Proceedings of the 16th International Conference on Soil Mechanics and Geotechnical Engineering

doi:10.3233/978-1-61499-656-9-3675

Mathematical and numerical modelling of geomaterials with special reference to catastrophic landslides and related phenomena

M. Pastor, M. Quecedo, E. Gonzalez, Ma.I. Herreros,
J.A. Fernandez Merodo, P. Mira & B. Haddad
Centro de Estudios y Experimentación de Obras Públicas, CEDEX, Madrid, Spain

1 INTRODUCTION

Mathematical and numerical models are a fundamental tool for predicting the behaviour of geostructures and their interaction with the environment. The term "mathematical model" refers to a mathematical description of the more relevant physical phenomena which take place in the problem being analyzed.
It is indeed a wide area including models ranging from the very simple ones for which analytical solutions can be obtained to those more complicated requiring the use of numerical approximations. In the case of Geomaterials, mathematical models have to account for one difficulty, which is the lack of homogeneity and the existence of discontinuities. The soils are made of solid particles, with voids filled with fluids which can move within the soil skeleton. This granular structure appears at higher scales than in other branches of science, and homogeneization does not always lead to accurate models. This is the case of inverse seggregation, where the continuum approach presents certain limitations. On the other hand, discrete models are still limited, and most practical applications concern one solid phase. Coupling of discrete particles with fluids surrounding them is one of the challenges which is presently being addressed. Therefore, a first division between discrete and continuum models can be made. In this work we will focus on the continuum approach, as in many cases pore pressures are needed to fully understand and describe the phenomena. We will assume that homoneneization is possible, and all the constituents coexist at any point of the geomaterial.

2 MATHEMATICAL MODELLING

The mathematical model will consist of a set of partial differential equations describing, for each phase and the mixture, the balance of mass, momentum and energy, complemented by suitable constitutive equations. Of course, constitutive equations must fulfill the requirements of the second law of thermodynamics, and can be derived from dissipation potentials which are related to a positive increment of entropy. Here it is worth noticing that there are two alternative frameworks, the eulerian and lagrangian, within which models are casted. Most of the work developped in geomaterials has been done within the latter. Most of both commercial and research codes are lagrangian, as in most cases the analyst is interested on deformation prior to failure, mechanism of failure and ultimate load. The goal is to avoid failure of the geostructure, modifying the design or reinforcing it in such a manner that failure will not take place under the design load conditions. But there are some other cases where failure can happen, and the analyst has to forecast its consequences. This is the case of fast catastrophic landslides and related phenomena, such as lahars. Here, there is no way to prevent the existence of pyroclastic surges and flows, and the only possible action is to model the consequences in order to build protection structures or propose suitable emergency plans. As an example, we could mention the case of debris flows in mountain areas, where several actions are taken, such as (i) prevent deforestation of source areas, (ii) build check dams to slow the flow and stop the larger boulders, and (iii) chanel or diverse the flow in populated areas. In all these cases, there is a phase transformation of the solid geomaterial which behaves in a fluid-like manner after failure has occured. Eulerian formulations have been favoured for modellers since the early stages of computational fluid dynamics. The situation is changing, and numerical techniques such as meshless methods, smooth particle hydrodynamics, etc, based on lagrangian formulations are gaining terrain nowadays. One of the main difficulties encountered in eulerian formulations is the existence of free surfaces and interfaces which separe different fluids. For instance, in the case of a landslide entering a reservoir, we have to keep track of the interface between reservoir water and air, water and the soil, and soil and air. According to the nature of their partial differential equations, mathematical models can be said to be (i) Elliptic, parabolic or hyperbolic, (ii) Linear or non-linear, and (iii) Uncoupled or coupled. In our case, in most cases we will find coupled, non-linear problems. Moreover, in some cases the nature of the problem can change from hyperbolic to elliptic or viceversa, causing illposedness and loss of unicity of the solution. This is the case of plasticity models when localized failure takes place. In order to avoid it, the description of the continum should be improved. Parabolic problems are encountered in transient seepage or heat transfer. Here internal fronts can develop, as in the case of a pyroclastic flow over ice and snow, which will melt and vaporize at earlier stages. Treatment of this internal fronts can be done in several alternative ways. Eulerian formulations are characterized by convective terms which, when being dominant, result on a hyperbolic problem. Indeed, due to their nonlinear nature (convective acceleration include quadratic products of velocity components), can result on the inception of discontinuities such as shock waves. Dynamic problems also described by hyperbolic partial differential equations. It is important to notice that there exist two alternative formulations including second or first order derivatives. While the former approach has been favoured by most modellers, it is worth mentioning that the latter presents important advantages from a numerical point of view. In comparison with the classical second order approach, few work has been done yet on the latter.
The purpose of this work is to present an overview of the different alternative mathematical and numerical models which can be applied to fast catastrophic landslides and other related problems such as waves caused by landslides. In the following sections, we will describe them in detail. Concerning mathematical modelling, we have chosen to start describing a general eulerian

model describing the coupling of soil skeleton and pore fluids, showing how some frequently used models are particular cases of the general formulation. From this general model, we will develop specialized models with different degrees of complexity which can be applied to the following situations:

