

# Dynamic shear moduli for clayey soils with various grain size distribution and plasticity

## Modules dynamiques de cisaillement pour les sols argileux avec granulométries et plasticités diverses

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### ABSTRACT

More than 50 undisturbed and remoulded clayey soil specimens were tested using hollow cylindrical torsional testing apparatus. The marine clay was sampled from 5 different harbours and coastal locations in western Japan. Sand and clay were mixed in various proportions for testing. The plasticity index of the sampled soils varied from 16 to 111 and the particle distribution curves were also variable. Based on experimental results, a unified formula representing the dynamic shear moduli and damping ratios for the clayey soils with various plasticity and sand contents is presented.

### RÉSUMÉ

Plus de 50 spécimens de sol argileux non perturbés et remariés ont été examinés à l'aide d'un appareil d'essai de torsion cylindrique creux. L'argile marine a été prélevée de 5 ports et d'endroits côtiers différents au Japon occidental. Le sable et l'argile ont été mélangés avec des proportions différentes pour l'essai. L'indice de plasticité des sols prélevés a varié de 16 à 111 et les granulométries ont aussi été changés. Basé sur les résultats expérimentaux, une formule unifiée pour la variation des modules dynamiques de cisaillement et des constantes d'amortissement avec la plasticité et le contenu de sable est présentée.

### 1 INTRODUCTION

Most soil strata do not consist of pure sand or clay but instead contain a mixture with various fines contents and plasticities.

Much work has been done to establish the formula for dynamic shear moduli and damping ratios for the equivalent linear seismic response analysis. The formulas for clayey soils were developed by Hardin and Black (1968), Kokusho et al. (1982) and Zen et al. (1987), etc. They were, however, based on the data for clay including small amounts of sand. It is therefore difficult to apply the formula to sandy clay containing fairly large amounts of sand, so-called intermediate soils.

In this research, an experimental formula estimating the dynamic shear modulus and damping ratio for clayey soils with various plasticities and sand contents has been proposed employing the hollow cylindrical torsional testing apparatus for undisturbed and disturbed clay sampled from 5 different harbour and coastal locations in western Japan. Sand and clay were artificially mixed at various contents to measure the shear modulus and the damping ratio for wide strain range.

### 2 MATERIAL PROPERTIES AND EXPERIMENTAL METHODS

Four samples of undisturbed marine clay were taken from Dejima in Hiroshima harbour. The undisturbed samples were taken with a thin walled tube 1m long, 75mm in diameter. Four kinds of marine clay were collected for the remoulded sample. Samples of Ariake A and B clay were obtained from the mouth of the Shirakawa River in Kumamoto and Higashiyoka in Saga prefecture on the coast of the Ariake Sea. Ariake C clay was taken from the mouth of the Rokkaku River in Saga and Onoda clay from Onoda harbor in Yamaguchi. Two artificial sand and clay mixtures of differing proportions, Onoda Mixture and Ariake C Mixture, were prepared. In order to prepare the remoulded sample, a clay slurry was mixed with an initial water content of twice the liquid limit, and was then poured into a consolidation vessel and preconsolidated with a vertical pressure of 50kPa.

The physical properties are summarized in Table1. The particle size distribution curves are illustrated in Fig.1 (a)(b)(c)(d). The numbers 100, 80, 60, 40 and 30 in the sample nomenclature represent the dry weight ratio of the Onoda clay or Ariake C

