Proceedings of the 16th International Conference on Soil Mechanics and Geotechnical Engineering

doi:10.3233/978-1-61499-656-9-2825

Probabilistic design of anchored sheet pile wall

Calcul probaliste de rideau de palplanches ancré

B.K. Low
School of Civil & Environmental Engineering
Nanyang Technological University
Singapore

ABSTRACT
Several geotechnical design approaches have gained importance in recent years, including the Limit State Design approach of Eurocode 7, and the Load and Resistance Factor Design (LRFD) approach in North America. Researchers have also tried to link the first order reliability method (FORM) with Eurocode 7 and LRFD, via partial factors based on calibration studies of FORM. However, a more flexible approach is to perform the design directly based on FORM, without the intermediary stage of calibrating partial factors. This paper describes a practical and relatively transparent reliability-based design procedure using anchored sheet pile wall as example. The procedure obtains the same solutions as the Hasofer-Lind method and FORM, but in a much simpler and direct manner. Specifically, the embedment depth of an anchored sheet pile wall will be determined. The uncertainties of the soil properties and dredge level will be modelled. The reliability-based design approach can reflect parametric sensitivities from case to case ; its advantages over design based on specified partial factors will be discussed.

RÉSUMÉ
Recemment, des plusieurs methodes de calcul geotechnical ont gagné importance, y compris l'approche de calcul à l'état limite d'Eurocode 7 et celle de calcul à facteur de charge et de résistance (LRFD) des Amériques du Nord. Les chercheurs ont aussi essayé de lier la méthode de fiabilité de premiere ordre (FORME) avec l'Eurocode 7 et le LRFD, par des facteurs partiels basés sur des études de calibrage de FORME. Cependant, une approche plus flexible est d'exécuter le calcul basé directement sur la methode FORME, sans l'étape intermédiaire de calibrage des facteurs partiels. Cet artcile décrit une procédure pratique et relativement transparente de calcul à base de fiabilité utilisant le rideau de palplanches ancré comme exemple. La procédure obtient les mêmes solutions comme la méthode Hasofer-Lind et celle de FORME, mais d'une manière beaucoup plus simple et directe. Spécifiquement, la profondeur de l'embedment d'un rideau de palplanches ancré sera déterminée. Les incertitudes des propriétés du sol et du niveau de drague seront modeler. La methode de calculs a base de fiabilité peut refléter des sensibilités paramétriques d'un cas a un cas. Ses avantages vis-a-vis les calculs basé sur les facteurs partiels spécifiés seront discutés.

1 INTRODUCTION

The lumped factor of safety approach has long been used by the geotechnical profession. More recently, code-specified partial factors have been used. Yet another possible approach is design based on a target reliability index which reflects the uncertainties of the parameters and their correlation structure. Among the various versions of reliability indices, the Hasofer-Lind index and FORM (first order reliability method) are more consistent, though widely perceived to be complicated. This paper illustrates reliability-based design by the efficient approach of Low & Tang (1997a, 2004) that achieves the same result as the Hasofer-Lind method and FORM.

The merits of reliability-based design are its ability to explicitly reflect correlation, uncertainties and sensitivities, and to automatically seek the most probable failure combination of parametric values case by case without using fixed partial factors.

The analytical formulations in a deterministic design are the basis of the performance function in a probabilistic-based design. Hence it is appropriate to briefly describe the deterministic approach in the next section, prior to extending it to a probabilistic-based design.

2 DETERMINISTIC ANCHORED WALL DESIGN BASED ON LUMPED FACTOR AND PARTIAL FACTORS

The deterministic geotechnical design of anchored walls based on the free earth support analytical model was lucidly presented in Craig (1997). An example is the case in Fig. 1, where the relevant soil properties are the effective angle of shearing resistance ϕ', and the interface friction angle δ between the retained soil and the wall. The characteristic values are $c' = 0$, $\phi' = 36°$ and $\delta = ½\phi'$. The water table is the same on both sides of the wall. The bulk unit weight of the soil is 17 kN/m^3 above the water table and 20 kN/m^3 below the water table. A surcharge pressure q_s = 10 kN/m^2 acts at the top of the retained soil. The tie rods act horizontally at a depth 1.5 m below the top of the wall.