- Inititation mechanisms of landslides triggered by rain, earthquakes, etc. Here we will not consider convective terms, and the model coincides with the much celebrated classical displacement-pressure formulations proposed by Zienkiewicz and coworkers at Swansea University. We will describe alternative formulations in terms of displacements and velocities.
- Propagation of fast landslides, where we will consider the following cases, according to the relative time scales of propagation and consolidation:
  - Drained avalanches of granular materials
  - Phenomena where coupled behaviour is relevant. A simplifying assumption will allow us to consider the movement as the superposition of propagation and consolidation.
  - Undrained landslides where the material can be described by specialized rheological models such as Bingham fluids.
- In many cases, the depth of the flowing soil is small in comparison with the length and width, and an integration along an axis normal to the terrain can be done. These depth integrated models present important saving of computer time and provide accurate and valuable information concerning propagation times, area affected by the landslide, etc. While the classical depth integrated models which are found in other domains such as coastal engineering do not present relevant slopes nor curvatures in the bottom this is not the case of landslides. The models have to be cast in a natural coordinate system. As the most important effect is that of curvature, which affects friction forces, the traditional approach can be followed provided the additional effect of centripete accelerations is included.
- Waves caused by landslides in reservoirs.
- Lahar initiation and propagation

3 NUMERICAL MODELLING

Of all models presented in the preceeding section, only few of them can be solved analytically. In most situations, it is necessary to use techniques to discretize the problem. Among these techniques, we can mention the following:

(i) The Finite Difference Method (FDM)
(ii) The Finite Element Method (FEM)
(iii) The Boundary Integral Method (BIEM)
(iv) The Finite Volume Method (FVM)
(v) Meshless methods (MM)

which are applied to solve continuum models. All of them present advantages and inconvenients. The Finite Element Method is perhaps the technique which is used in a vaste majority of cases, either for Lagrangian or for Eulerian formulations. From a historical point of view, it is worth mentioning that Finite Differences were the first discretization technique used to solve an engineering problem (Assuan dam). The situation changed with the development of Finite Elements, and it appeared a division between Computational Fluids Mechanics problems, which were solved via Finite Differences and Solid Mechanics problems, where Finite Elements were more popular. The situation changed in the middle eighties, and the realm of Finite Elements expanded over the Fluid Mechanics domain, mainly because of the use of unstructured grids and adaptive remeshing which allowed capture of shocks and discontinuities in a natural yet elegant manner. Another advance which allowed this expansion was the effective treatment of the convective terms bu the Finite Element community. Concerning Fluid Dynamics, it can be said that the main techniques used today are FD, FVM and FEM. In the case of geomechanics, these methods have been used to study the movement of fluidized soil masses. As far as solid problems are concerned, the BIEM has been used in the case of infinite domains, at it provides a simple way to treat absorbing boundary conditions avoiding reflections on the boundaries of the computational domain. One efficient way consists of decomposing the domain into an interior and an exterior subdomains, the latter being modelled via BIEM. It is worth mentioning that in some cases, explicit lagrangian formulations have been implemented into FD codes for soils, but, still, a wide majority of codes use FEM. During the past years, meshless methods have become more used, and it can be said that in the near future they will provide efficient alternatives to both solid and fluid mechanics formulations.

3.1 *Application to Landslides (i) Initiation*

In the case of landslides, the Finite Element Method is the technique which has been more used for all triggering mechanisms (rain, earthquakes, external loading, soil degradation,...) It is important to notice here that there are two main mechanisms of failure: localized and diffuse. The first is characteristic of dense granular soils and overconsolidated clays, while the latter is more typical of loose soils where liquefaction takes place. Therefore, numerical simulations have to implement (i) suitable elements for localized failure, and (ii) realistic constitutive models able to describe liquefaction failure. Concerning localized failure, it must be said that has attracted the attention of many researchers in the last decade, who have investigated the mathematical and numerical difficulties. The problem becomes ill posed, exhibiting a dependence on mesh size. The type of element used in the analysis is also of paramount importance, and it is found that limit loads are overpredicted and spurious mechanisms of failure are found in the analysis. Liquefaction has been throughly studied, and many constitutive models able to reproduce it can be found today in the literature. However, it must be pointed out that once the soil has liquefied, it behaves in a fluid like manner. The generated pore pressures can have a dissipation time larger than that of propagation, and the mobilized mass of soil moves with velocities which may reach 60 Km/h over large extents of terrain.

3.2 *Application to Landslides (ii) Propagation*

From a methodological point of view, propagation of catastrophic landslides and other related phenomena have to be studied using Computational Fluid Dynamic tools. Here, convective terms play a paramount role, and the standard Boubnov Galerkin technique cannot be used. Several alternatives have been proposed during the past, such as (a) Upwinding (b) Least squares (c) Godunov based methods and (d) Taylor Galerkin method. Another important problem is that of fluid incompressibility, which prevents the use of elements with the same interpolation functions for velocities and pressures and the use of explicit schemes. Several alternatives exist to circumvent this problem, such as the fractional step method, where only pressures are obtained in an implicit manner. Eulerian formulations present the difficulty of having to track interfaces between flowing materials along the mesh. The Level Set method provides a suitable way to track one or several interfaces. We will show here some examples of waves generated by landslides in reservoirs and bays. Finally, we will adress some simplifications such as the so-called "depthintegrated models", where we have included sissipation of pore water pressures. The rheology of the flowing soil mass influences both the internal friction and the friction with the terrain over which it flows. Some examples of catastrophic flows described in the literature will be presented, such as Aberfan and Las Colinas.