Table 1: Test conditions and specimen description

| Sample name             |            | $FC(\%)$ | $PC_{2mm}(\%)$ | $PC_{5mm}(\%)$ | $w_L(\%)$ | $I_P$ | $w_L^*(\%)$ | $I_P^*$ | $w_i(\%)$ | $e_i$ | $\sigma'_{mc}(\text{kPa})$ |
|-------------------------|------------|----------|----------------|----------------|-----------|-------|-------------|---------|-----------|-------|----------------------------|
| Onoda Mixture           | OC100      | 84.5     | 29.0           | 37.5           | 84.04     | 49.5  | 84.04       | 49.5    | 59.7      | 1.564 | 66.7,100,133.3             |
|                         | OC80       | 67.0     | 23.2           | 30.0           | 69.18     | 44.8  | 60.49       | 37.8    | 49.4      | 1.278 | 66.7,100,133.3             |
|                         | OC60       | 50.0     | 17.4           | 21.5           | 63.15     | 40.6  | 46.13       | 27.9    | 43.7      | 1.182 | 66.7,100,133.3             |
|                         | OC40       | 32.5     | 11.6           | 14.0           | 55.41     | 34.0  | 25.99       | 13.5    | 28.3      | 0.807 | 66.7,100,133.3             |
|                         | OC30       | 25.3     | 8.7            | 11.0           | 47.03     | 26.4  | 17.37       | 9.1     | 24.2      | 0.726 | 66.7,100,133.3             |
| Ariake C Mixture        | ACC100     | 100.0    | 53.5           | 61.5           | 155.34    | 111.0 | 155.34      | 111.0   | 114.1     | 2.867 | 66.7,100,133.3             |
|                         | ACC80      | 79.5     | 42.8           | 49.0           | 138.37    | 98.3  | 123.70      | 84.7    | 91.9      | 2.405 | 66.7                       |
|                         | ACC60      | 59.7     | 32.1           | 37.0           | 124.25    | 88.3  | 88.96       | 63.5    | 69.9      | 1.944 | 66.7,100,133.3             |
|                         | ACC40      | 39.9     | 21.4           | 24.0           | 108.48    | 77.0  | 53.68       | 34.3    | 49.6      | 1.365 | 66.7                       |
|                         | ACC30      | 29.8     | 16.1           | 18.5           | 95.45     | 67.3  | 39.68       | 22.9    | 39.5      | 1.113 | 66.7,100,133.3             |
| Undisturbed Dejima clay | C-5 T-4    | 99.1     | 25.0           | 67.0           | 116.75    | 82.2  | 116.75      | 82.2    | 121.4     | 3.040 | 66.7                       |
|                         | C-5 T-12   | 96.6     | 35.8           | 51.0           | 134.10    | 97.4  | 134.10      | 97.4    | 119.1     | 3.360 | 66.7                       |
|                         | C-8 T-2    | 99.9     | 54.8           | 75.0           | 113.80    | 84.5  | 113.80      | 84.5    | 124.7     | 3.484 | 66.7                       |
|                         | C-8 T-11   | 43.0     | 8.0            | 13.0           | 38.08     | 16.0  | 28.53       | 6.5     | 37.6      | 1.104 | 66.7                       |
| Remoulded clay          | Ariake A   | 98.3     | 42.0           | 75.2           | 89.09     | 47.7  | 89.09       | 47.7    | 82.3      | 2.128 | 66.7,100                   |
|                         | Ariake B   | 97.9     | 21.3           | 52.0           | 83.47     | 43.6  | 83.47       | 43.6    | 59.2      | 2.213 | 66.7                       |
|                         | Itsukaichi | 98.6     | 50.2           | 77.0           | 109.59    | 75.1  | 109.59      | 75.1    | 92.4      | 2.523 | 66.7,100                   |

