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Elastoplastic models for soils and soft rocks

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ABSTRACT
The basic structure of constitutive models for remoulded or reconstituted soil behaviour is recalled first, making reference to elasto viscoplastic models and to incrementally non-linear laws. More recent developments on the modelling of natural soils and bonded geomaterials is presented next as an extension of the previous theories. Numerical simulations of tests on cemented soils are compared to experimental results. Finally a possible way of taking weathering into account is suggested. Weathering effects are investigated by comparing experimental test results and theoretical predictions for oedometric conditions. It is shown that, for certain values of the constitutive parameters, compaction bands can occur within the specimens.

1 INTRODUCTION

Constitutive modelling of soils, in the modern sense of the term, started in the early sixties at Cambridge under the direction of Ken Roscoe. The goal was the formulation of a mathematical law able to reproduce the observed behaviour of soils in laboratory tests and to predict their behaviour in boundary value problems related to geotechnical engineering. The reference materials were selected in such a way that their behaviour was relatively simple and repetitive (i.e., such that similar specimens subject to the same loading conditions behave similarly). Kaolinitic clay reconstituted in the laboratory starting from slurry was chosen as the representative material for clays. Quartz sand with rounded particles and more or less uniform particle size, such as Leighton Buzzard, was instead the representative material for sands and coarse geomaterials in general. Following an original conjecture of Drucker et al. (1957), the theory of elastoplasticity with hardening or softening was chosen as the conceptual framework within which suitable models for soil behaviour could be developed. The original Cam Clay model (Schofield and Wroth, 1968) and its modification (Roscoe and Burland, 1968) were able to describe the behaviour of remoulded kaolin in radial (isotropic or oedometric compression) and shear triaxial tests, in drained and undrained conditions, in normally consolidated or overconsolidated states, within the same conceptual framework. This result had a major impact on the geotechnical community and can now be considered as a change of paradygm for the successive development of Soil Mechanics research.
Few years later, on the same premises, a bunch of models (Lade, 1977; Vermeer, 1978; Nova and Wood, 1979) were capable of reproducing the observed behaviour of virgin sand with an acceptable degree of accuracy. Adachi and Oka (1982) introduced time in the description of clay behaviour, so that viscous effects could be modelled. To this latter topic a following section is devoted.
In the following, for space reasons, only the basic principles of traditional elastoplasticity will be recalled.

2 MODELS FOR UNCEMENTED SOILS

Consider remoulded clay or freshly deposited ('virgin') sand specimens. When one of these is subjected to a cycle of loading-unloading and reloading, some common features of the experimental results can be observed:

- soil behaviour is non-linear and irreversible, i.e. only part of the strains occurring in the loading phase can be recovered upon unloading;
- in a cycle of unloading-reloading, instead, the observed behaviour is more or less reversible and characterised by a larger stiffness than upon virgin loading;
- when the stress state reaches the level at which unloading started, a marked stiffness change is observed and the soil behaves as it would be virgin, i.e. as if the unloading-reloading cycle would have not been performed;
- furthermore, soil behaviour is very much sensitive to the value of the isotropic effective pressure at which it is tested, i.e. stiffness and strength depend very much on it.

The simplest way to cope with such observations is to assume that soil behaviour can be considered as elastic-plastic with hardening.

It is assumed that strain rates $\dot{\varepsilon}_{hk}$ can be decomposed into a reversible (elastic) part and a permanent (plastic) component:

$$\dot{\varepsilon}_{hk} = \dot{\varepsilon}^{e}_{hk} + \dot{\varepsilon}^{p}_{hk} \tag{1}$$

