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Nonlinear sensitivity of laterally loaded long piles in non-homogeneous soil - effect of boundary conditions

Sensibilité non-linéaire de longs pieux chargés latéralement dans un sol non-homogène - effet des conditions limites

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ABSTRACT
In the last few decades pile foundations became one of the most important sources of resistance against lateral loads. The paper presents the study of single laterally loaded piles, embedded in non-homogeneous soil and subjected to cyclic loading, using the sensitivity analysis approach. The sensitivity of maximum lateral deflection due to the changes of the design variables is investigated in a theoretical fashion. The design variables considered are the properties of both the pile and soil. The sensitivity results are given in a graphical form that reveals how the changes in the design variables that are crucial for the performance of the system are distributed spatially along the pile axis. The paper investigates the effect of the pile head boundary condition on the sensitivity results. A comparative analysis between piles with different head constraints is presented. It is shown that the sensitivity results connected with the pile bending stiffness are the results mostly affected by the change in head condition.

RÉSUMÉ
Durant les dernières décennies les fondations de pieux sont devenues une des plus importantes sources de résistance contre les charges latérales. Cet article présente une étude des pieux individuels chargés latéralement, installés dans un sol non homogène et sujets aux charges cycliques. La sensibilité de déflexion maximale latérale due aux changements des variables du plan est examinée théoriquement. Les variables du plan considérées sont les propriétés du pieu et du sol. Les résultats de sensibilité sont donnés sous forme graphique qui révèle comment les changements des variables du plan, cruciales pour la performance du système, sont distribuées spatialement le long du pieu. L'article examine l'effet de la condition de la tête du pieu sur les résultats de sensibilité. Une analyse comparative des pieux ayant têtes soumises aux différentes contraintes est présentée. L'analyse montre que les résultats de sensibilité associés avec la rigidité du pieu sont les résultats les plus influés par le changement de la condition de la tête du pieu.

1 INTRODUCTION

The structural systems can be classified based on the type of kinematic constraints imposed on the structures. Each type of constraint implies a possibility of development of the specific internal forces and deformability performance. In case of laterally loaded piles, the deformation of the pile-soil system when embedded in the same soil and subjected to the same load is different for a free head pile than it is for a fixed head one. A free head pile is unrestrained against rotation at the pile head while a fixed head pile is fixed against rotation at its top (Evans and Duncan, 1982).

The influence of the constraints on the behavior of the system can also be extended to the effect of changes of the parameters of the system on the changes of the performance of the system. This effect can be investigated using sensitivity analysis (Kleiber et al., 1997). Previous sensitivity studies were applied to laterally loaded piles embedded in homogeneous soil (Budkowska and Priyanto, 2003). The paper presents the sensitivity analysis of long single piles subjected to lateral cyclic horizontal loads embedded in non-homogeneous soil. Moreover, it concentrates on the effect of the head constraint on the sensitivity results.

The top lateral deflection is chosen to describe the system performance and its sensitivity to the changes of the design parameters is investigated. The design variables (parameters) considered are the properties of both the pile and soil. The sensitivity formulation, previously developed by the authors (Hafez and Budkowska, 2004) will be briefly discussed. It will then be applied to piles with different pile head conditions (free and fixed) to examine the effect of the head constraint on the sensitivity results.

2 NONLINEAR SENSITIVITY FORMULATION

The studied pile is a single pile embedded in non-homogeneous stratified soil consisting of a layer of soft clay overlaying a layer of sand. Nonlinear models describing the behavior of soils are needed for the development of the sensitivity formulation. The well known "Matlock-Reese" soil reaction-pile deflection (*p-y*) curves developed experimentally for many homogeneous soils are used. However, application of *p-y* methodology to stratified soils requires an appropriate modification of the *p-y* approach.

2.1 *p-y curves for homogeneous soils*

The *p-y* curves proposed by Matlock (1970) for soft clay and those proposed by Reese et al. (1974) for sand were used in our study. Both models, shown in Figure 1, were for piles embedded in homogeneous soils below water table subjected to cyclic load. In Figure 1(a) for clay, p_{ult} is the ultimate soil resistance per unit length of the pile and y_{50} is the deflection at one-half of that ultimate soil resistance. The ultimate soil resistance p_{ult} depends on x, the spatial variable starting from the soil surface and directed downwards along the pile axis. The ultimate soil resistance p_{ult}, becomes constant at the depth of reduced resistance of clay, x_{rc}.

The *p-y* curves (Figure 1(a)) describe the nonlinear behavior of clay through three stages of soil behavior. The *p-y* relationships governing these different stages are functions of the following parameters; ε_{50}, the strain corresponding to one-half of the compressive strength of clay, b, the diameter of the pile, the submerged unit weight of clay, and c, the undrained cohesion.

The *p-y* curves for sand (Figure 1(b)) describe the nonlinear behavior of sand through four different stages. The *p-y* relationships governing these different stages are functions of the following parameters; b, ϕ, the friction angle of sand, γ'_s, the submerged unit weight of sand and k, a constant representing the modulus of subgrade reaction in the first linear stage in the curve. The ultimate soil resistance of the sand also depends on the depth of reduced resistance of sand, x_{rs}.