In Fig. 1, the active earth pressure coefficient K_a is based on the Coulomb-wedge closed-form equation, which is practically the same as the Kerisel-Absi (1990) active earth pressure coefficient. The passive earth pressure coefficient K_p is based on polynomial equations fitted to the values of Kerisel-Absi (1990), for a vertical wall and a horizontal retained soil surface.

The required embedment depth *d* of 3.29 m in Fig. 1—for a lumped factor of safety of 2.0 against rotational failure around anchor point *A*—agrees with Craig (1997, Example 6.9).

Using the alternative limit state design approach, with a partial factor of 1.2 for the characteristic shear strength, one enters $\tan^{-1}(\tan\phi'/1.2) = 31°$ in the cell of ϕ', and changes the embedment depth *d* until the summation of moments is zero. A required embedment depth *d* of 2.83 m is obtained, again in agreement with Craig (1997).

The partial factors in limit state design are applied to the characteristic values, which are themselves conservative estimates and not the most probable or average values. Hence there is a two-tier nested safety: first during the conservative estimate of the characteristic values, and then when the partial factors are

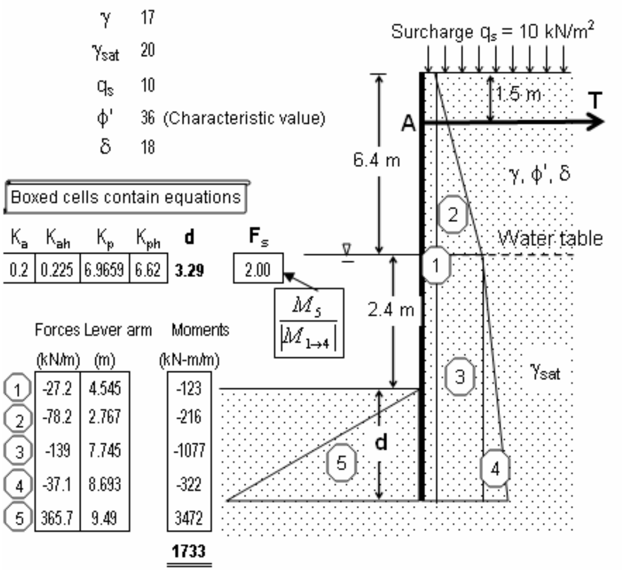

Figure 1. Deterministic design of embedment depth *d* based on a lumped factor of safety of 2.0

applied to the characteristic values. This is evident in Eurocode 7 where Section 2.4.3 clause (5) states that the characteristic value of a soil or rock parameter shall be selected as a cautious estimate of the value affecting the occurrence of the limit state. Clause (7) further states that characteristic values may be lower values, which are less than the most probable values, or upper values, which are greater, and that for each calculation, the most unfavorable combination of lower and upper values for independent parameters shall be used.

The above Eurocode 7 recommendations imply that the characteristic value of ϕ' (36°) in Fig. 1 is lower than the mean value of ϕ'. Hence in the reliability-based design of the next section, the mean value of ϕ' adopted is higher than the characteristic value (36°) of Fig. 1.

It should be borne in mind that while characteristic values and partial factors are used in limit state design, mean values (not characteristic values) are used with standard deviations and correlation matrix in a reliability-based design.

