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Distributed parameter sensitivity of piles in nonlinear sand subjected to cyclic bending moment

Sensibilité de paramètre distribué des pieux sujets au moment cyclique dans un sable nonlinéaire

N. Rahman & B.B. Budkowska
Department of Civil and Environmental Engineering, University of Windsor, Canada

ABSTRACT
The paper presents the formulation and numerical investigations of the sensitivity of maximum lateral deflection of a long pile embedded in p-y sand below the water table subjected to cyclic loading. The bending moment M_p of discrete variability is applied to the pile head. The cyclicity of the pile-soil system is tackled in a quasi-static fashion. The parameters involved in the p-y soil relationship as well as the pile bending stiffness are considered as the design variables of a continuous type. The system is analysed in the scope of variational calculus based on the virtual work principle. The virtual load method is employed in the determination of the first variation of generalized deformation. The formulation leads to establishing the new sensitivity functions that are spatial integrands connected with each of the design variables. The nummerical investigations are focused on the determination of sensitivity integrands of maximum deflection caused by the changes of the design variables of the system. The characteristics of spatial senstivities associated with each design variable of the system are discussed in detail.

RÉSUMÉ
L'article présente la formulation et les investigations numériques de la sensibilité de déflexion maximale latérale d'un long pieu installé dans un sable p-y sous la nappe phréatique sujet à une charge cyclique. Le moment Mp de variabilité discrète s'est appliqué à la tête du pieu. La cyclicité du système de pieu-sol est abordée d'une manière quasi-statique. Les paramètres inclus dans le rapport p-y du sol ainsi que la rigidité du pieu sont considérés comme variables du plan du type continu. Le système est analysé dans le domaine du calcul de variation sur la base du principe de travail virtuel. La méthode de charge virtuelle est utilisée pour déterminer la variation première de la déformation généralisée. La formulation mène à l'établissement des nouvelles fonctions de sensibilité qui sont des intégrants spatiales associées avec chacune des variables du plan. Les investigations numériques sont concentrées sur la détermination des intégrants de la sensibilité de déflexion maximale due aux changements des variables de plan du système. Les caractéristiques des sensibilités spatiales associées avec chacune des variables de plan du système sont examinées en détail.

1 INTRODUCTION

The sensitivity of engineering systems to variations of their parameters is one of the basic aspects in the treatment of engineering systems. In order to be able to give a unique formulation, the mathematical model is postulated to be known. Sensitivity analysis is considered to be part of the solution (at the design stage) to a practical problem to know the sensitivity prior to its implementation or to reduce the sensitivity systematically if this turns out to be necessary. Sensitivity considerations have provided a fundamental motivation for the use of feedback and are mainly responsible for its development and what is called control theory and adaptation.

The behaviour of any real system changes with time in an often unpredictable way caused by environmental, material or operational influences. The potential for changes is typically expressed in terms of ranges of permissible changes of a group of parameters called design variables.

The choices for application of sensitivity methods in civil engineering, automotive, machine design, aerospace, thermodynamics, physical chemistry and economics are growing in numbers (Kleiber et al., 1997). As a result of the growing pace of applications, research on sensitivity analysis is increasingly driven by real-life problems. The research on sensitivity performance of soil-structure interaction systems embedded in nonlinear homogenous media subjected to various types of loading (Budkowska and Liu, 2004, Budkowska and Priyanto, Part I and Part II, 2003) is suitably inscribed in the general development of sensitivity analysis and takes part in the solutions of real-life engineering problems at the stage of the design, rehabilitation, improvement, monitoring, maintenance etc.

The sensitivity theory of distributed parameters of nonlinear systems can also be incorporated into new emerging research areas as well as to the on-going research in cognate areas. The former is focused on the assessment of vulnerability of the civil engineering infrastructure systems due to the threats of explosive loadings of a blasting type acting on transportation systems such as bridges, infrastructure systems, public facility buildings, etc. (Shuster, 2004, Fortner, 2004, Gocevski and Pietruszczak, 2004). The latter refers to the research on updating modeling of soil-structure interaction systems that aims to incorporation local features of soil to existing models as well as taking into account the effect of installation methods on the behaviour of the system (Kim et al. 2004, Murchison and O'Neil, 1984). This area of research also embraces structural investigations and applications of new materials, such as FRP (Fiber Reinforced Polymers), to a soil-structure infrastructure system (El Naggar, 2002).