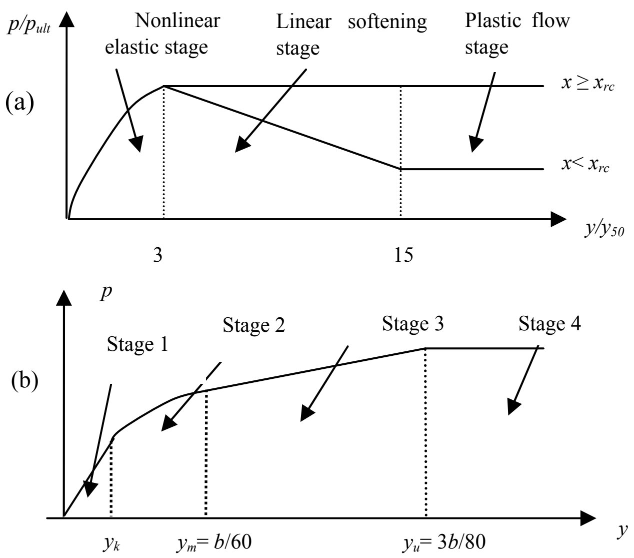


Figure 1 (a) Cyclic behavior of soft clay after Matlock (1970) (b) Cyclic behavior of sand after Reese et al. (1974)

2.2 *Modification of p-y curves*

The *p-y* models described above should be modified for the studied pile embedded in non-homogeneous soil. The pile extends a depth H_1 in soft clay followed by a depth H_2 in sand. To develop the *p-y* curves for this case, the equivalent thickness method proposed by Georgiadis (1983) was used to account for the non-homogeneity of the soil.

In this method, the upper soft clay layer is treated as if the soil consists altogether of soft clay. For the lower sand layer, an equivalent depth of sand, h_2, is found such that the value of the sum of the ultimate soil resistance for the equivalent sand and the upper soft clay are equal at the interface between the two layers. This allows for the determination of a local coordinate system of the lower sand layer that is analyzed by means of the *p-y* model for homogeneous sand.

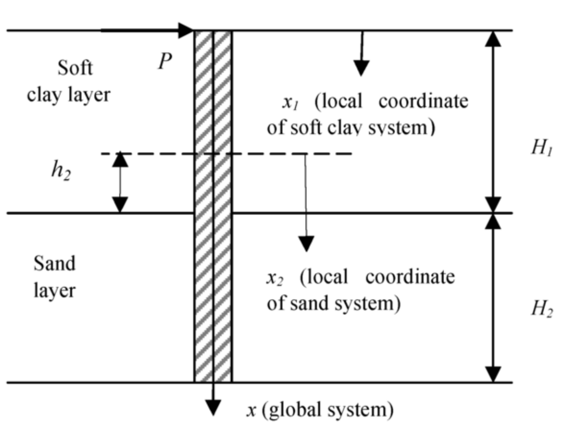


Figure 2. The coordinate systems of a pile embedded in a non-homogeneous soil

Accordingly, the local coordinates of the clay start at the ground surface while those of the sand start at a distance h_2 above the interface between the two layers as shown in Figure 2. These local coordinates are used to determine the proper depth x used in the *p-y* relationships.

2.3 *Theoretical formulation*

First-order sensitivity analysis of the pile soil system is performed using the distributed parameter sensitivity technique. This is done in the framework of variational calculus with the aid of the virtual work principle employing the adjoint method. Accordingly, some new spatial functions called sensitivity operators that are integrands can be determined. These sensitivity operators are of crucial importance in providing quantitative information on the expected changes of the performance of the system caused by the changes of the parameters (design variables) in the presence of unchangeable load conditions.

The lateral deflection at the pile head represents the performance of the system in our case. The design variables are the *p-y* relationship parameters and the bending stiffness of the pile, EI. They are given in the following vector $\boldsymbol{d} = \{EI, b, \gamma'_c, \varepsilon_{50}, c, \gamma'_s, \phi, k\}^{\mathrm{T}}$. The variation of the lateral top deflection δy_t due to the variation of the design variables $\delta\boldsymbol{d}$ is sought.

The theoretical formulation is developed using the adjoint method, which involves a primary and an adjoint pile. Both piles are of length l and the adjoint pile is subjected to a unit load, 1_a at the pile head. It has the same material and geometry, bounded to the same boundary conditions and is in the same state of deformation of the primary pile. Using the virtual work principle, the following equation can be written:

$$1_a \delta y_t = -\int_0^l M_a \delta y'' dx + \int_0^l p_a \delta y dx \tag{1}$$

where M_a, p_a are the moment and the soil reaction of the adjoint structure subjected to the unit load, respectively, and δy, $\delta y''$ are the variations of deformations imposed on the primary structure.

