and a single undrained triaxial compression test on the particular normally consolidated clay in question. These observations, reported in 1947, also form part of the background for Critical State Soil Mechanics.

Rutledge's Hypothesis and the counterpart to the Cam Clay model, namely the Granta Gravel model (Schofield and Wroth

1968), do not work for granular materials in the range of conventional geotechnical stress magnitudes, because the consolidation curve and the strength curve cross each other at low pressures. However, as the stresses increase, significant particle crushing begins, and the compressibility of the sand increases when plotted on an e - log(stress) diagram. The e - log(stress) diagrams for Cambria sand with initial relative densities of 30, 60, and 90% are shown in Fig. 4, and they indicate that the isotropic consolidation and the strength curves become parallel, and the sand behavior begins to resemble that of a normally consolidated clay at high pressures. Results from triaxial compression and extension tests performed under drained and undrained conditions are shown in these diagrams. Results of isotropic and one-dimensional compression tests with initial void ratios similar to those of the triaxial test specimens are also shown in the diagrams.

Examination of Fig. 4 indicates that the extension and compression strengths plot along separate lines at low pressures. The compression deviator stresses are higher than the extension values. At these low stresses the specimens are dilating, which increases the void ratio. As the stresses increase the specimens start to undergo volumetric compression at an increasing rate, due to particle crushing and grain rearrangement, and the two strength lines merge together and form a single compressive strength line for all four types of triaxial tests. The compressive strength of the Cambria sand therefore depends only on the void ratio at failure and is independent of pore pressure and test type. Fig. 4 shows that this is true for all initial relative densities.

Comparison of the three e - log(stress) diagrams also shows that the stress magnitude at which the strength lines from compression and extension merge together increases with increasing relative density. This appears to be coincident with the stress magnitude at which particle breakage begins and large volumetric strains occur. For higher stress magnitudes the Cambria sand at all initial relative densities exhibit characteristics similar to those observed by Rutledge (1947) for normally consolidated clays. The isotropic and the one-dimensional compression curves (from Yamamuro et al. 1996) in all cases become parallel to the strength curves at high pressures. The two different compression curves do not quite run parallel in the beginning, apparently because the break points on the curves, corresponding to the preconsolidation pressure for clay, do not occur at the same pressure, but the two curves for each of the three initial relative densities do tend to merge at higher pressures.

The inferred capability of the unifying behavior is that high pressure strength predictions can be made from an isotropic or one-dimensional compression test and a single high pressure triaxial test. The resulting compressive strength is used to locate a strength line parallel to the compression line on the e log(stress) diagram. For drained tests the maximum deviator stress occurs at failure, and it is possible to predict the effective stress secant friction angle from this deviator stress and the confining pressure at the same void ratio. For the drained test the compressive strength occurs simultaneously with the maximum effective stress ratio.

However, this is not true for undrained tests at high pressures. In these tests, the maximum deviator stress occurs well inside and before the maximum effective stress ratio. Therefore, effective stress failure conditions cannot be predicted for undrained tests. The instability line for the sand goes through the point of maximum deviator stress (Bopp and Lade 1997, Yamamuro and Lade 1997), but since only consolidation stress conditions are represented by the compression line (and not the effective stresses at the maximum deviator stress), the location of the instability cannot be predicted from these results either. However, the compressive strength for a given consolidation pressure can be inferred for undrained conditions.



Fig. 4. Void Ratio vs. Log(Stress) for Istropic and One-Dimensional Compression Tests and Maximum Deviator Stress for Drained and Undrained Triaxial Compression and Extension Tests on Loose, Medium Dense, and Dense Cambria Sand.

### 7 CONCLUSIONS

The present investigation was performed to conduct a thorough examination of the effect of initial relative density on the drained and undrained stress-strain, volume change or pore pressure, and strength behavior of granular materials over a large range of confining pressures.

Specimens exposed to isotropic pressures above 15 MPa had essentially the same void ratio after consolidation, regardless of initial relative density. Above 15 MPa the initially loose, medium dense, and dense specimens tested in compression exhibited similar stress-strain and volume change characteristics when sheared under drained conditions.

The effect of initial relative density on the drained Mohr-Coulomb secant friction angle is most pronounced in the low pressure regime. The friction angles in compression and extension decreased with increasing confining pressure to reach similar minimum values at mean normal stresses at failure in the range from 3.5 to 11 MPa. Beyond these pressures the friction angles in compression increased slightly, but they maintained very similar values, near 33<sup>0</sup>, for all initial relative densities. The friction angles in the extension tests reduced with increasing confining pressure, and they approached, but did not reach the same value within the high pressures employed in this study. The extension friction angles were higher than those for compression at low confining pressures, but they crossed over in the medium pressure range and became lower, in the range from  $29^{\circ}-32^{\circ}$ , than the compression friction angles at high pressures. Effective stress friction angles for undrained compression and extension tests varied systematically in the range from 32° to 35<sup>°</sup> with slightly higher values for the compression tests.

Interpretation of all results from the high pressure testing study in terms of total stresses shows that the sand behaves as a normally consolidated clay. Thus, Rutledge's Hypothesis applies to the sand, i.e. when plotted on a void ratio - log(stress) diagram, the compressive strengths from compression and extension triaxial tests, drained and undrained, all plot on the same curve, and this curve is parallel to the consolidation curve over a large range of stresses in this diagram.

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