In order to establish whether plastic strains occur under a given stress increment $\dot{\sigma}'_{ij}$, it is defined a loading function $f(\sigma'_{ij}, p_k)$ that depends on the state of effective stress and on a set of variables p_k. These are known as hidden variables or hardening parameters and depend on the 'history' of the soil volume element via the experienced plastic strains.
If

$$f(\sigma'_{ij}, p_k) < 0 \tag{2}$$

only elastic strain increments can take place, whatever is the stress rate. If

$$f(\sigma'_{ij}, p_k) = 0 \tag{3}$$

two possibilities exist. If the stress increment is such that the loading function takes a negative value after this is applied to the soil volume element ($df < 0$), only elastic strains occur. If on the contrary the value of the loading function does not change after the stress increment ($df = 0$), then plastic strains as well as elastic take place. Plastic strain rates are assumed to be derivable from a plastic potential (flow rule)

$$\dot{\varepsilon}_{ij}^{p} = \Lambda \frac{\partial g}{\partial \sigma'_{ij}} \tag{4}$$

where Λ is a plastic multiplier that can be determined from the hardening rules, i.e. from the evolution rules of the hardening parameters:

$$\dot{p}_s = \frac{\partial p_s}{\partial \varepsilon_{hk}^{p}} \dot{\varepsilon}_{hk}^{p} = \frac{\partial p_s}{\partial \varepsilon_{hk}^{p}} \Lambda \frac{\partial g}{\partial \sigma'_{hk}} \tag{5}$$

By imposing the condition $df = 0$ (Prager's consistency rule)

$$df = 0 = \frac{\partial f}{\partial \sigma'_{ij}} \dot{\sigma}'_{ij} + \frac{\partial f}{\partial p_s} \frac{\partial p_s}{\partial \varepsilon_{hk}^{p}} \Lambda \frac{\partial g}{\partial \sigma'_{hk}} \tag{6}$$

$$\Lambda = -\frac{\frac{\partial f}{\partial \sigma'_{ij}} \dot{\sigma}'_{ij}}{\frac{\partial f}{\partial p_s} \frac{\partial p_s}{\partial \varepsilon_{hk}^{p}} \frac{\partial g}{\partial \sigma'_{hk}}} \tag{7}$$

Plastic strain rates are eventually determined via Equation (4). The various elastoplastic constitutive models differ essentially in the choice of the expressions of the loading function, the plastic potential and the hardening rules.

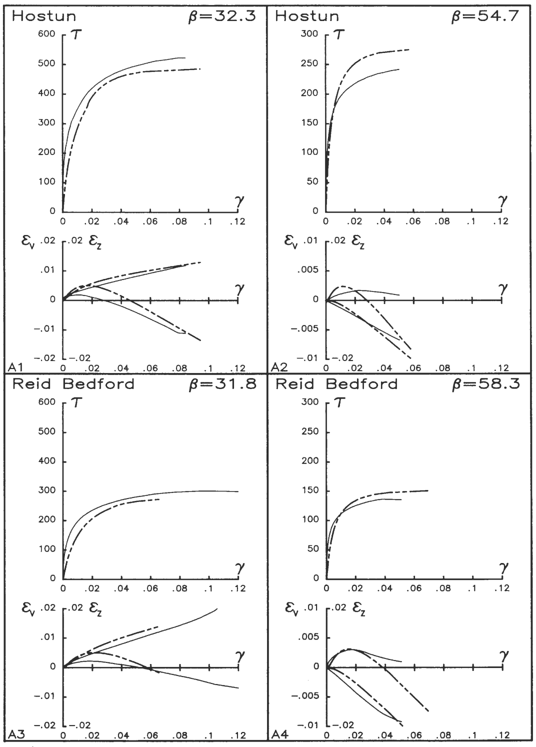


Figure 1. Comparison of experimental and predicted results in four drained hollow cylinder tests on sand: constant cell pressure and constant angle between major principal and vertical stress expressed in degrees (After Nova, 1988)

A delicate point concerns the modelling of the elastic behaviour. Upon unloading and reloading soil behaviour is not fully reversible. This fact has important consequences when cyclic loading is considered. Neither the observed ratchetting in drained tests nor pore pressure increase in undrained tests (possibly leading to cyclic liquefaction) can be modelled by an elastic constitutive law. On the other hand, for the sake of simplicity, most models assume that the behaviour in unloading-reloading is fully reversible.