To obtain the sensitivity operators, the expressions of the first variation of internal forces ($\delta M = M,_{y''} \delta y'' + M,_{\boldsymbol{d}} \delta\boldsymbol{d}$; $\delta p = p,_y \delta y + p,_{\boldsymbol{d}} \delta\boldsymbol{d}$) are equated to zero since the load applied during the analysis is kept constant. In addition, the relationship between the bending moment and the second derivative of lateral deflection ($M = -EIy''$) is used. Expanding the vector $\boldsymbol{d}$, the following equation can be reached:

$$\begin{aligned}
1_a \delta y_t = & -\int_0^{H_1} [M_a \left(\frac{1}{EI}\right) y'' EI] \left[\frac{\delta EI}{EI}\right] dx - \int_0^{H_1} [p_a \left(\frac{\partial p}{\partial y}\right)^{-1} \frac{\partial p}{\partial b} b] \left[\frac{\delta b}{b}\right] dx \\
& - \int_0^{H_1} [p_a \left(\frac{\partial p}{\partial y}\right)^{-1} \frac{\partial p}{\partial \gamma'_c} \gamma'_c] \left[\frac{\delta \gamma'_c}{\gamma'_c}\right] dx - \int_0^{H_1} [p_a \left(\frac{\partial p}{\partial y}\right)^{-1} \frac{\partial p}{\partial c} c] \left[\frac{\delta c}{c}\right] dx - \\
& \int_0^{H_1} [p_a \left(\frac{\partial p}{\partial y}\right)^{-1} \frac{\partial p}{\partial \varepsilon_{50}} \varepsilon_{50}] \left[\frac{\delta \varepsilon_{50}}{\varepsilon_{50}}\right] dx - \int_{h_2}^{h_2+H_2} [M_a \left(\frac{1}{EI}\right) y'' EI] \left[\frac{\delta EI}{EI}\right] dx \\
& - \int_{h_2}^{h_2+H_2} [p_a \left(\frac{\partial p}{\partial y}\right)^{-1} \frac{\partial p}{\partial b} b] \left[\frac{\delta b}{b}\right] dx - \int_{h_2}^{h_2+H_2} [p_a \left(\frac{\partial p}{\partial y}\right)^{-1} \frac{\partial p}{\partial \gamma'_s} \gamma'_s] \left[\frac{\delta \gamma'_s}{\gamma'_s}\right] dx - \\
& \int_{h_2}^{h_2+H_2} [p_a \left(\frac{\partial p}{\partial y}\right)^{-1} \frac{\partial p}{\partial \phi} \phi] \left[\frac{\delta \phi}{\phi}\right] dx - \int_{h_2}^{h_2+H_2} [p_a \left(\frac{\partial p}{\partial y}\right)^{-1} \frac{\partial p}{\partial k} k] \left[\frac{\delta k}{k}\right] dx
\end{aligned} \tag{1}$$

For more details refer to Hafez and Budkowska, 2004. Equation (1) can be written in a symbolic form as:

$$
\begin{aligned}
1_a \delta y_t = & -\int_0^{H_1}(S_{EI})_c(\delta EI_N)_c\,dx - \int_0^{H_1}(S_b)_c(\delta b_N)_c\,dx - \\
& \int_0^{H_1}(S_{\gamma_c})_c(\delta\gamma'_{cN})_c\,dx - \int_0^{H_1}(S_c)_c(\delta c_N)_c\,dx - \int_0^{H_1}(S_{\varepsilon 50})_c(\delta\varepsilon_{50N})_c\,dx \\
& - \int_{h_2}^{h_2+H_2}(S_{EI})_s(\delta EI_N)_s\,dx - \int_{h_2}^{h_2+H_2}(S_b)_s(\delta b_N)_s\,dx - \\
& \int_{h_2}^{h_2+H_2}(S_{\gamma_s})_s(\delta\gamma'_{sN})_s\,dx - \int_{h_2}^{h_2+H_2}(S_\phi)_s(\delta\phi_N)_s\,dx - \int_{h_2}^{h_2+H_2}(S_k)_s(\delta k_N)_s\,dx
\end{aligned}
\qquad (2)
$$

where $(S_{(..)})_c$ and $(S_{(..)})_s$ denote the normalized sensitivity operators for clay and sand, respectively, corresponding to each design variable $_{(..)}$ which are given between [ ] in Eq. (1). The symbols $(\delta(..)_N)_c$ and $(\delta(..)_N)_s$ denote the normalized variations of design variables for clay and sand, respectively, corresponding to each design variable (..), which are given between $\left[-\right]$ in Eq. (1).

The graphical presentation of the sensitivity operator of each parameter allows for the detection of how and where the change of this parameter affects the change of the lateral head deflection at the pile top. Each operator is normalized with respect to the initial value of its parameter. These operators depend on the internal forces and the derivatives of *p* with respect to *y*. These derivatives are obtained from the *p-y* relationships that have different expressions for each soil stage. The local coordinates of each layer are taken into consideration as shown in the equation.