In the paper, the performance of a pile structure embedded in sand located below the water table subjected to bending moment of a cyclic type is explored in the framework of sensitivity theory of nonlinear systems with design variables being the distributed type, that is, the continuous functions of space variables. The approach presented allows to analyse the sensitivity of the pile-soil interaction system in a continuous fashion which is superior to the discrete approach.

The performance of the system is described by means of a performance functional which contains maximum generalized deformations. The determination of changes of maximum generalized deformations is based on the variational form of virtual work principle (Washizu, 1976). It requires the involvement of an adjoint system that is suitably loaded by an appropriate generalized unit load. The changes of the performance functional of the investigated system (called the primary system) are formulated in the vicinity of the deformed system. Employing some constitutive constraints imposed on the pile-soil system, the changes of maximum generalized deflections are determined by means of a number of new functions called the sensitivity integrands that are associated with the change of each of the design variables.

The sensitivity integrands are spatial functions of key importance to establish those places where changes of the specific design variables affect mostly the performance of the system. The sensitivity integrands directly reveal in a quantitative fashion where, how and how far in spatial terms, the changes of each design variable affect the performance of the system. The distributions of sensitivity integrands are of substantial value for the purpose of design, monitoring of deteriorating performance or where the rehabilitation measures should be undertaken.

2 BRIEF DESCRIPTION OF THE SOIL MODEL EMPLOYED IN THE INVESTIGATIONS

The explored soil-pile system is subjected to bending moment M_p that is applied to the pile head. The analysis of the system is conducted in the coordinate system x, y attached to the pile head. The axis x is directed downward whereas axis y which defines lateral deflection of the system is aimed at the right.

The soil adjacent to the pile is described by the p-y model of Reese et al. (1974) in which p stands for lateral soil reaction whereas y means lateral deflection. The model itself is well known and discussed in detail in Reese et al. (1974). In this paper, only basic equations of the model are specified. In general, at arbitrary depth x, the soil reaction p depends on the ultimate soil resistance p_c, a deflection y, modulus of subgrade reaction k, width of the pile b and functions A_c, B_c that allow the involvement of a cycle effect to the soil response p.

The p-y model of sand located below the water table allows for the analysis of cyclic load in a quasi-static fashion. Consequently, this means that an inertia term is absent in the basic equation of the system.

The ultimate soil resistance p_c (that depends on the depth x, an angle of internal friction ϕ and the co-efficient of active lateral earth pressure of Rankine's type, K_a) requires two smooth functions p_{ct} and p_{cd} that are distributed along the pile axis. The function p_{ct} is defined within the interval $0 \le x \le x_r$, whereas the second function p_{cd} is valid for $x \ge x_r$. The condition of the continuity of the ultimate soil resistance p_c (described by the functions p_{ct} and p_{cd}) allows for the determination of the location x_r that defines the transition of function p_{ct} into p_{cd}.

The p-y model is described by means of a set of equations that are valid within certain intervals of lateral deflection y. The specific values of lateral deflections (that define the boundaries of a particular type of soil reaction p curve) are denoted as y_k, y_m and y_u. The values of y_m and y_u are constants and are equal to b/60 and 3b/80, respectively. The values of y_k are determined as the intersections of two families of p-y curves which define linear and parabolic p-y relationships. The standard set of p-y relationships that may develop at arbitrary depth x depending on the value of deflection y are the following:

$$p = kxy \qquad \text{for } y \le y_k \qquad (1)$$

$$p = B_c p_c \left(\frac{60}{b} y\right)^{0.8\left(\frac{A_c}{B_c}-1\right)} \qquad \text{for } y_k \le y \le y_m \qquad (2)$$

$$p = p_c\left[B_c + \frac{48}{b}\left(y - \frac{b}{60}\right)(A_c - B_c)\right] \qquad \text{for } y_m \le y \le y_u \qquad (3)$$

In both Eqs. (2) and (3) the ultimate soil resistance p_c is described by p_{ct} and p_{cd} for $x \le x_r$ and $x \ge x_r$, respectively. The examples of p-y curves are shown in Fig. 1 with specified points corresponding to the characteristic values of $y = y_k$, $y = y_m$ and $y = y_u$.

