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Localization in geomechanics

Ioannis Vardoulakis
Faculty of Applied Mathematics and Physics, Department of Mechanics, National Technical University of Athens, Athens Greece

ABSTRACT
In this note we summarize briefly the origins of localization theory and we provide an overview of recent contributions in the subject of shear-banding as applied to Geomechanics.

1. INTRODUCTION

Localization theory is a natural extension of Mohr's strength of materials theory, which was published in the year 1900. The mathematical formulation of bifurcation and post-bifurcation phenomena and related instabilities constitutes the basis of a contemporary continuum theory of failure (cf. Vardoulakis & Sulem 1995).

Localization of deformation leads to a change of scale of the problem, so that phenomena occurring at the scale of the grain cannot be ignored anymore in the mechanical modeling process of the macroscopic behavior of the material. Thus in order to describe correctly localization phenomena it appears necessary to resort to the so-called continuum models with micro-structure. These observations have prompted the extension of the classical continuum mechanics descriptions of solids by resorting to the so-called Cosserat- or Gradient models. These generalized continua usually contain additional kinematical degrees of freedom and/or higher deformation gradients, which account somehow for the material's micro-structure. The description of statics and kinematics of continuous media with microstructure has been studied systematically by many authors in the past. This work was revived in the last two decades of the 20th century, in order to address properly the problem of localization.

Figure 1. Dual shear bands in perlite (Sarakina Melos Island, Greece[1])

2. STATE-OF-THE-ART

2.1 *Shear banding in Soil Mechanics*

Localization phenomena are central in Soil Mechanics and in years past they have been studied extensively by many researchers worldwide. Recently several papers were published summarizing the state-of-the-art on localization of the deformation ("shear banding") in soils (Saada et al. 1999, Nemat-Nasser & Okada 2001, Desrues & Chambon 2002, Wood 2002, Lade 2002 & 2003, Gudehus & Nuebel 2004, Desrues & Viggiani, 2004). Notably, the experimental work of Dr Jacques Desrues and his co-workers at the Laboratoire 3S in Grenoble has substantially contributed to our understanding of localization phenomena in Soil Mechanics.

One basic property of localization phenomena is some degree discontinuity of the deformation. Today we know that prior to *localization* the governing partial differential equations of the underlying quasi-static rate-boundary-value problem are elliptic and exclude discontinuous solutions. At the onset of localization these equations are changing type and from elliptic they turn hyperbolic. *Slip-lines* and *shear-bands* are thus identified with the characteristic lines of the governing hyperbolic partial differential equations.

From the theoretical point of view the so-called Thomas-Hill-Mandel shear-band model was introduced in the early '60s and it was widely publicized by the paper of Rudnicki & Rice (1975). This model constitutes even today the starting point for further developments either in the direction of the so-called strong discontinuities approach (Larson et al. 1996, Borja 2002) or of the weak discontinuities approach (Weir & Young 2003).

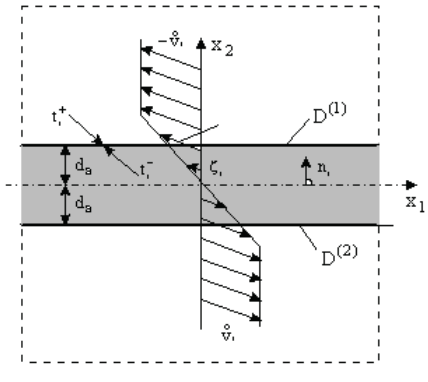

Figure 2. The Thomas-Hill-Mandel shear-band model

[1] Vardoulakis I. *Behavior of Granular Materials*. In: *Handbook of Materials Behavior Models* (Jean Lemaitre Ed.) Chapter 11.4, Academic Press, 2001.