3 FROM DETERMINISTIC TO RELIABILITY-BASED ANCHORED WALL DESIGN

There are conceptual and computational barriers in reliability analysis by the Hasofer-Lind method and FORM. This is because the classical approaches of these methods—well-documented in Ditlevsen 1981, Ang & Tang 1984, and Baecher & Christian 2003—require frame-of-reference rotation and coordinate transformation, and iterative numerical derivatives using less ubiquitous special-purpose programs. This paper applies the intuitive expanding dispersion ellipsoid perspective of Low & Tang (1997a, 2004), in the context of reliability-based anchored wall design. The ellipsoidal perspective in the original coordinate space of the random variables leads to an efficient automatic constrained optimization reliability approach in the ubiquitous spreadsheet platform. The approach obtains the same result as the Hasofer-Lind method and FORM, but is operationally more direct and transparent.

The anchored sheet pile wall will be designed based on reliability analysis (Fig. 2). As mentioned earlier, the *mean value* of ϕ' in Fig. 2 is larger than the *characteristic value* of Fig. 1. In total there are six random variables, with mean and standard deviations as shown. These random variables are assumed to be normally distributed. Some correlations among parameters are assumed, as shown in the correlation matrix (crmatrix). For example, it is judged logical that the unit weights γ and γ_{sat} should be positively correlated, and that each is also positively correlated to the angle of friction ϕ', since $\gamma' = \gamma_{sat} - \gamma_w$.

The analytical formulations based on force and moment equilibrium in the deterministic analysis of Fig. 1 are also required in a reliability analysis, but are expressed as limit state functions or performance functions: "= Sum (Moments$_{1\rightarrow5}$)".

The matrix formulation (Ditlevsen 1981) of the Hasofer-Lind index β is:

$$\beta = \min_{\underline{x} \in F} \sqrt{\left(\underline{x} - \underline{\mu}\right)^T \underline{C}^{-1} \left(\underline{x} - \underline{\mu}\right)} \tag{1a}$$

where $\underline{x}$ is a vector representing the set of random variables x_i, $\underline{\mu}$ the vector of mean values μ_i, $\underline{C}$ the covariance matrix, and F the failure domain.

A more convenient equivalent formulation of Eq. (1a) is:

$$\beta = \min_{\underline{x} \in F} \sqrt{\left[\frac{x_i - \mu_i}{\sigma_i}\right]^T \left[\underline{R}\right]^{-1} \left[\frac{x_i - \mu_i}{\sigma_i}\right]} \tag{1b}$$

by virtue of

$$\underline{C}^{-1} = \left[\underline{\sigma}\right]^{-1} \left[\underline{R}\right]^{-1} \left[\underline{\sigma}\right]^{-1} \tag{1c}$$

in which $\underline{R}$ is the correlation matrix, σ_i the standard deviations, $[\underline{\sigma}]$ the diagonal standard deviation matrix. Low & Tang (1997b and later) used Eq. (1b) in preference to Eq. (1a) because the correlation matrix $\underline{R}$ is easier to set up, and conveys the correlation structure more explicitly than the covariance matrix $\underline{C}$.

The array formula for Eq. (1b) in cell β of Fig. 2 is:

=sqrt(mmult(transpose(nx),mmult(minverse(crmat),nx))) (2)

followed by "Enter" while holding down the "Ctrl" and "Shift" keys. In the above formula, *mmult*, *transpose* and *minverse* are Microsoft Excel's built-in functions, each being a container of program codes for matrix operations.

Given the uncertainties and correlation structure in Fig. 2, we wish to find the required total wall height *H* so as to achieve a reliability index of 3.0 against rotational failure about point "*A*". Initially the column *x** was given the mean values. Microsoft Excel's built-in constrained optimization tool Solver was then used to minimize β, by changing (automatically) the *x**