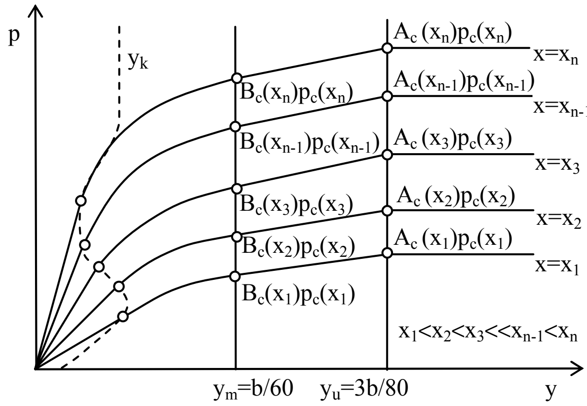

Figure 1. Examples of p-y curves for model of sand located below the water table subjected to cyclic loading developed at various depths x_i with marked characteristic deflections y_k, y_m, y_u.

Equations (1) – (3) show that depending on the value of the lateral load M_p the resulting deflection lines that develop in the soil can intersect various soil physical states that can develop within the soil medium depending on the magnitude of lateral deflection y. The possible scenarios of deflection lines that developed along the pile axis are classified as curves A, B, C and D. They are shown in Fig. 2. The curve A is developed when the soil along the entire length of the pile is in a linear elastic state. The application of a larger value of M_p is associated with an increase of lateral deflection (curve B) that passes through the nonlinear elastic and linear elastic soil phases. A further increase of M_p generates curve C that intersects soil being in a linear elastic, nonlinear elastic and bilinear elastic state. Finally, the application of a suitably large load M_p allows the develop of a considerable deflection (curve D) that runs through linear elastic, nonlinear elastic, bilinear and plastic soil phases.

The possibilities of the development of various soil states caused by the increased value of bending moment M_p and corresponding deflection curves with marked value of characteristic deflections y_k, y_m and y_u are shown in Fig. 2.

For completeness of discussion on p-y soil model the variabilities of functions A_c and B_c along the pile length are shown in Fig. 3.

3 SENSITIVITY OF DISTRIBUTED PARAMETERS OF THE PERFORMANCE OF PILE-SOIL INTERACTION SYSTEM

The investigated pile-soil system is subjected to moment M_p applied to the pile head. The behaviour of the soil that surrounds the pile is described by Eqs. (1) – (3). The pile structure is considered as a beam element, which satisfies the following equation:

$$EIy'' = -M \qquad (4)$$

where EI = bending stiffness, M = internal bending moment.

The interaction of pile-soil system obeys the differential equation given as:

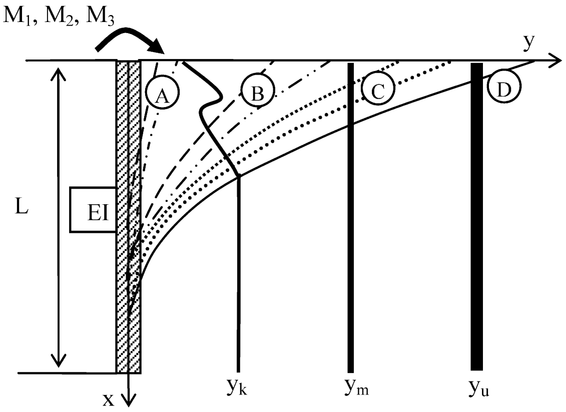

Figure 2. Development of a soil single phase deflection line (curve A) and soil multiphase deflection curves B, C and D along the pile axis resulting from an application of a continuously increased bending moment M_p.