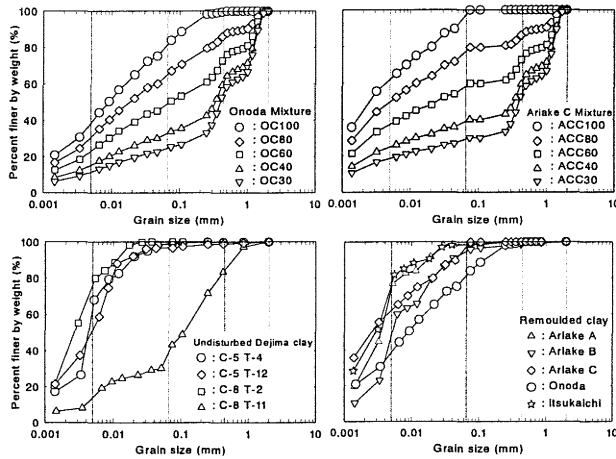


Figure 1. Particle size curves of tested samples

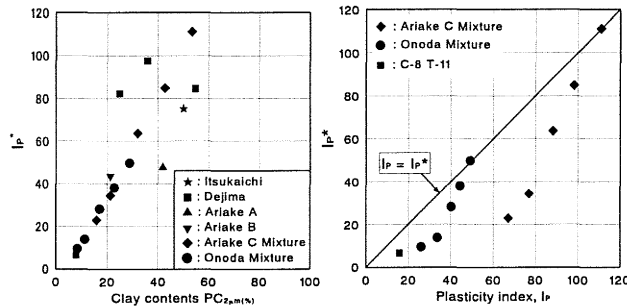


Figure 2. Relationship between  $I_p^*$  and clay content

clay to the total weight in the soil mixture. In other words, OC100 and ACC100 represent Onoda clay and Ariake C clay respectively. The fines content  $FC$ , clay content  $PC$ , plasticity index  $I_p$ , initial water content  $w_i$  and initial void ratio  $e_i$  of the soil mixtures decreased with increasing sand content.  $PC_{2\mu m}$  and  $PC_{5\mu m}$  represent the clay content defining clayey particle diameter as  $2\mu m$  and  $5\mu m$  or less respectively.

Four samples, C-8 T-11, OC40, OC30 and ACC30 were classified as Sand by the Method of Classification for Geomaterials for Engineering Purposes (JGS 0051-2000). However, these samples show a very high plasticity index range from 16 to 77. Intermediate soils are classified as having a fines content range from 20% to 50% and plasticity index range from 0 to 30. Therefore, in addition to those 4 samples OC60 was also classified as an intermediate soil.

The symbols  $w_L^*$  and  $I_p^*$  were new parameters defined in this research as the liquid limit and the plasticity index respectively of the sand and clay mixtures. According to the method of JIS A 1205, the sample passing a  $0.425mm$  sieve is normally used to measure liquid limit and plastic limit. There was a concern that the material properties would be insufficiently reflected in the plasticity index of soils with a high sand content. In order to represent the plasticity of soil mixtures, liquid and plastic limits were determined for samples passing a  $2.0mm$  sieve using the method of JIS A 1205, and the resulting plasticity index was defined as  $I_p^*$ .  $I_p^*$  was equated with the plasticity index obtained from the method of JIS A 1205 for the samples with fines content higher than 85%. The range of the data spread of  $I_p^*$  was less than 5% for all samples and was slightly larger than that of  $I_p$ . Fig.2 shows the relationship between  $I_p^*$  and the clay content defined with a clay particle diameter of less than  $2\mu m$  for all sample. It can be seen that  $I_p^*$  increases linearly with increasing clay content. Thus, it is considered that  $I_p^*$  is useful in representing the consistency limit for sand and clay soil mixtures. Fig.3 provides a plot of the plasticity index against  $I_p^*$  for 11 samples. The difference in the value between

plasticity index and  $I_p^*$  increases with decreasing plasticity index, and increasing sand content.

The hollow cylindrical specimens with initial dimensions of 75mm outer diameter, 35mm inner diameter and 100mm height were trimmed from the blocks of clay and set on the hollow cylindrical torsional triaxial shear apparatus. In order to obtain a high degree of saturation, a backpressure of 100kPa was applied for 2 hours.  $B$ -values of more than 0.95 was observed in all specimens used in the tests. These specimens were consolidated anisotropically at a constant lateral coefficient of earth pressure  $K=0.5$ . As shown in Table.1, three different confining stresses and effective vertical stresses  $\sigma_{vc}'$  of 100kPa, 150kPa and 200 kPa (i.e. effective mean principal stress  $\sigma_{mc}'$  of 66.7kPa, 100kPa, and 133.3 kPa) were applied to the specimens. For the purpose of normalization of the effect of consolidation time on tests, the 2t method was applied to determine the end of the consolidation time. Generally the time for the end of primary consolidation is determined by the 3t method (JSSMFE, 1979). However, since it was impossible to place a filtration sheet along the circumference of the hollow cylindrical specimen and the time for consolidation required was very long, the 2t method was introduced. Specimens were tested under a sinusoidal torsional cyclic load using a hydraulic actuator at a frequency of 0.1Hz in undrained conditions according to JGS 0543-2000.