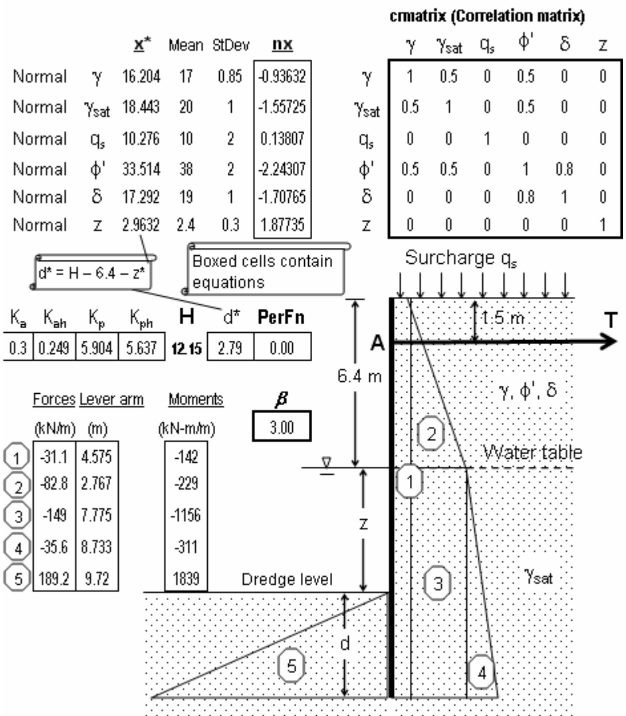

Figure 2. Design total wall height for a reliability index of 3.0 against rotational failure. Dredge level and hence *z* and *d* are random variables.

column, subject to the constraint that the cell *PerFn* be equal to zero. The solution (Fig. 2) indicates that a total height of 12.15 m would give a reliability index of 3.0 against rotational failure. With this wall height, the mean-value point is safe against rotational failure, but rotational failure occurs when the mean values descend\ascend to the values indicated under the x^* column. These x^* values denote the *design point* on the limit state surface, and represent the most likely combination of parametric values that will cause failure. The distance between the mean-value point and the design point, in units of directional standard deviations, is the Hasofer-Lind reliability index.

The expected embedment depth is $d = 12.15 - 6.4 - \mu_z = 3.35$ m. At the failure combination of parametric values the design value of z is $z^* = 2.9632$, and $d^* = 12.15 - 6.4 - z^* = 2.79$ m. This corresponds to an 'overdig' allowance of 0.56 m. Unlike Eurocode 7, this 'overdig' is determined automatically, and reflects uncertainties and sensitivities from case to case in a way that specified 'overdig' cannot.

The *nx* column indicates that, for the given mean values and uncertainties, rotational stability is, not surprisingly, most sensitive to ϕ' and the dredge level (which affects z and d and hence the passive resistance). It is least sensitive to uncertainties in the surcharge q_s, because the average value of surcharge (10 kN/m^2) is relatively small when compared with the over 10 m thick retained fill. Under a different scenario where the surcharge is a significant player, its sensitivity scale could conceivably be different. It is also interesting to note that at the design point where the six-dimensional dispersion ellipsoid (explained in next section) touches the limit state surface, both unit weights γ and γ_{sat} (16.20 and 18.44) are lower than their corresponding mean values, contrary to the expectation that higher unit weights will increase active pressure and hence greater instability. This apparent paradox is resolved if one notes that smaller γ_{sat} will (via smaller γ') reduce passive resistance, smaller ϕ' will cause greater active pressure and smaller passive pressure, and that γ, γ_{sat}, and ϕ' are logically positively correlated.

In a reliability-based design (such as the case in Fig. 2) one does not prescribe the ratios *mean*/x^*—such ratios, or ratios of *(characteristic values)*/x^*, are prescribed in limit state design—but leave it to the expanding dispersion ellipsoid to seek the most probable failure point on the limit state surface, a process which automatically reflects the sensitivities of the parameters. Besides, one can associate a probability of failure for each target reliability index value. The ability to seek the most-probable design point without presuming any partial factors and to automatically reflect sensitivities from case to case is a desirable feature of the reliability-based design approach.

The spreadsheet-based reliability-based design approach illustrated in Fig. 2 is a more practical and relatively transparent intuitive approach that obtains the same solution as the classical Hasofer-Lind method for correlated normals and FORM for correlated nonnormals (section 6). Unlike the classical computational approaches, the present approach does not need to rotate the frame of reference or to transform the coordinate space.