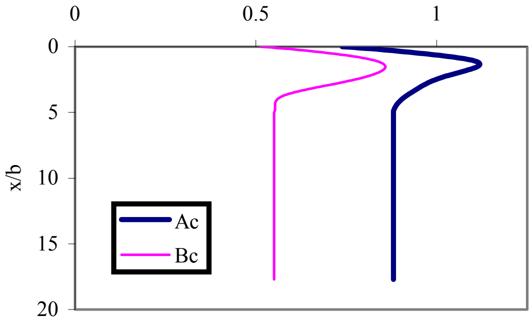

Figure 3. Distributions of dimensionless functions A_c and B_c representing the effect of cyclicity in p-y sand located below the water table subjected to cyclic loading.

$$EIy^{IV} + p(y) = 0 \tag{5}$$

where p(y) = soil reaction which is a function of a deflection y.

The physical parameters contributing to p-y relationships of soil (ϕ, K_a, b, γ') and the bending stiffness of the pile EI of Eq. (4) are considered as the design variables of distributed type. The detailed derivation of the sensitivity analysis of distributed parameter is presented in Budkowska and Priyanto, Part I (2003). Here only brief summary will be given. The investigations can be conducted by means of the direct differentiation method or employing variational formulation with involvement of Lagrangian multipliers (Kleiber et al., 1997, Budkowska, Part I and Part II, 1997, Budkowska and Szymczak, 1992).

Both methods lead to conclusion that sensitivity performance of a system can be explored in the scope of virtual work principle based on virtual load method (Haug et al., 1986, Kleiber et al., 1997). Consequently, both methods require introducing an adjoint (called also a conjugated) system that is suitably loaded by a generalized unit load. In general, for nonlinear system, it is of primary importance to model the adjoint system in a way that a generalized virtual load is applied to deformed system.

The primary system is defined by its geometry, type of load and the point of its application, boundary conditions and physical parameters such as EI, ϕ, K_a, b, γ' and their variations δEI, $\delta\phi$, δK_a, δb, $\delta\gamma'$. The soil design variables are arranged in a vector **r**. The system behaviour is described by Eqs. (1) – (5).

The changes (variations) of the generalized maximum displacement $\delta\mathbf{u}$ of the system caused by the changes of the design variable vector $\delta\mathbf{r}$ are sought.

Employing the virtual load method (Washizu, 1976) for variation of generalized displacement $\delta\mathbf{u}$ (with components δy_T, $\delta\varphi_T$) allows us to formulate the sensitivity performance with distributed design variables.

Thus,

$$\bar{1}\delta\mathbf{u} = \int_0^L -\bar{M}\delta y''dx + \bar{p}\delta y dx \tag{6}$$

The determination of variations $\delta y''$ and δy of the investigated system requires to consider the relationships between the changes of internal forces (δM, δp) and the changes of generalized deflections $\delta y''$ and δy in the form as:

$$\delta M = \frac{\partial M}{\partial EI}\delta EI + \frac{\partial M}{\partial y''}\delta y'' \tag{7}$$

$$\delta p = \frac{\partial p}{\partial \mathbf{r}}\delta\mathbf{r} + \frac{\partial p}{\partial y}\delta y \tag{8}$$

Bearing in mind the postulate that the changes of the generalized displacements are developed in the presence of a constant load, therefore the changes of internal forces are prohibited. Thus, the variations of internal forces must vanish. Consequently, this allows to determine the variations of generalized deflections expressed by some functions that are explicitely associated with the appropriate design variables as:

$$\delta y'' = -\frac{\partial M}{\partial EI}\frac{\partial y}{\partial M}\delta(EI) \tag{9}$$

$$\delta y = -\frac{\partial p}{\partial \mathbf{r}}\frac{\partial y}{\partial p}\delta\mathbf{r} \tag{10}$$

Substitution of the variations of deflections of primary structure (given by Eqs. (9), (10)) to Eq. (6) allows us to arrive at the sought sensitivity integrands affecting the changes of the generalized displacements $\delta\mathbf{u}$. The final form of sensitivity performance equation is the following:

$$\bar{1}\delta\mathbf{u} = \int_0^L \left(\begin{matrix} P_{EI}\delta EI_N + P_\phi \delta\phi_N + P_{Ka}\delta K_{a_N} + P_{\gamma'}\delta\gamma'_N \\ + P_b\delta b_N + P_k\delta k_N \end{matrix} \right) dx \tag{11}$$

where the variations of the design variables are normalized with respect to their initial values for the purpose of consistency of units of all sensitivity integrands.