### 3 THE EFFECT OF PLASTICITY INDEX AND CONFINING STRESS ON INITIAL SHEAR MODULUS

The equivalent shear modulus  $G_{eq}$  corresponding to the single amplitude shear strain  $(\gamma)_{SA}=0.0001\%$  was defined as the initial shear modulus  $G_0$ . Fig.4 shows change of the initial shear modulus due to the difference of the plasticity index and  $I_p^*$  obtained from the test carried out at  $\sigma_{mc}'=66.7kPa$ . In this figure, the  $G_0$  vs.  $I_p$  and  $G_0$  vs.  $I_p^*$  relationships are represented with unfilled and filled circles respectively. The relationship between the initial shear modulus and the plasticity index shows different lines for each mixture. On the other hand, it can be seen that the initial shear modulus and  $I_p^*$  have a unique relationship.

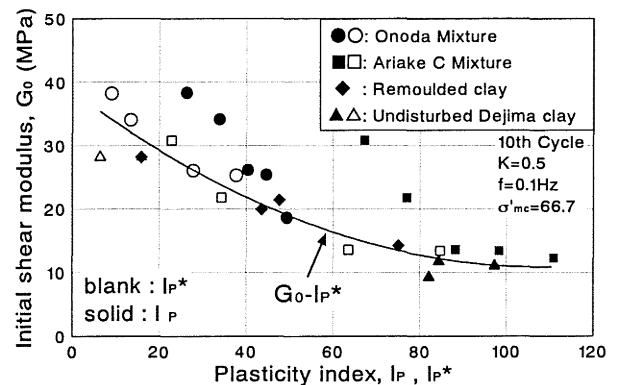


Figure 4. Initial shear modulus vs. plasticity index and  $I_p^*$

The initial shear modulus of normally consolidated clay is often represented by the following formula with the confining stress  $\sigma_c'$  and the void ratio  $e$ .

$$G_0 = A \cdot F(e) \cdot (\sigma_c')^n \quad (1)$$

where  $A$  and  $n$  are experimental constants and  $F(e)$  is a formula of the void ratio. There have been various experimental formulae for the initial shear modulus of normally consolidated clay proposed by many researchers. Zen *et al.* (1987) gave the following equation based on the idea that the initial shear modulus should not be represented by a function containing both the confining stress and void ratio which already have a unique relationship. Rather, it should be represented by a function of the

confining stress only in order to estimate an initial shear modulus for normally consolidated clay.

$$G_0 = A \cdot (\sigma_c')^n \quad (2)$$

In equation (2), in the case of the value of plasticity index being higher than 30, the experimental constants  $A$  and  $n$  were given as  $A=285-2I_p$  and  $n=1$ , respectively. Based on this idea, the form of equation (2) was selected for the formula to indicate the initial modulus. In order to determine the experimental constant  $n$ , the initial shear modulus obtained from tests carried out at 3 different effective mean principal stress  $\sigma_{mc}'$  of 66.7kPa, 100kPa and 133.3kPa are shown versus  $\sigma_{mc}'$  on the logarithmic graph in Fig.5. This figure shows that the  $\log G_0$  vs.  $\log \sigma_{mc}'$  relationship is nearly straight in all samples, and the slopes of these lines are 1 on average. Thus, the value of  $n$  is determined to be 1.