4 INTUTIVE PERSPECTIVES OF RELIABILITY INDEX

As a multivariate normal dispersion ellipsoid expands, its expanding surfaces are contours of decreasing probability values, according to the established probability density function of the multivariate normal distribution:

$$f(\underline{x}) = \frac{1}{(2\pi)^{\frac{n}{2}}|C|^{0.5}} \exp\left[-\frac{1}{2}\left(\underline{x}-\underline{\mu}\right)^T C^{-1}\left(\underline{x}-\underline{\mu}\right)\right]$$

$$= \frac{1}{(2\pi)^{\frac{n}{2}}|C|^{0.5}} \exp\left[-\frac{1}{2}\beta^2\right] \quad (3)$$

where β is defined by equation (1a) or (1b), without the "min". Hence, to minimize β (or β^2 in the above multivariate normal distribution) is to maximize the value of the multivariate normal probability density function, and to find the smallest ellipsoid tangent to the limit state surface is equivalent to finding the most probable failure point (the *design point*). This intuitive and visual understanding of the *design point* is consistent with the more mathematical approach in Shinozuka (1983, equations 4, 41, and Fig. 2), in which all variables were transformed into their standardized forms and the limit state equation was written in terms of the standardized variables. The expanding ellipsoidal perspective in the original space of the random variables is discussed further in Low & Tang (2004) and Low (2005).

5 POSITIVE RELIABILITY INDEX ONLY IF MEAN-VALUE POINT IS IN SAFE DOMAIN

In Fig. 2, if a trial H value of 10 m is used, and the entire "x*" column given the values equal to the "mean" column values, the performance function *PerFn* exhibits a value –448.5, meaning that the mean value point is already inside the unsafe domain. Upon Solver optimization with constraint *PerFn = 0*, a β index of 1.34 is obtained, which should be regarded as a negative index, i.e., –1.34, meaning that the *unsafe* mean value point is at some distance from the nearest safe point on the limit state surface that separates the safe and unsafe domains. In other words, the computed β index can be regarded as positive only if the *PerFn* value is positive at the mean value point. For the case in Fig. 2, the mean value point (prior to Solver optimization) yields a positive *PerFn* for $H > 10.6$ m. The computed β index increases from 0 (equivalent to a lumped factor of safety equal to 1.0, i.e. on the verge of failure) when H is 10.6 m to 3.0 when H is 12.15 m, as shown in Fig. 3.

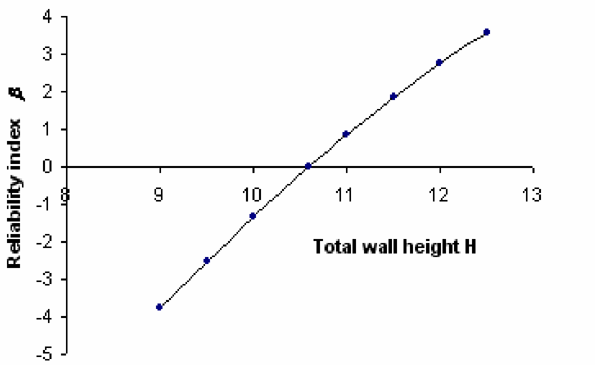

Figure 3. Reliability index is 3.00 when H = 12.15 m. For H smaller than 10.6 m, the mean-value point is in the unsafe domain, for which the reliability indices are negative.