4 NUMERICAL INVESTIGATIONS OF SENSITIVITY PERFORMANCE OF A LONG PILE LOADED BY MOMENT M_P

The essential features of the distributed parameter sensitivity analysis presented in Section 3 are supported by numerical investigations conducted for a single isolated pile. The free head long pile subjected to analysis is embedded in a sand located below the water table to which the bending moment M_p is applied. The required systems for analysis (that is primary and adjoint) are shown in Fig. 4. As discussed, the Reese et al. (1974) sand model provides suitable resistance to cyclic loading. The length of the pile is equal to 17.9 m that is equivalent to 9T (T=relative stiffness factor defined by Evans and Duncun (1982)).

The numerical values of the input data employed in investigations are the following: EI=60,000 kN-m^2, k=16,300 kN/m^3, ϕ=32^0, K_a=0.31, γ=9.04 kN/m^3 and b=0.406 m.

The pile structure is subjected to bending moment M_p of discrete variability. One of the objectives of these studies is to show how the magnitude of the applied moment M_p affects the sensitivity performance caused by the changes of the design variables of the pile-soil system.

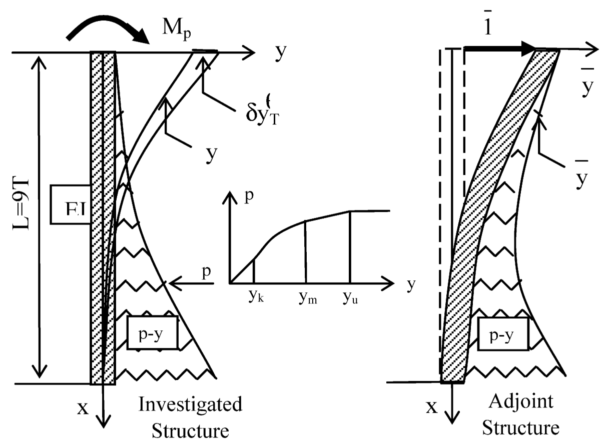

Figure 4. Investigated pile-soil structure and corresponding adjoint structure with suitable unit generalized load used in analysis δy_T.

The set of values of M_p is chosen such to allow the development on the soil surface all the soil's physical phases that the model has to offer. The knowledge on the soil physical state developed is assessed by projecting the pile head lateral displacement y_T on the p-y curve constructed for x=0. The values determined for y_T are projected on p-y curve for soil surface and associated values of M_p are marked on p-y curve. They are shown in Fig. 5. The sensitivity integrands $P^{My}_{(***)}$ due to the changes of the design variables $(_{***})$ that affect the variation of lateral deflection δy_T when the pile-soil system is subjected to M_p are expressed in units of force whereas the normalized sensitivity variations are scalars. Equation (11) explains that $P^{My}_{(***)}$s are spatial functions that require integrating with respect to depth x to give a numerical value of δy_T.

The numerical investigations are done by means of the finite difference program COM624P (Wang and Reese, 1993). For the problem investigated the sensitivity integrands P^{My}_{EI}, P^{My}_{k}, P^{My}_{ϕ}, P^{My}_{Ka}, P^{My}_{γ}, P^{My}_{b} affecting the changes of the pile head lateral deflection δy_T due to the changes of δEI, δk, $\delta\phi$, δK_a, $\delta\gamma$, δb for specified values of M_p are presented in Figs. 6-11.

5 DISCUSSION ON NUMERICAL RESULTS OF SENSITIVITY PERFORMANCE

The results in Figs. 6-11 show that the changes of maximum lateral deflections due to the changes of the design variables of the system depend on:

- the behaviour of the system, that is, how the system is defined in terms of a generalized load vs. generalized displacement,
- the magnitude of load that is applied to the system.

The examination of the distributions of sensitivity integrands presented in Figs. 6-11 reveals that:

- each design variable of the system affects the changes of the performance in a different fashion, the spatial variability of sensitivity integrands depends on how a specific design variable enters the behavioural equation of the system.