3 SENSITIVITY APPLICATION

The sensitivity analysis is applied to two cases of single long piles with different boundary conditions; a free head and a fixed head pile. The pile is embedded in nonhomogeneous soil; the top 40% is soft clay while the bottom 60% is sand. The sensitivity operators are calculated for constant initial values of the design variables. The following typical initial values are used: EI = 55,400 kNm^2; b = 406 mm; ϕ = 33^o; k = 16,300 kN/m^3; γ'_s = 10 kN/m^2; γ'_c = 7.5 kN/m^2; c = 18 kN/m^2; ε_{50} = 0.02. The laterally loaded pile is solved as a beam on elastic foundation involving nonlinear modeling of the soil-pile interaction response (*p-y* curves) using the finite difference program COM624P (Wang and Reese, 1993).

3.1 *Differences between free and fixed head piles*

According to the used *p-y* methodology, the *p-y* curves do not depend on the pile head condition. Matlock (1970) performed some tests of a pile in soft clay restraining the pile head in one test and allowing it to rotate in another test. The *p-y* curves that were derived from each of the loading conditions were essentially the same. Thus, these experimental *p-y* curves will predict within reasonable limits the response of a pile whose head is free to rotate or fixed against rotation (Reese and Van Impe, 2001).

However, according to Evans and Duncan (1983), the determination of whether the pile embedded in homogeneous soil is a short or long pile depends on the pile head condition. For each condition (free and fixed) a different relative stiffness factor T, on which the classification of length depends, can be obtained. The values of T are available for homogeneous soils only. To deal with our case of non-homogeneous soil the following steps were taken.

For the free head pile embedded in non-homogeneous soil, the relative stiffness factor was calculated for a homogeneous layer of clay, T_c, and a homogeneous layer of sand, T_s. Numerical studies were performed using an average value T_{av} (T_{av}= $(T_c+T_s)/2$). The pile was shown to behave as a typical long pile for all cases of non-homogeneous soil (ranging from the special case of a soil consisting of 100% clay and 0% sand to a case consisting of 0% clay and 100% sand) and for all the range of applied loads starting from a length equal to $8T_{av(free)}$(l = 16m).

The same procedure was followed for the fixed head pile. It behaved as a long pile starting from a length equal to $7T_{av(fixed)}$ (l = 12.4m). Two piles, one with a free head and the other with a fixed head, each of length 16 m are accordingly chosen for our analysis to satisfy a long pile for both cases. The thickness of the clay layer is 6.4 m followed by the sand layer.

In addition, the pile behavior is affected by the head condition. The internal forces and the deformability performance of the pile differ for a free and for a fixed head pile. This in turn will affect the sensitivity results as will be seen in the following section. The moment distribution and the deflection of the primary pile for the free and fixed head pile are shown in Figures 3 and 4, respectively.

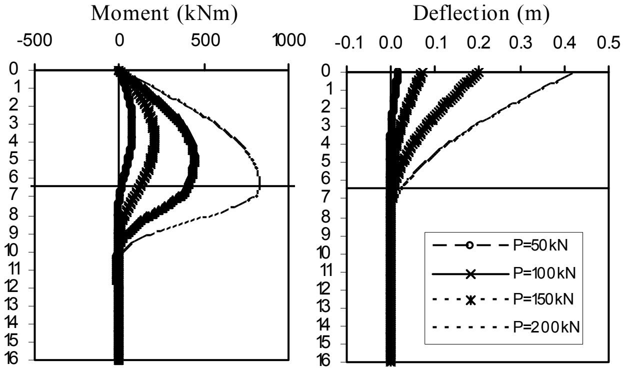


Figure 3. Moment distribution and deflection of the free head pile

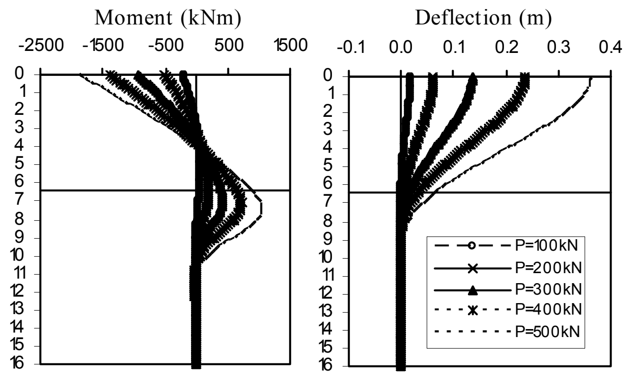


Figure 4. Moment distribution and deflection of the fixed head pile

3.2 *Sensitivity results*

The sensitivity results are given in the form of graphical presentation of the sensitivity operators. Since the system behavior is nonlinear, sensitivity operators are calculated for each increment of load. The range of lateral loads P, applied at the pile head, was chosen up to the load that causes the deflection of the pile head to reach the flow stage of soft clay. Accordingly all soil stages will be covered in the analysis. This deflection was reached at P = 200kN for the free head pile and P = 500kN for the fixed head pile.