6 RELIABILITY-BASED DESIGN INVOLVING CORRELATED NONNORMALS

The two-parameter normal distribution is symmetrical and, theoretically, has a range from $-\infty$ to $+\infty$. For a parameter that admits only positive values, the probability of encroaching into the negative realm is extremely remote if the coefficient of variation (Standard deviation/Mean) of the parameter is 0.20 or smaller, as for the case in hand. Alternatively, the lognormal distribution has often been suggested in lieu of the normal distribution, since it excludes negative values and affords some convenience in mathematical derivations. Figure 4 shows an efficient reliability-based design when the random variables are correlated and follow lognormal distributions. The required Rackwitz-Fiessler (1978) equivalent normal evaluations (for m^N and σ^N) are conveniently relegated to a function created in the VBA programming environment of Microsoft Excel. More

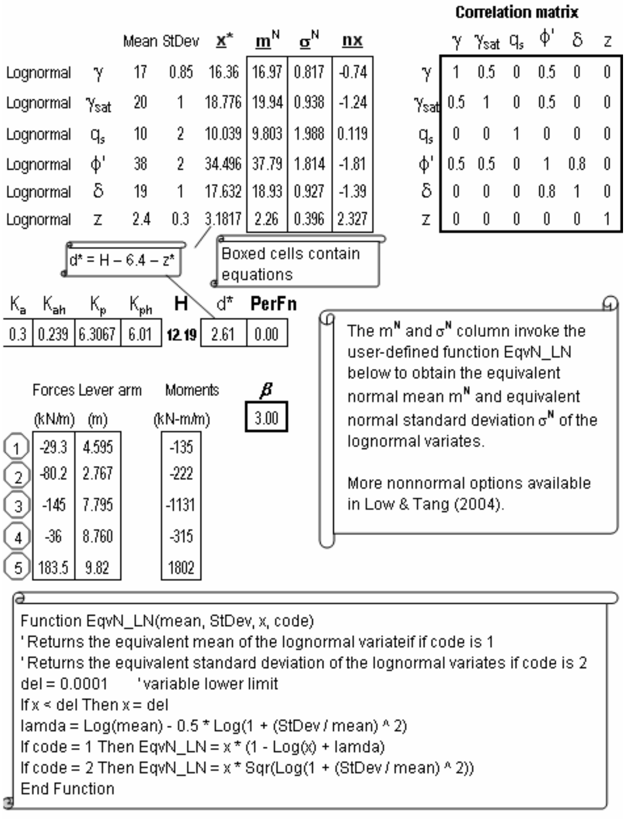

Figure 4. Reliability-based design involving correlated lognormal random variables

nonnormal options and the computational approach encapsulated in Fig. 4 are described in Low and Tang (2004).

For the case in hand, the required total wall height H is practically the same whether the random variables are normally distributed (Fig. 2) or lognorrmally distributed (Fig. 4). Such insensitivity of the design to the underlying probability distributions may not always be expected, particularly when the coefficient of variation (standard deviation/mean) or the skewness of the probability distribution is large.

7 FINITE ELEMENT RELIABILITY ANALYSIS VIA RESPONSE SURFACE METHODOLOGY

Programs can be written in spreadsheet to handle implicit limit state functions (e.g., Low et al. 1998, Low 2003, and Low & Tang 2004 p.87). However, there are situations where serviceability limit states can only be evaluated using stand-alone finite element or finite difference programs. In these circumstances, reliability analysis and reliability-based design by the present approach can still be performed, provided one first obtains a response surface function (via the established response surface methodology) which closely approximates the outcome of the stand-alone finite element or finite difference programs. Once the closed-form response functions have been obtained, performing reliability-based design for a target reliability index is straightforward and fast. Examples of coupled response surface method and spreadsheet-based reliability analysis were illustrated in Li (2000) and Vipman et al. (2000).

8 SUMMARY AND CONCLUSIONS

An efficient reliability-based design approach was illustrated for an anchored wall. The correlation structure of the six variables was defined in a correlation matrix. Normal distributions and lognormal distributions were considered in turn (Figs. 2 & 4), to investigate the implication of different probability distributions. The procedure is able to incorporate and reflect the uncertainty of the passive soil surface elevation.