The simple use of elementary Hooke's law is not possible, however. When a soil specimen is compacted, its elastic stiffness increases both in compression and shear. This could be accounted for, at first sight, by taking both the bulk modulus and the shear modulus as linearly dependent on the isotropic pressure. As shown for instance by Zytinski et al. (1979), this assumption violates the conservation of energy principle. A more complex elastic law must be assumed capable to admit the existence of an elastic potential. In this way, the strain state depends only on the stress state and not on the way it was achieved. Energy can neither be extracted from the specimen nor dissipated in closed loop tests. Few models have been proposed to fulfil the condition of the existence of the elastic potential. Among them we can recall Lade and Nelson (1987), Molenkamp (1988) or Borja and Tamagnini (1998).

In the eigthies and nineties of the last century more and more refined models based on hardening elastoplasticity were produced. The ability of predicting the experimental behaviour in monotonic tests different from those used for calibrating the constitutive parameters considerably increased. As an example Fig. 1 shows the predictions at the Cleveland Workshop (Saada and Bianchini, 1988) of one of these models (Nova, 1988) in a hollow cylinder test.

To take account that plastic strains occur even in unloading-reloading, traditional plasticity was extended either introducing a set of nested yield surfaces (Mroz et al., 1978), or mapping the behaviour within the yield locus onto the elastoplastic behaviour of a suitably defined point on the bounding surface (Dafalias and Herrmann, 1982) or introducing the concept of generalised plasticity (Pastor et al., 1990).

A completely different philosophy was followed by Darve (1978), Kolymbas (1984) and Chambon et al. (1994) who exploited the concept of incremental non-linearity of the constitutive law. This allowed Darve (1988) to predict correctly the behaviour of a sand specimen sheared in a cubic apparatus at constant isotropic and deviator stress but varying Lode angle.

3 MODEL FOR BONDED SOILS AND SOFT ROCKS

Natural soil behaviour is more complex than that of reference geomaterials. Diagenetic bonds are developed between grains affecting their behaviour in various ways. For instance soils acquire tensile strength and, since bonds are often fragile in nature, they often develop a collapsible structure, giving rise to unexpected instabilities. Despite such differences, the mathematical structure of the constitutive models describing bonded geomaterials can be obtained by modifying the original models for unbonded soils slightly. This fact conjectured by Nova (1986) and qualitatively demonstrated by Leroueil and Vaughan (1990), was exploited by a number of authors to model various kind of soft rocks such as shale, tuff, calcarenite, marl or chalk (see e.g. Shao & Henry, 1991; Nova, 1992; Kavvadas et al., 1993; Gens & Nova, 1993).

Soil bonding evolves with time. Uncohesive sediments are gradually transformed into sedimentary rocks (diagenesis). On the other hand, given enough time, even the hardest rock such as granite can be transformed into a residual soil by the action of weathering. Nova and Lagioia (1996) postulated that both diagenesis and weathering effects could be described in the same framework, by taking the parameters characterising bond strength as time dependent. In particular weathering effects were thoroughly investigated by Nova and co-workers (Nova , 2000; Nova et al., 2003).

From a macroscopic viewpoint, the existence of bonds is essentially reflected by the occurrence of a non-zero tensile strength. Moreover the size of the "initial" elastic domain is not only controlled by the maximum past pressure but also by the degree of cementation.

The constitutive law for remoulded soils can be accordingly adapted to describe the behaviour of cemented soils by introducing two new parameters, p_t and p_m, whose meaning is illustrated in Figure 2. The former is linked to the tensile strength of the natural soil, while the latter controls the growth in size of the initial elastic domain. In fact those parameters should be intimately connected, since both effects are linked to the same physical reason. It is assumed in the following that those are proportional to each other:

$$p_m = \alpha p_t \tag{8}$$

The evolution law of p_t, and then of p_m via Equation (8), can be assumed to be (Gens and Nova, 1993):

$$\dot{p}_t = -\rho p_t \left|\dot{\varepsilon}_v^p\right| \tag{9}$$

The role played by the preconsolidation pressure p_c for uncemented soils is now played by p_s.