The THM shear-band model is a material structure bounded by two stationary discontinuity surfaces of the velocity gradient (Fig.2). The THM model yields the Rice bifurcation condition, expressed as (Rice, 1973, 1976, Vardoulakis 1976)

$$\Gamma_{ik} g_k^{(\alpha)} = 0 \tag{1}$$

where $\Gamma_{ik} = C_{ijkl}^{u} n_j n_l$ coincides with the so-called **acoustic tensor** of the upper-bound linear comparison solid[2] for the direction n_i. Continuous shear-band bifurcations exist, if for some characteristic direction n_i the acoustic tensor of the upper-bound, linear comparison-solid is singular; i.e. if there is n_i such that

$$\det(\Gamma_{ik}) = 0 \tag{2}$$

The existence of real solutions of the bifurcation condition, Eq. (2), coincides, within a (dynamic) acceleration-waves analysis, with the condition for non-traveling body waves. This is interpreted as a manifestation of "deformation trapping", since at the considered state of shear-band bifurcation, the solid has lost also its ability to transmit body waves and energy is therefore "trapped" inside the material by the formation of these "narrow zones of intense shear". In the contrary, the condition $\det(\Gamma_{ik}) > 0$ guaranties the existence of real body waves and energy radiation.

For elasto-plastic solids, the threshold to the bifurcation stress, which satisfies the characteristic Eq. (2) is usually expressed in terms of tangent hardening modulus H_t. Within a simple setting a flow theory of plasticity for frictional, dilatant materials Eq. (2) yields to the so-called Mandel solution for the critical dimensionless hardening modulus (Mandel, 1996):

$$h = \frac{|\sigma_1 + \sigma_2|}{2G} H_t \quad , \quad h = h_B = \frac{(\mu_B - \beta_B)^2}{8(1-\nu)} \tag{3}$$

Thus continuous shear-band bifurcation in plane-strain deformations of non-associative frictional-dilatant materials, is predicted in the hardening regime of the mobilized friction coefficient.

Since H_t is a decreasing function of the plastic hardening parameter, we seek the orientation (given by the direction of the normal to the shear-band boundaries) for which the value of H_t is maximum. Thus the critical hardening modulus for shear-band bifurcation is computed as the solution of a constrained maximization problem (Rudnicki & Rice 1975). Computational results for the critical hardening modulus and the corresponding critical orientation angles of shear-bands for various constitutive models for non-associative, frictional elasto-plastic materials were computed by Molenkamp (1985). Ortiz et. al. (1987) provided numerical procedures for evaluating the above constrained maximization problem, whereas analytic solutions of it are given by Bardet (1991) and Bigoni & Hueckel (1990, 1991). Benallal and Comi (1996) presented a very elegant geometric analysis of the localization condition utilizing the spectral theory of 4th order tensors.

One of the major questions raised by the bifurcation analysis approach has been the validation or not of the classical solutions concerning the shear-band orientation with respect to the principal axes of stress (Tatsuoka et al. 1990). To this end three competing theoretical predictions were to be tested against the experiment[3]:

$$\theta_C = 45^\circ + \frac{1}{2}\phi_p \, , \quad \theta_R = 45^\circ + \frac{1}{2}\psi_p \tag{4}$$

$$\theta_{A-V} = 45^\circ + \frac{1}{2}\left(\frac{\phi_p + \psi_p}{2}\right) \tag{5}$$

The first solution is the classical Coulomb solution whereas the second one is the Roscoe solution. The third solution was first proposed by Arthur et al. (1977) on the basis of experimental observations and was subsequently proven theoretically within the frame of shear-band bifurcation analysis and was supported experimentally by Vardoulakis (1980); Fig.3.

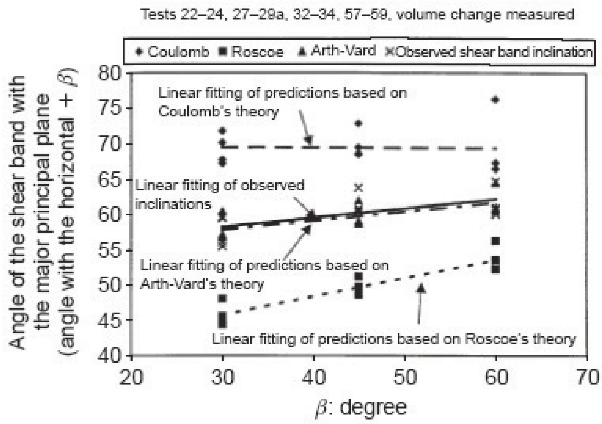

Figure 3. Comparison of theoretical and experimental results (Saada et al. 1999)

2.2 *Post-bifurcation deformation localization*

In contemporary Geomechanics we are not only interested in states of *incipient failure* but also in finding ways to trace the deformation in the so-called *post-failure* regime.