The load was applied in increments of 25kN for the free head pile and 50kN for the fixed head one. For each increment of load the sensitivity operators were calculated at discrete close points along the length of the pile. The sensitivity operators are plotted for each design variable for both the free and fixed head piles.

The distribution of sensitivity operators $(S_{\varepsilon 50})_c$, $(S_{\gamma' c})_c$, and $(S_c)_c$ (Figures 5-7) presents the changes of pile-head lateral deflection δy_t caused by the change of normalized design variables ε_{50}, γ'_c, and c respectively. These variables are connected with the clay layer only. Each figure is divided into two for clarification of results due to the difference in the order of magnitude of the sensitivity operators for different load increments. The distribution of the sensitivity operators connected with the sand properties (γ'_s , ϕ , k) start from a depth x = 6.4 m which is the beginning of the sand layer (Figures 8-10). The operators EI and b are connected with both the sand and the clay and are shown in Figures 11 and 12.

To study the effect of the boundary condition on the sensitivity results, a comparison is made between the results of the free head pile and the fixed head one. Comparison can be deflection-based or load-based. In deflection-based comparison, we compare free and fixed head piles having almost the same deflection at the pile head, i.e. we compare between piles that are subjected to loads causing the soil to be in the same soil stage. In the load-based comparison we compare the behavior of the free and fixed head piles when they are both subjected to the same load.

For deflection-based comparison, Figures 5 to 12 can be easily used (for example we can compare load P = 200kN for a free pile with P= 500kN for a fixed head pile since they cause almost the same deflection and cause the soil to be in the same stage). From the figures, it can be observed that, in general, distributions differ slightly between fixed and free head piles for all operators (doesn't differ in shape of distribution but in numerical value) except for EI where there is a major difference in the distribution of S_{EI} between free and fixed head piles.

The graphical presentation of S_{EI} for a fixed and free head pile is shown in Figure 12. The Figure shows that for a free head pile variation of EI at the ground surface will not affect the pile head deflection δy_t while variation of EI at the ground surface will cause a considerable variation in the pile head deflection δy_t for a fixed head pile. The negative sign for both cases shows that increase in EI causes a decrease in δy_t as expected.

This uniqueness of S_{EI} can be attributed to the dependence of S_{EI} on the moments of the adjoint and primary piles and its independence on the p-y relationships ($(S_{EI})_c = (S_{EI})_s = S_{EI}$). The other operators do not depend on moment distribution and depend on the p-y relationships and accordingly on the deflection (refer to Eqs. 1 and 2). As shown in Figures 3 and 4, the moment differs considerably between free and fixed head piles while the deflection pattern doesn't differ considerably between the two cases. The p-y curves are identical as explained above.

If comparison is load-based, then for the same load, the deflection of the fixed head pile will be less than that of the free head pile (for example compare the piles when they are both subjected to load = 100kN). The soil for the fixed-head pile will be in an earlier stage of soil behavior and the numerical values of the sensitivity operators will be less in general.

Therefore we can say that sensitivity of top deflection to change in parameters is less for a fixed head pile than for a free-head one in general for load-based comparison. However, there are two exceptions. The first is S_{EI} where the distribution pattern between free and fixed head piles is completely different. The second is the distribution of $(S_{\varepsilon 50})_c$ where there are different signs of $(S_{\varepsilon 50})_c$ for different stages of soil.

For example, if we compare $(S_{\varepsilon 50})_c$ at load P = 100 kN, then the soft clay soil at the surface will be in a nonlinear elastic stage for the fixed head pile while it will be in the linear softening stage for the free head pile. As shown in Figure 5, the sign of the operator is positive for the fixed head pile while it will be negative for the free head one at the ground surface . Thus an increase in ε_{50} at the surface will cause an increase in the top deflection for the fixed head pile while it will cause a decrease in the top deflection for the free head one.