Reliability index is the shortest distance between the mean-value point and the limit state surface—the boundary separating safe and unsafe combinations of parameters—measured in units of directional standard deviations. It is important to check whether the mean-value point is in the safe domain or unsafe domain before performing reliability analysis. This is done by noting the sign of the performance function (*PerFn*) in Figs. 2 and 4 when the x^* columns were initially assigned the mean values. If the mean value point is safe, the computed reliability index is positive; if the mean-value point is already in the unsafe domain, the computed reliability index should be considered a negative entity, as illustrated in Fig. 3.

The differences between reliability-based design and design based on specified partial factors were discussed. The merits of reliability-based design are thought to lie in its ability to explicitly reflect correlation structure, standard deviations, probability distributions and sensitivities, and to automatically seek the most probable failure combination of parametric values case by case without relying on fixed partial factors. Corresponding to each desired value of reliability index there is also a reasonably accurate simple estimate of the probability of failure.

The spreadsheet-based constrained-optimization reliability approach can be coupled with stand-alone finite element or other numerical packages, via the established response surface methodology.

REFERENCES

Ang, H.S. & Tang, W.H. 1984. *Probability concepts in engineering planning and design, vol. 2-Decision, risk, and reliability*. John Wiley, New York.

Baecher, G.B. & Christian, J.T. 2003. *Reliability and statistics in geotechnical engineering*. John Wiley.

Craig, R.F. 1997. *Soil Mechanics*, 6th ed., Chapman & Hall, London.

Ditlevsen, O. 1981. *Uncertainty modeling: with applications to multidimensional civil engineering systems*. McGraw-Hill, New York.

ENV 1997-1 1994. *Eurocode 7: Geotechnical design. Part 1: General rules*. Brussels: CEN, European Committee for Standardization.

Hasofer, A.M. & Lind, N.C. 1974. Exact and invariant second-moment code format. *J. Engrg. Mechanics*, 100:111-121, ASCE, New York.

Kerisel, J. and Absi, E. 1990. *Active and Passive Earth Pressure* Tables, 3rd ed., A.A. Balkema, Rotterdam.

Li, Guojun 2000. Soft clay consolidation under reclamation fill and reliability analysis. *Ph.D. thesis*. Nanyang Technological University, School of Civil & Environmental Engineering, Singapore.

Low, B.K. & Tang, W.H. 1997a. Efficient reliability evaluation using spreadsheet. *J. Engrg. Mechanics*, 123(7):749-752, ASCE.

Low, B.K. & Tang, W.H. 1997b. Reliability analysis of reinforced embankments on soft ground. *Canadian Geotechnical Journal*, Vol. 34, No. 5, 672-685.

Low, B.K., Gilbert, R.B. & Wright, S.G. 1998. Slope reliability analysis using generalized method of slices. *Journal of Geotech. and Geoenviron. Engrg*, ASCE, New York, Vol. 124, No. 4, 350-362.

Low, B.K. 2003. Practical probabilistic slope stability analysis. *Proceedings, Soil and Rock America,* M.I.T., Cambridge, Massachusetts, June 2003, Verlag Glückauf GmbH Essen, Vol. 2, 2777-2784.

Low, B.K. & Tang, W.H. 2004. Reliability analysis using object-oriented constrained optimization. *Structural Safety*, Elsevier Science Ltd., 26(1): 69-89.

Low, B.K. 2005. Reliability-based design applied to retaining walls. *Geotechnique*, Feb. 2005.

Rackwitz, R. & Fiessler, B. 1978. Structural reliability under combined random load sequences. *Computers & Structures*, Elsevier, 9:484-94.

Shinozuka, M. 1983. Basic analysis of structural safety. *Journal of Structural Engineering*, ASCE, 109(3): 721-740.

Vipman Tandjiria, Teh, C.I. & Low, B.K. 2000. Reliability analysis of laterally loaded piles using response surface methods. *Structural Safety*, Elsevier Science Ltd., Vol. 22, No. 4, pp.335-355.