The THM approach to the shear-band problem involves the consideration of a classical constitutive model and the examination of the existence of discontinuity planes for the velocity gradient, which in turn are identified with the shear-band boundaries. Since the formulation of the problem does not contain a material property with the dimension of length, it is not possible to produce a statement about the shear-band thickness. Moreover at the state of shear-band bifurcation the underlying quasi-static problem is changing type, undergoing an elliptic-to-hyperbolic transition. In the post-bifurcation regime we deal in general with mathematically ill-posed problems (Schaeffer 1992, Shearer et al. 2003), which need some degree of regularization.

There is ample experimental evidence that shear-bands in granular materials engage a significant number of grains. Starting with Roscoe (1970) experimental observations suggest that the width of shear-bands is a small multiple of grain diameter (Scarpelli & Wood 1982, Vardoulakis & Graf 1985, Oda & Kazama 1998, Alshibli & Sture 1999). In order to be able to predict theoretically the dimensions of the shear-band, the grain size must be

[2] cf. Raniecki & Bruhns (1981).

[3] In these expressions the subscript p denotes "peak" values; i.e. limiting values of the friction and dilatancy angles.

introduced into the constitutive model. Thus, in order to trace the deformation in the post-bifurcation regime one has to account for the microstructure of the material by resorting to the so-called higher-order continuum theories like the Cosserat Continuum Theory. This idea was widely publicized by the paper Mühlhaus & Vardoulakis (1987) and has meanwhile matured in a variety of large scale numerical simulations, which account for higher continuum effects (Papanastasiou & Vardoulakis 1992, Oka et al. 2000, Zervos et al. 2001a & b, Chambon et al., 2001 & 2004, Matsushima et al. 2002, Pamin et al. 2003, Simone et al. 2004, Khoei et al. 2004).

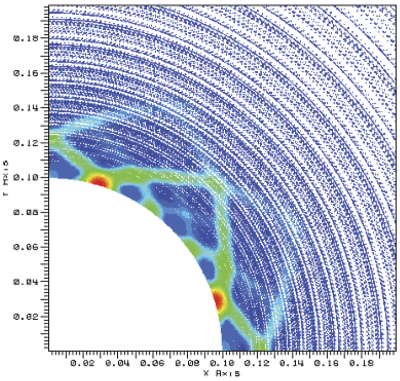

Figure 4. Robust post-failure shear-banding computations using a 2nd gradient plasticity F.E. model (Zervos et al. 2001 b)

2.3 *Constitutive factors affecting shear-banding*

Shear-banding is a rather complex phenomenon, where a number of constitutive factors influence the result. In that direction one should mention the recent studies on the effect of incremental non-linearity (Kolymbas & Herle 2003, Chambon & Roger 2003), and on the combined effect of incremental non-linearity and Cosserat micro-structure (Tejchman & Gudehus 2001). Other important factors which influence shear banding are anisotropy (Tatsuoka 1990, Iizuka et al. 1992) and non-coaxiality (Papamichos & Vardoulakis 1994, Hashiguchi & Tsutsumi 2003). Vardoulakis & Georgopoulos (2004) considered the range of validity of the Gutierrez-Ishihara (2000) modification of Taylor's "stress-dilatancy" hypothesis and concluded that the stress-dilatancy rule breaks down if in a process an abrupt rotation of principal axes is imposed, as this is the case at the onset of localization. During this phase the corresponding stress function decreases monotonously, whereas the dilatancy oscillates between large negative and large positive values.