As proposed by Kim and Lade (1984) and Desai et al. (1986) in similar contexts, it will be assumed that the expressions of the constitutive functions are formally the same as those employed for uncemented soils. The arguments of the functions will be however p^* and η_{ij}^* defined as follows:

$$p^* = p' + p_t \tag{10}$$

$$\eta_{ij}^* = \frac{s_{ij}}{p^*} \tag{11}$$

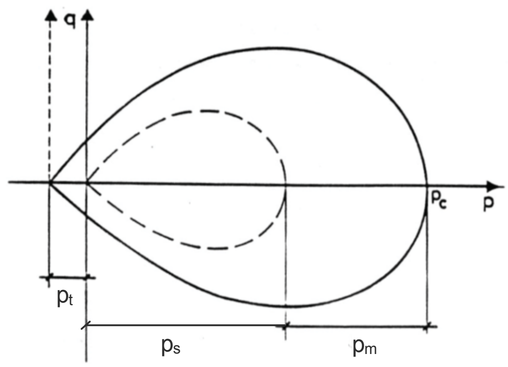


Figure 2. Modification of elastic domain due to cementation

A dummy variable can be introduced for mathematical convenience:

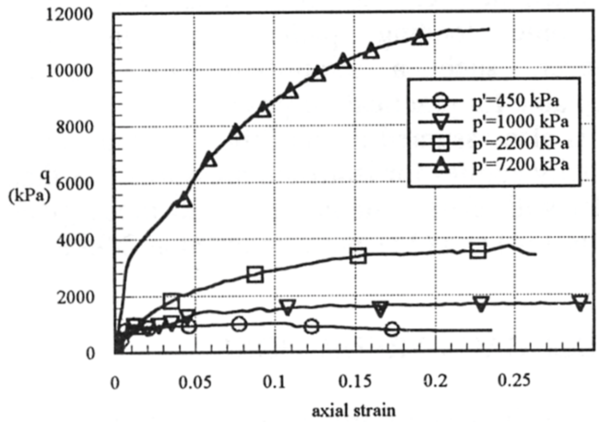
$$p_c = p_t + p_s + p_m \tag{12}$$

Experimental and calculated behaviour for drained tests on calcarenite at constant isotropic pressure are shown in Figures 3.

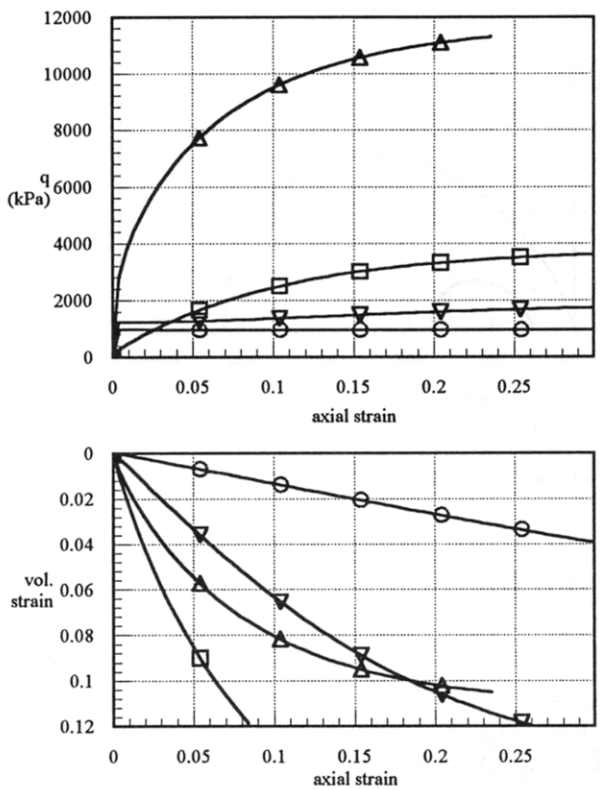
Similar results can be obtained for other soft rocks, such as different types of calcarenite (Lagioia and Nova, 1995), tuff and chalk (Nova and Lagioia, 1996). Figure 4 shows a comparison for experimental and calculated behaviour in undrained tests on chalk (after Nova and Lagioia, 1996 – data after Leddra, 1988) after K_0 consolidation. It is interesting to note how the model can reproduce the complicate effective stress path followed by the material element