The quantitative values of the sensitivity integrands are indicators of the influence of the changes of the design variables on the sensitivity performance of the system. Regarding the investigated pile-soil system, it is seen that the effect of changes to δEI on δy_T is developed in a very consistent fashion that increases progressively together with the increase of load M_p. This is due to the simple form of a constitutive model employed for pile structure. The design variables associated with the p-y soil model are accounted for a variety of ways. Moreover, they contribute in a different fashion to the behaviour of the soil; that is, their contributions depend on which physical soil's phase is able to develop within a soil depth when subjected to load M_p.

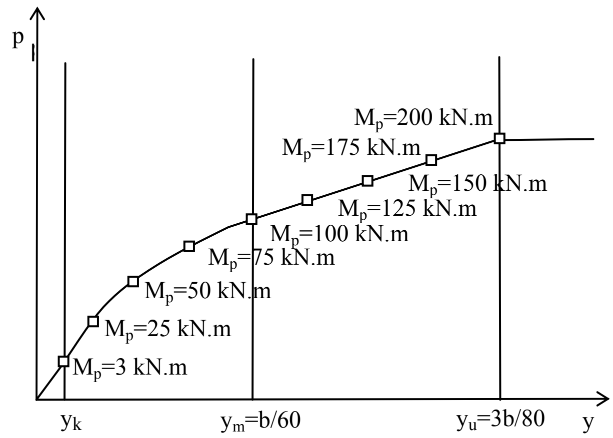

Figure 5. The p-y curve for soil surface (x=0) with projected values of y_T and corresponding values M_p.

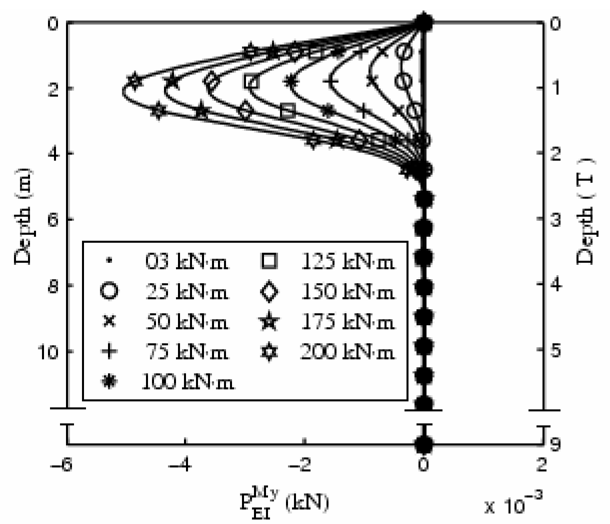

Figure 6. Distributions of P^{My}_{EI} (affecting δy_T due to the changes of δEI) along the pile depth for specified values of M_p.

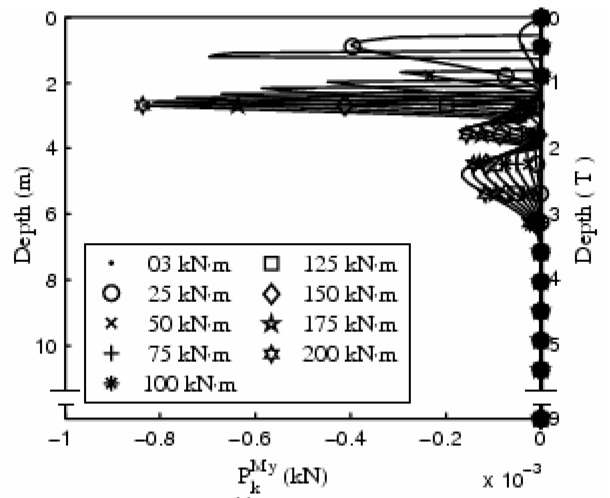

Figure 7. Distributions of P_k^{My} (affecting δy_T due to the changes of δk) along the pile depth for specified values of M_p.