2.4 *Micromechanical considerations*

Oda & Kazama (1998) remark that: "..that a shear band grows through buckling of columns together with rolling at contacts; it can be said that the thickness of a shear band is determined by the number of particles involved in a single column...". Indeed from the micro-mechanical point of view an important structure that appears to dominate localized deformation is the formation and collapse (buckling) of grain columns, as this was demonstrated experimentally by Oda and was explained theoretically by Satake (1998). These load-carrying columns belong to the so-called "competent grain fraction" (Dietrich 1976, Vardoulakis 1989, Staron et al.2001) and their current length reflects more or less the current shear band thickness.

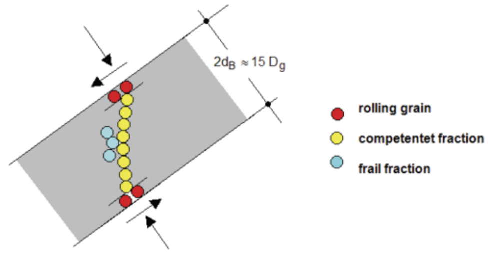

Figure 5. The Oda-Satake micromechanical model of the shear-band in granular material

As indicated in Figure 5 columns of grains transmit the intergranular forces constituting the "competent fraction" of the grains; these columns are supported by the lesser loaded grains, called the "frail grain fraction". The buckling of the granular columns results in the observed shear-band contraction (Vardoulakis & Georgopoulos 2004). We notice that softening of the shear-band could be partially due to the "roller-bearing" effect of rolling grains at shear-band boundaries.

The study of the material behavior at the shear-band scale is done by resorting micromechanical models and to Discrete Element simulations (Calvetti 2003). Characteristically one should refer here to the recent micromechanical studies that pertain to the questions of the relevance or not of couple stresses (Bardet & Vardoulakis 2001, Ehlers et al. 2003, Kruyt 2003) of the importance of incremental non-linearity (Kishino, 2002) and of grain crushing (Cheng et al. 2004).

2.5 *Cataclastic shear-banding and compaction bands*

At elevated confinement, suppressed dilatancy may lead to grain crushing or "cataclasis" inside the shear-band (El Bied et al. 2002; Fig. 6), which in turn leads to substantial permeability reduction (Papamichos et al. 1993), due to the shifting of the harmonic mean of the grain sizes towards the fines end of the sieve curve.

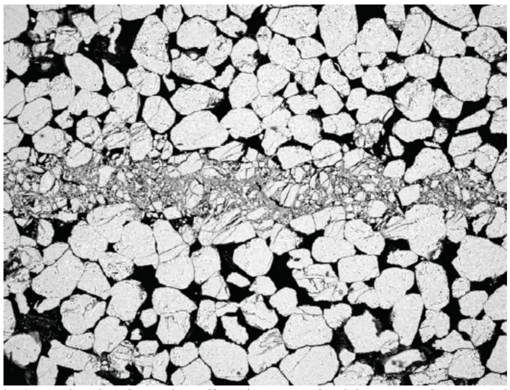

Figure 6. Cataclastic shear-banding in Fontainebleau sandstone (el Bied et al., 2002)

Vardoulakis & Sulem (1995, sect. 5.8) introduced first the link between grain crushing and compaction layering as an additional localization instability. Compaction bands in porous rocks have been studied extensively both from the experimental and the theoretical point of view (Haimson 2001, Rudnicki 2002, Holcomb & Olsson 2003).

2.6 *Undrained shear banding*

Undrained shear banding is a theoretical possibility discussed extensively in the literature (cf. Runesson et al. 199, Zhang & Schrefler 2002). Physically undrained shear-banding is not always possible, since a prerequisite for this phenomenon is the formation of pore-water pressure shocks across the shear band boundaries, which in ordinary (slow) processes cannot be sustained (Vardoulakis 1996). However in rapid loading conditions, as this is the case in load-controlled experiments, internal flow patterns are reported, consisting of a set of shear-bands (Han & Vardoulakis 1991; Fig.7).