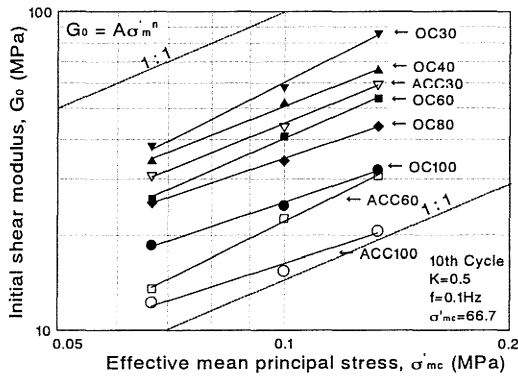


Figure 5.  $\log G_0$  vs.  $\log \sigma_{mc}'$  relationship for determination of experimental constant  $n$

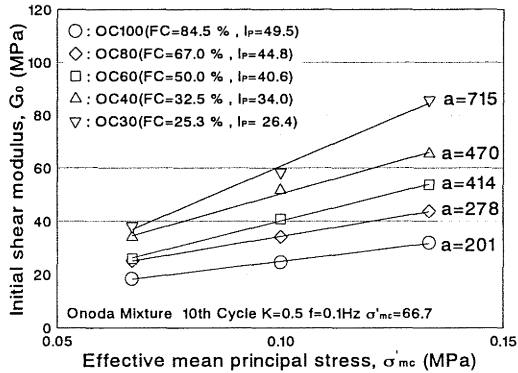


Figure 6. Relationship between initial shear modulus and effective mean principal stress for Onoda Mixture

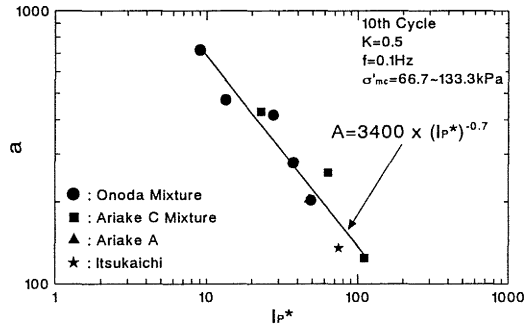


Figure 7.  $\log a$  vs.  $\log I_p^*$  relationship for determination of experimental constant  $A$

Fig.6 shows the relationships between the initial shear modulus and effective mean principal stress to be straight, and the value of  $a$  is shown on the right side of each line indicating the slope of each line. It can be seen that the degree of the confining stress dependency on the initial shear modulus increases

with increasing sand content, and with decreasing plasticity index. In Fig.7, the values of  $a$  from Fig.6 are plotted against  $I_p^*$  on the double logarithmic graph. It can be seen that the  $\log a$  vs.  $\log I_p^*$  relationship can be approximated by a straight line. The experimental constant  $A$  of equation (2) is determined by the following formula by the approximate straight line shown in Fig.7 for the observed range of  $I_p^*$  from 6.5 to 111.

$$A = 3400 \cdot (I_p^*)^{-0.7} \quad (I_p^* = 6.5 \sim 111) \quad (3)$$

Substitution of equation (3), and a value of  $n = 1$  obtained above, into equation (2) leads to equation (4) which can be used to calculate the initial shear modulus of the normally consolidated clayey soils with various grain size distribution and plasticity independently of the classification of the clay or the sand.

$$G_0 = 3400 \cdot (I_p^*)^{-0.7} \cdot \sigma_{mc}' \quad (I_p^* = 6.5 \sim 111) \quad (4)$$

#### 4 THE EFFECT OF PLASTICITY INDEX AND CONFINING STRESS ON SHEAR MODULI AND DAMPING RATIO

The variation of the normalized shear modulus  $G_{eq}/G_0$  and the damping ratio obtained from the tests carried out at the effective mean principal stress  $\sigma_{mc}'=66.7$ kPa along with  $\log(\gamma)_{SA}$  is shown for samples of Onoda Mixture in Fig.8 for the single amplitude strain range from 0.0001% to 1.0%. The tendency of the strain-dependency on both the shear modulus and damping ratio becomes more noticeable with increasing sand content, and with decreasing plasticity index. It may be considered that the continuous change of the strain-dependent curve of both the shear modulus and the damping ratio is attributable to the proportion of the sand contained in the soils.