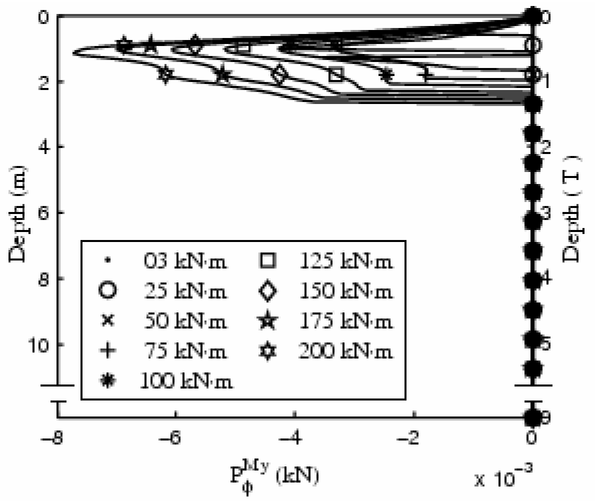

Figure 8. Distributions of P_ϕ^{My} (affecting δy_T due to the changes of $\delta\phi$) along the pile depth for specified values of M_p.

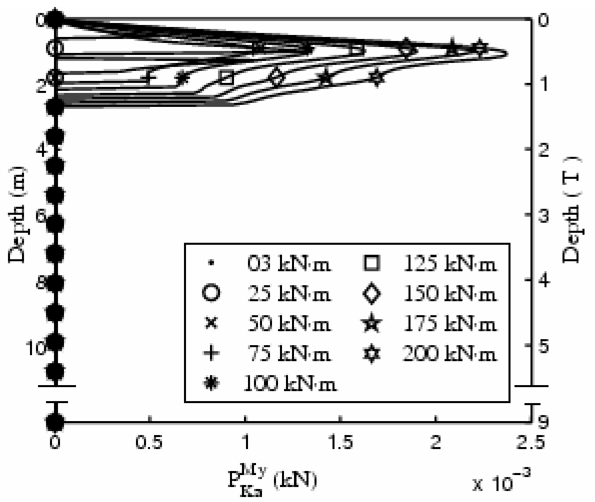

Figure 9. Distributions of P_{Ka}^{My} (affecting δy_T due to the changes of δK_a) along the pile depth for specified values of M_p.

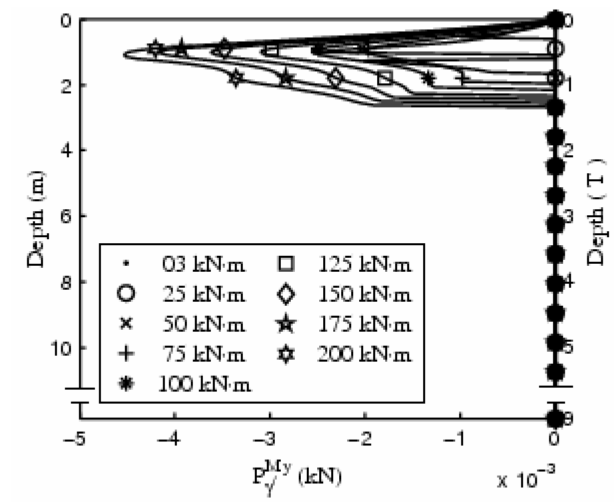

Figure 10. Distributions of P_γ^{My} (affecting δy_T due to the changes of $\delta\gamma$) along the pile depth for specified values of M_p.

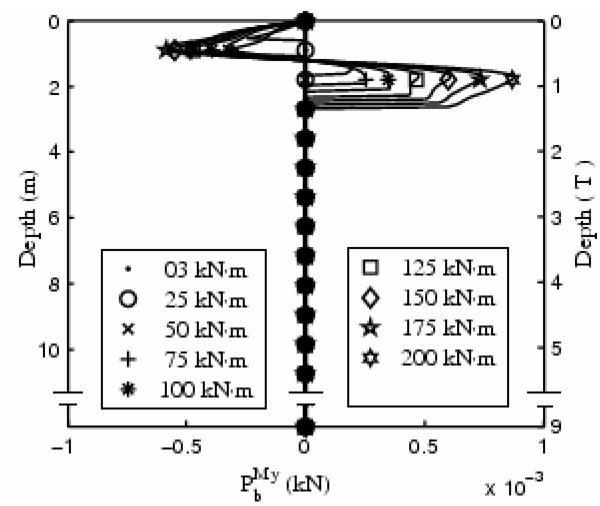

Figure 11. Distributions of P_b^{My} (affecting δy_T due to the changes of δb) along the pile depth for specified values of M_p.