Undrained dilatant hardening, as it is observed in kinematically controlled experiments, should not be considered as a stabilizing factor. It has been shown that undrained dilatant hardening becomes unstable at about maximum shear stress (Rice, 1975; Vardoulakis, 1986, 1996). This is known in the pertinent literature as a 'flutter'-type instability (Loret et al., 1997; Benallal & Comi, 2002, 2003). Stability of dilatant hardening and progressive strain localization under (globally) undrained conditions is addressed recently by Oka et al. (2002) and Lu et al. (2003, 2004).

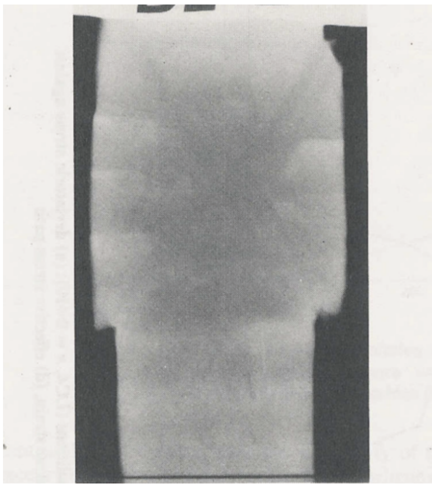

Figure 7. Localization patterning in a load-controlled, globally undrained biaxial test (Han & Vardoulakis, 1991)

2.7 *Dynamic shear-band deformation and thermal pressurization*

Back-analyses of catastrophic landslides like the Vaiont slide (Voight & Faust 1982) and the observations from the active fault drilling operations (Otsuki et al. 2003, Cornet 2004, Sulem et al. 2004 a & b) renewed the interest of the Geomechanics and Geophysics communities on the role of temperature and fluids in active faulting (Lachenbruch, 1980; Mase and Smith, 1985, Rice 2003). Rapid shear heating tends to increase pore pressure and to decrease the effective compressive stress and the shearing resistance of the fault material (Vardoulakis 2000). On the other hand dilatancy tends to decrease pore pressure (Garagash & Rudnicki 2003).

Fault zones are often characterized by large amounts of clay minerals, which form well-defined structures within the fault zone. These clay minerals inside the fault gouge are widely believed to affect significantly the mechanical behavior of faults, since as is the case for normally consolidated clays, they tend to contract when heated (Campanella, & Michell 1968, Hueckel & Pellegrini 1981). Based on these observations Vardoulakis (2002a), re-formulated the set of equations that govern the motion of a rapidly deforming shear-band, starting from first principles and discussed the dynamics of catastrophic landslides or creeping faults (Vardoulakis 2002 b)

2.8 *Shear-band patterning*

An interesting question which arises with sets of parallel faults as observed in extensional zones is that of fault spacing. This is for example observed in delta zones under gravity extension. These faults are generally attributed to the failure of the gliding upper layer triggered by the existence of a detachment lower layer. In this direction notable are the papers by Lesniewska & Mroz, (2000), Wolf et al. (2003) and Nuebel & Huang (2004).

3. STABILITY-INSTABILITY-CONTROLABILITY

Nowadays it is recognized that there is a large variety of bifurcations and instabilities of material and/or of geometric nature, which are leading to various modes of failure, strictly inside the domain in stress-space, which is bounded by the Mohr-Coulomb failure criterion, but also inside the domain defined by Rice's localization condition, Eq. (3). The existence of such instabilities has been conjectured from the theory of non-associated elasto-plasticity (Vardoulakis 1986, Han & Vardoulakis 1991) and from the "controllability" theory (Imposimato & Nova 1998). Some of these modes of collapse can be called "diffuse failure modes" and they seem to be the leading mechanisms in certain types of slope failures (Laouafa & Darve 2002), debris flows and submarine slides in deltaic zones (di Prisco & Imposimato 2002), which can be simulated presently successfully by the finite element method (cf. Zhang et al. 2002).

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