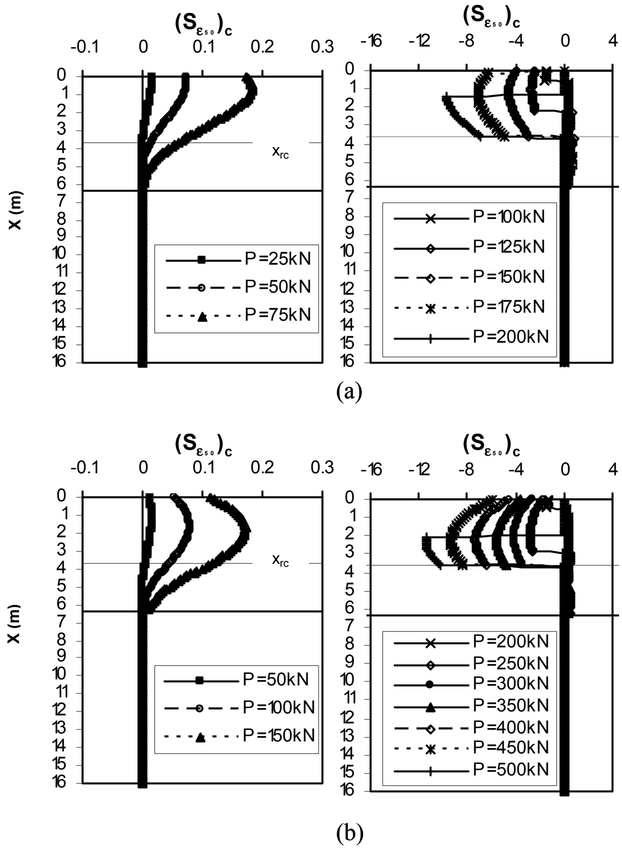


Figure 5. Distribution of $(S_{\varepsilon 50})_c$ for (a) free head pile (b) fixed head pile

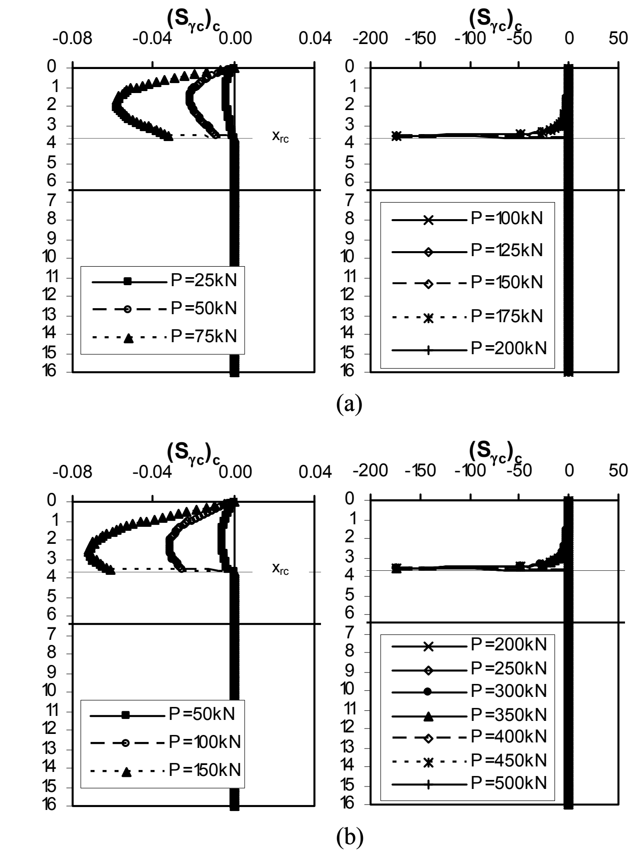


Figure 6. Distribution of $(S_{\gamma' c})_c$ for (a) free head pile (b) fixed head pile