4 MODELLING WEATHERING EFFECTS

By keeping the same mathematical structure, the model sketched above was recently refined to model the effects of weathering on the mechanical behaviour of soft rocks (Nova, 2000; Nova et al., 2003). A more realistic non-linear elastic law was introduced (Borja and Tamagnini, 1998). Degradation was



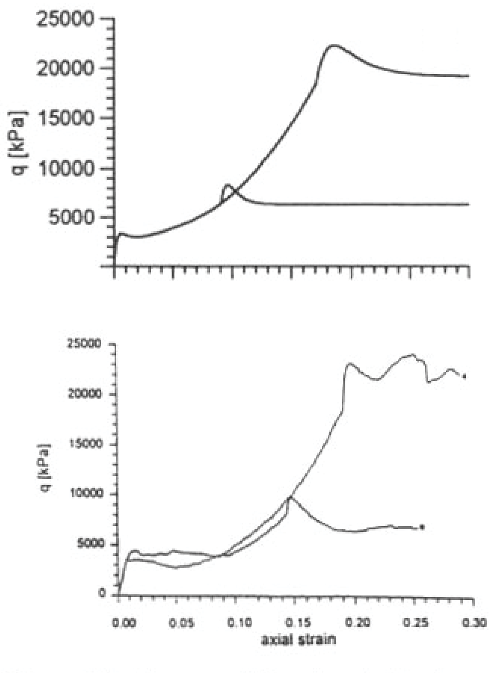
(a)



(b)

Figure 3. a) Natural Calcarenite: constant isotropic pressure tests : experimental data from Coop and Atkinson (1993); b) calculated results (after Lagioia and Nova, 1993)

made to depend on volumetric and deviatoric plastic strains. A different expression (but similar shape) for plastic potential and loading function was chosen (Lagioia et al., 1996). Most importantly, the effects of weathering have been modelled by assuming that the cementation parameters p_m and p_t depend not only on plastic strains but also on non-mechanical variables, such as time, the degree of chemical attack (Nova and Castellanza, 2001) or temperature (Nova et al., 2004).



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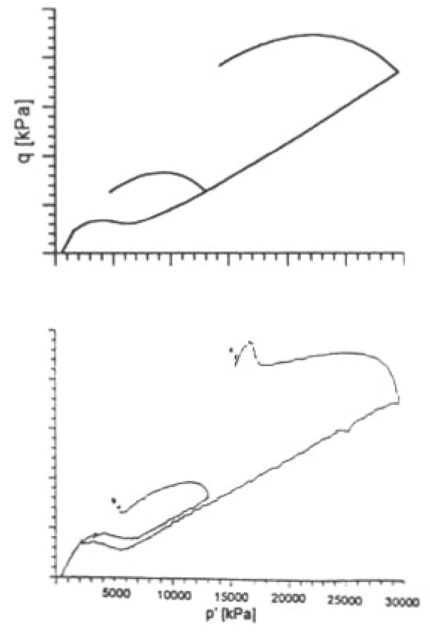
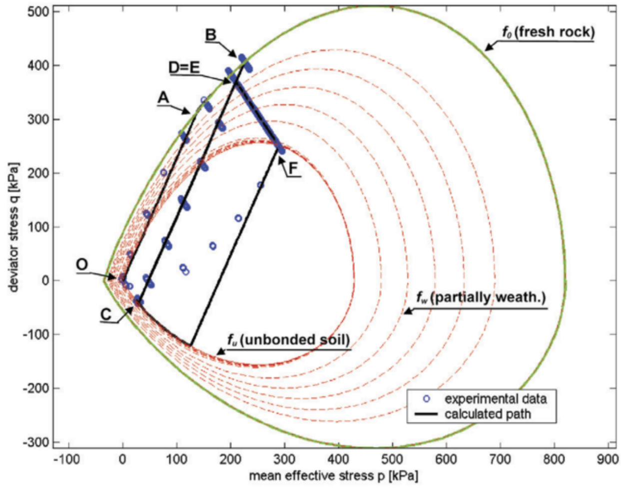


(b)

Figure 4. Undrained tests after K_0 consolidation on Stevn's Klint chalk: calculated results (above) from Nova and Lagioia (1996) and experimental data from Leddra (1988). a) stress strain relationship b) stress path.