The linear elastic soil phase is only dependent on design variable k which is a modulus of subgrade reaction. The distributions of P_k^{My}, shown in Fig. 7, for various values of M_p are recommended to be analysed with the aid of Fig. 3, that explains a rapid increase in P_k^{My} values even for small values of load M_p. This is connected with the fact that each deflection line being itself continuous enters a specific soil phase within the soil medium at a well defined value of lateral deflection. The physical boundaries that separate one soil phase from another are developed within a soil medium for y_m=b/60 and y_u=3b/80. The p-y relation, although continuous for arbitrary depth x, does not have continuous derivatives with respect to lateral deflection y at the points of transition from one phase to another. The described features affect the smoothness and generate specific regularities of sensitivity integrands. Another important aspect (besides numerical values of sensitivity integrands) of key practical significance is the depth up to which the sensitivity integrands can develop and where they reach their maxima. The diagrams presented in Figs. 6-11 show that for a long pile embedded in p-y sand below the water table subjected to cyclic loading in the presence of M_p the changes of the design variables that affect the maximum performance of the pile-soil system are developed (starting from the soil surface) within the depth of 3T for δk; 2.5T for δEI and up to 1.3T for the remaining changes of the design variables (δϕ, δK_a, δγ, δb). This result is of a substantial value from the point of view of the design, rehabilitation, improvement, monitoring, maintenance, etc.

The question that arises is the following: why does the spatial distributions of P_{EI}^{My} and P_{k}^{My} reach to a much larger depth of soil whereas the remaining $P_{(***)}^{My}$ are developed much closer to the soil surface. The answer to this question can be found in the p-y relationship and the constitutive relationship for the pile material. All the design variables of soil in question (ϕ, K_a, γ, b) are associated with nonlinear and bilinear soil phases.

6 CONCLUSIONS

The paper presents the sensitivity analysis of distributed parameters of maximum lateral deflection of long pile embedded in p-y sand below the water table subjected to bending moment M_p of cyclic type. The parameters of the system that define the behaviour of the soil and the pile are taken as the design variables of a continuous type. Employing the variational form of virtual load method, the first variation of the maximum performance caused by the changes of the design variables is determined. The sensitivity of maximum generalized deformation contains new spatial functions called the sensitivity integrands that are associated with the changes of each of the design variables.

The numerical investigations of long pile subjected to bending moment M_p of discrete variability allow the determination of the quantitative values as well as spatial distributions of sensitivity integrands $P_{(***)}^{My}$ associated with each design variable ($_{***}$) for various values of load M_p.

The presented results of sensitivity analysis lead to the following conclusions:

1. In general, the sensitivity performance of the system due to the changes of the design variables depends on the behaviour of the system, that is, how the relationship between the generalized load and generalized deformation is formulated.
2. For the analysed long pile embedded in the p-y sand loaded by moment M_p of discrete variability, the sensitivity of maximum lateral deflection due to the changes of the pile bending stiffness EI and the modulus of subgrade reaction k is spread within the depth 2.5T and 3T, respectively (starting from the soil surface).
3. The sensitivity integrands affecting the maximum lateral deflection due to the changes of p-y soil parameters (excluding modulus of subgrade reaction k) are developed up to the depth of 1.3T starting from the soil surface.
4. The maximum values of sensitivity integrands $P_{(***)}^{My}$ due to the changes of sand p-y design variables are located at the depth (0.6-0.8)T measured from the soil surface.
5. The maximum values of P_{EI}^{My} affecting δy_T caused by the change of stiffness EI are located at the depth 1.1T measured from the soil surface.
6. In the investigated case the deflection lines as well as internal forces are developed up to the depth of 4T. However, the sensitivity integrands have the nonzero values at a much smaller depth, particularly for physical parameters associated with the p-y soil model. This fact is of key importance for the design, improvement, rehabilitation and maintenance of the pile-soil system.
7. The portion of soil where the applied load M_p generates the lateral deflection larger than 3b/80 defines the development of soil plastic flow. Then all sensitivity integrands $P_{(***)}^{My}$ due to the changes of soil design variables are equal to zero since the p-y relationship is not unique.

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