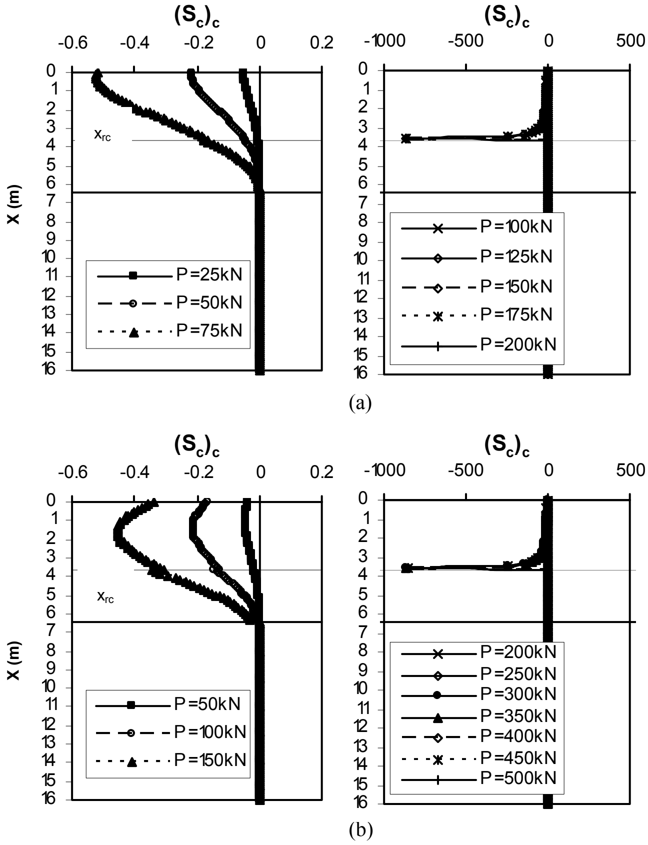


Figure 7. Distribution of $(S_c)_c$ for (a) free head pile (b) fixed head pile

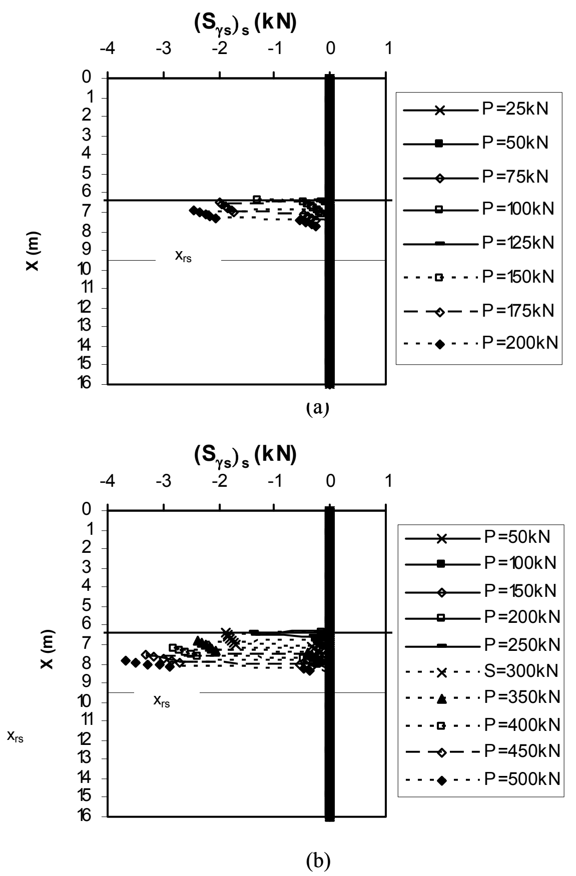


Figure 8. Distribution of $(S_{\gamma' s})_s$ for (a) free head pile (b) fixed head pile

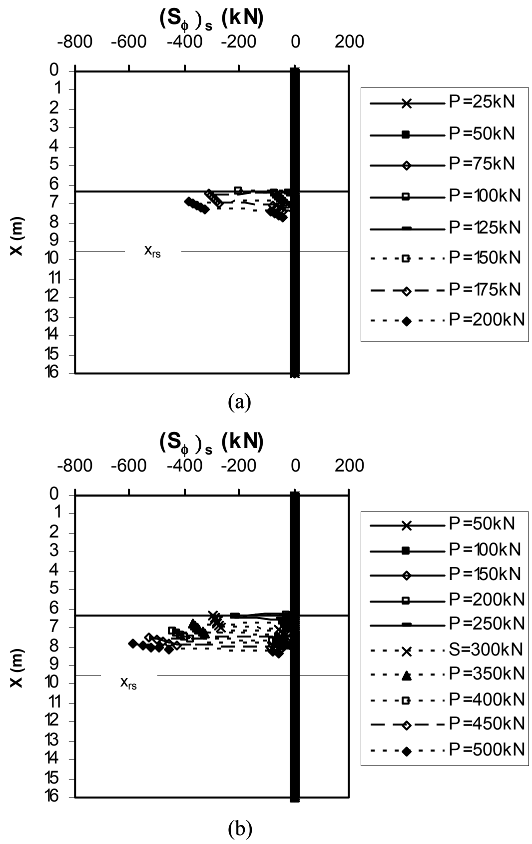


Figure 9. Distribution of $(S_\phi)_s$ for (a) free head pile (b) fixed head pile