As an example, Figure 5 shows the experimental and the calculated behaviour of a specimen of lime cemented silica sand subject to oedometric loading, chemical degradation by seepage of acid at constant vertical stress, and unloading after full degradation.



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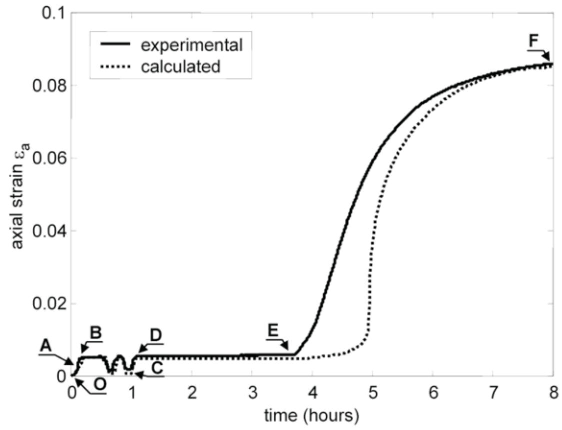
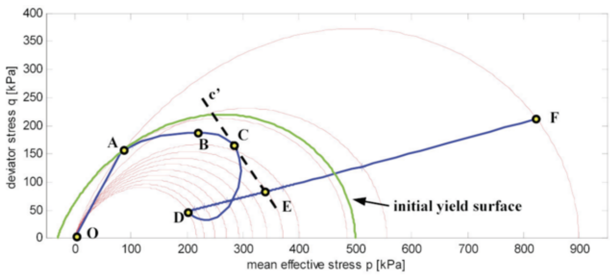


(b)

Figure 5, Lime cemented silica sand: oedometric loading followed by chemical attack at constant axial load and final unloading: a) stress path; b) variation of axial strain with time (after Nova et al., 2003).



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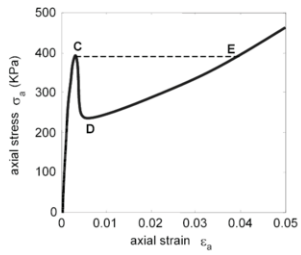

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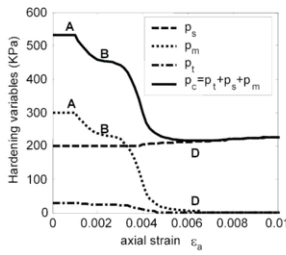

(c)

Figure 6. Predicted behaviour in a strain-controlled oedometric test on a bonded material: (a) stress path in the q:p plane (Point C corresponds to the maximum value of axial stress σ_a); (b) stress-strain response in the σ_a:ε_a plane; (c) evolution of internal variables (after Nova et al., 2003).

It can be further shown that, with a convenient set of parameters, the model can describe unstable phenomena. These include compaction bands (Nova, 2003; Castellanza and Nova, 2003) or even implosions under isotropic loading and the associated pore pressure build up when drainage is not allowed for (Arroyo et al., 2004). For instance Fig.6 shows the stress path of an oedometric test on a cemented geomaterial with strong degradation. At peak (point C) the formation of compaction band is possible.

5 CONCLUSIONS

Although relatively simple and characterised by a limited number of constitutive parameters, elastoplastic models are capable of reproducing the observed behaviour of many different geomaterials: soils (gravel, sand, silt and clay) and soft rocks (calcarenite, tuff, chalk and marl). Many interesting phenomena can be described. In particular several types of instability (loose sand liquefaction, drained and undrained shear bands, compaction bands in cemented soils, implosion under isotropic loading) can be predicted.

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