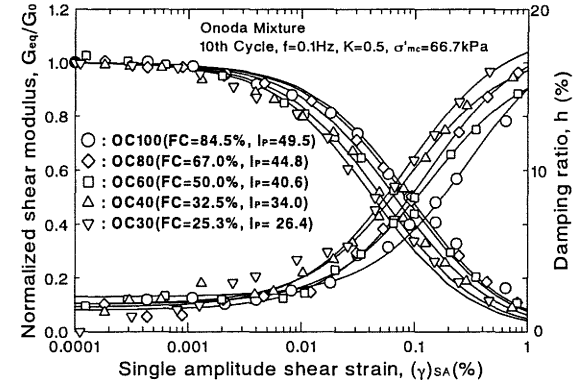


Figure 8. Strain-dependent changes of shear modulus and damping ratio for Onoda Mixture

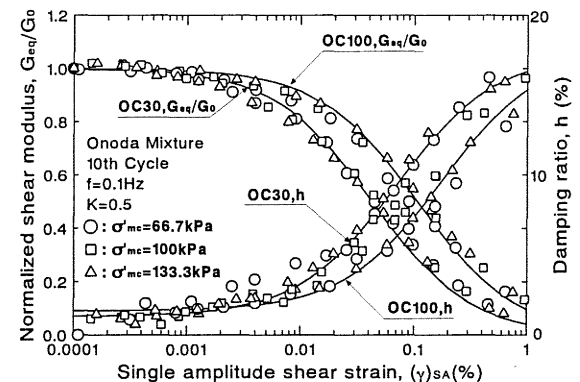


Figure 9. The effect of confining stress on  $G_{eq}/G_0$  vs.  $\log(\gamma)_{SA}$  relationship and  $h$  vs.  $\log(\gamma)_{SA}$  relationship

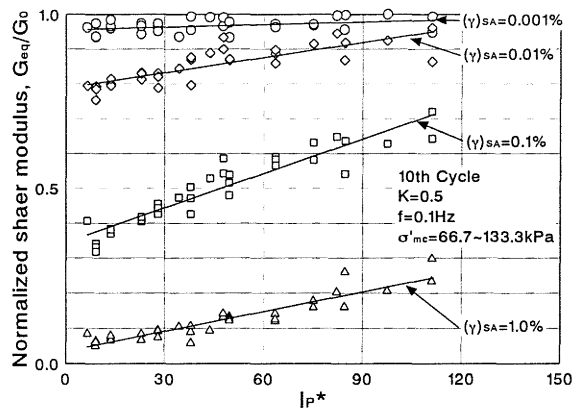


Figure 10. Relationship between  $G_{eq}/G_0$  and  $I_P^*$  for different single amplitude of shear strain

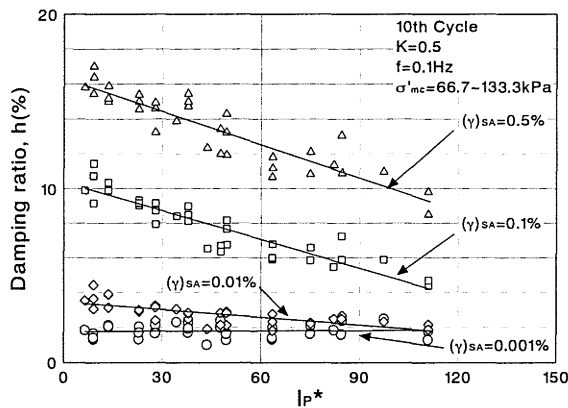


Figure 11. Relationship between damping ratio and  $I_P^*$  for different single amplitude of shear strain

Table 2: Value of  $a_G$ ,  $b_G$ ,  $a_h$  and  $b_h$  for a typical strain levels

| $(\gamma)_{SA} (\%)$ | $G_{eq}/G_0$ |       | $h (\%)$ |        |
|----------------------|--------------|-------|----------|--------|
|                      | $a_G$        | $b_G$ | $a_h$    | $b_h$  |
| 0.0001               | 0            | 1     | 0.00350  | 0.994  |
| 0.001                | 0.00024      | 0.957 | -0.00049 | 1.781  |
| 0.005                | 0.00098      | 0.853 | -0.00532 | 2.600  |
| 0.01                 | 0.00144      | 0.790 | -0.01493 | 3.482  |
| 0.025                | 0.00244      | 0.633 | -0.03329 | 5.652  |
| 0.05                 | 0.00298      | 0.497 | -0.04202 | 7.668  |
| 0.1                  | 0.00329      | 0.346 | -0.05557 | 10.404 |
| 0.25                 | 0.00295      | 0.173 | -0.06456 | 14.238 |
| 0.5                  | 0.00223      | 0.094 | -0.06370 | 16.357 |
| 1.0                  | 0.00187      | 0.036 | -0.05587 | 17.999 |