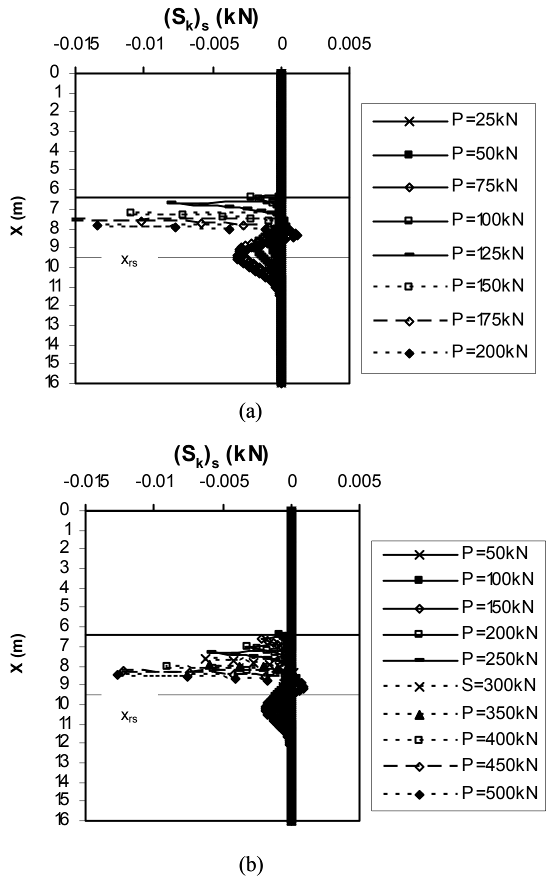


Figure 10. Distribution of $(S_k)_s$ for (a) free head pile (b) fixed head pile

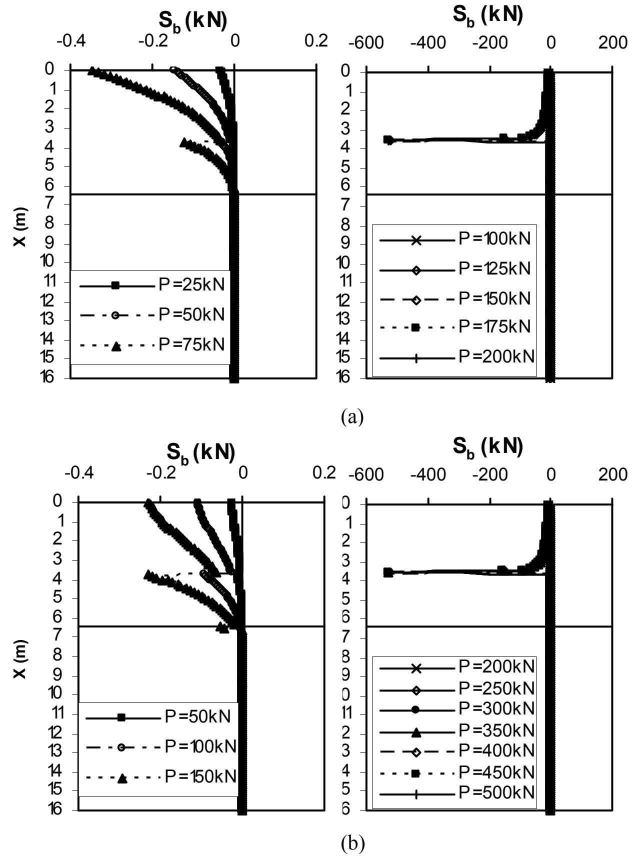
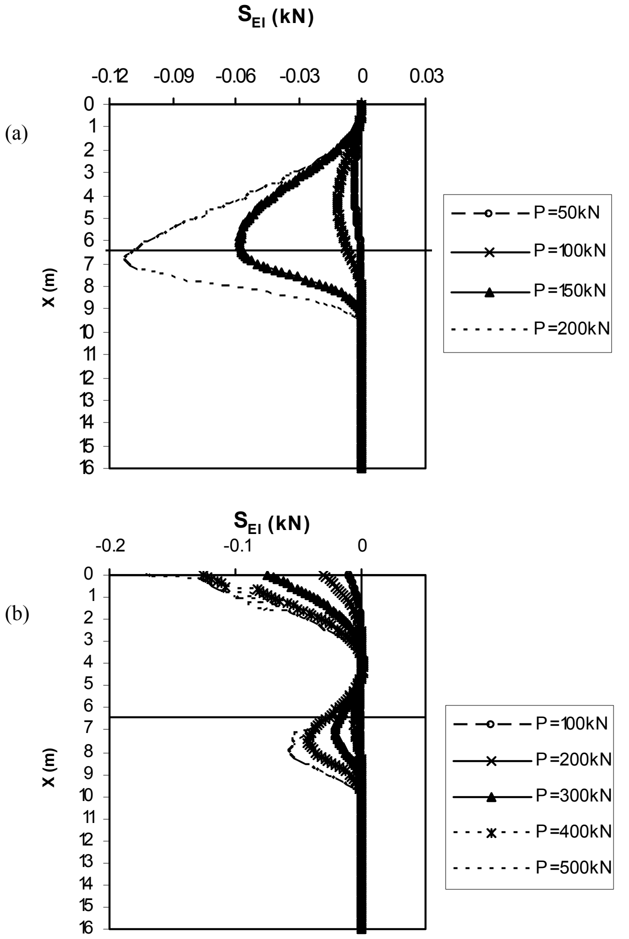


Figure 11. Distribution of S_b for (a) free head pile (b) fixed head pile



.Figure 12. Distribution of S_{EI} for (a) free head pile (b) fixed head pile

4 SUMMARY AND CONCLUSIONS

The sensitivity of the lateral top deflection of single piles subjected to lateral horizontal cyclic loads due to the variation of design variables was investigated. The pile was embedded in non-homogeneous soil consisting of a layer of soft clay overlying a layer of sand. The design variables studied were the physical parameters of both soils and the pile's bending stiffness.

The theoretical formulation for sensitivity was developed using the virtual work principle. The sensitivity formulation was then applied to piles with different head constraints (free and fixed) to study the effect of the head constraint on the sensitivity results. The results are given for both free and fixed head piles in the form of sensitivity operators that are graphically plotted along the pile length. This graphical presentation allows for the detection of how and where the change of each design variable will affect the lateral top deflection of the soil.

A comparison was carried out between the results of the free and fixed head pile where the following can be concluded:

1- The head constraint affects mostly the results connected with the pile bending stiffness.
2- If comparison is deflection-based, then, in general, the there will be a small difference in the shape and numerical values between results except for the results connected with *EI*.
3- If comparison is load-based, then the fixed head pile lateral top deflection will be less sensitive for variation of design variables in general with the exception of design variables *EI* and ε_{50}, the strain corresponding to one-half of the compressive strength of clay.

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