Fig.9 shows both  $G_{eq}/G_0$  vs.  $\log(\gamma)_{SA}$  relationship and  $h$  vs.  $\log(\gamma)_{SA}$  relationship obtained from the tests for 2 samples, OC100 and OC30, of Onoda Mixture with 3 different effective mean principal stresses  $\sigma'_{mc}$  of 66.7kPa, 100kPa and 133.3kPa. The results of the tests shown in this figure indicate an insignificant effect of the confining stress on the strain-dependent change of both shear modulus and damping ratio. This result agrees with that obtained by Kokusho *et al.* (1982) etc., although the result reported by Zen *et al.* (1987) indicates that the strain-dependency on the shear modulus of clayey soils with low plasticity index less than 30 such as OC30 should be affected by the confining stress.

The relationship between the normalized shear modulus and  $I_P^*$  obtained from all tests carried out in this research is shown in Fig.10, where the normalized shear modulus corresponding to four steps of the single amplitude of shear strain  $(\gamma)_{SA}$  of 0.001%, 0.01%, 0.1% and 1.0% read from  $G_{eq}/G_0$  vs.  $\log(\gamma)_{SA}$  graph. It can be seen that the normalized shear modulus increases with an increase in  $I_P^*$  and can be approximated by a

straight line for each single amplitude of shear strain. Based on this result, the normalized shear modulus for every single amplitude of shear strain is obtained from the following formula with the slope  $a_G$  and the intercept of the vertical axis  $b_G$  of the approximately straight lines shown in Fig.10.

$$G_{eq}/G_0 = a_G \cdot I_P^* + b_G \quad (I_P^* = 6.5 \sim 111) \quad (5)$$

Fig.11 shows the value of damping ratio against  $I_P^*$  for all specimens for 4 single amplitude shear strains  $(\gamma)_{SA}$  of 0.001%, 0.01%, 0.1% and 0.5% read from the  $h$  vs.  $\log(\gamma)_{SA}$  graph. It can be observed that the damping ratio decreases with increasing  $I_P^*$  and it can also be approximated by a straight line for each single amplitude shear strain. At an arbitrary strain level, the damping ratio is obtained by the following formula with the slope  $a_h$  and the intercept of the vertical axis  $b_h$  of the approximately straight lines shown in Fig.11.

$$h = a_h \cdot I_P^* + b_h \quad (I_P^* = 6.5 \sim 111) \quad (6)$$

Table.2 shows the list of values of  $a_G$ ,  $b_G$ ,  $a_h$  and  $b_h$  for a typical strain levels. It can be considered that the normalized shear modulus and damping ratio of clayey soils with various grain size distribution and plasticity can be estimated by using equation (5), (6) and the values shown in Table.2.

## 5 CONCLUSIONS

New formulae predicting the initial shear modulus, normalized shear modulus and damping ratio of clayey soils with various grain size distributions and plasticity indices have been derived from a series of hollow cylindrical torsional cyclic simple shear tests. The following conclusions were reached based on the experimental investigation.

1. The initial shear modulus and the degree of strain-dependency of both the shear modulus and the damping ratio were significantly affected by the sand content. It was found that the initial shear modulus, the normalized shear modulus and the damping ratio showed great correlation to a new plasticity index  $I_P^*$  defined in this paper.

2. New formulae have been proposed which can be applied to clayey soils with various grain size distributions and plasticity indices. The initial shear modulus can be predicted using equation (2). Furthermore, both the normalized shear modulus and damping ratio at the arbitrary strain levels can be predicted using the values shown in Table.2 or equation (4) and (5), respectively.

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