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The evolution of crushing in granular materials and its effect on their mechanical properties

L'évolution de l'écrasement en matériaux granulaires et de son effet sur leurs propriétés mécaniques

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ABSTRACT
Granular materials forming part of civil engineering structures crush as a result of static and dynamic loads. In this study the crushing of a sand under a combination of compression and shear loads is studied in the laboratory in the ring shear apparatus. The amount of crushing of the sand is evaluated using fractals. The effect of crushing on the angle of internal friction and the hydraulic conductivity of the sand is also presented.

RÉSUMÉ
Matériaux granulaires faisant partie d'écrasement de structures de génie civil en raison des charges statiques et dynamiques. Dans cette étude l'écrasement d'un sable sous une combinaison des charges de compression et de cisaillement est étudié dans le laboratoire dans l'appareillage de cisaillement d'anneau. La quantité d'écrasement du sable est évaluée en utilisant des fractals. L'effet de l'écrasement sur l'angle du frottement interne et la conductivité hydraulique du sable est également présenté.

1 INTRODUCTION

Granular materials form part of engineering structures such the base of flexible pavements, highway embankments, and foundations. The granular materials forming part of these structures are subjected during their engineering lives to either static and/or dynamic loads. As a result of these loads, particle abrasion and particle breakage occur (Lee and Farhoomand, 1967; Lade et al., 1996; Coop, 1999; Bolton, 1999; and Raymond, 2000). According to Lee and Farhoomand (1967) and Coop (1999), particle breakage or crushing seems to be a general feature for all granular materials. Grain crushing is influenced by grain angularity, grain size, uniformity of gradation, low particle strength, high porosity, and by the stress level and anisotropy (Ramamurthy, 1968; McDowell et al., 1996; Bohac et al., 2001).

When a granular mass is subjected to a compressive load, the particles resist the load through a series of contacts between the grains. The particles with highly loaded contacts are usually aligned in chains (Cundall and Strack, 1979). Crushing starts when these highly loaded particles fail and break into smaller pieces that move into the voids of the original material. This migration causes the settlement of a granular assembly. Also, on crushing, fines are produced and the grain size distribution curve becomes less steep. Consequently, with continuing crushing, the granular material becomes less permeable and more resistant to crushing (Cedergren, 1994). Grain size distribution is a suitable measure of the extent of crushing (Lade et al., 1996).

In this study a sand will be subjected to crushing in the ring shear apparatus. The level of crushing of the sand will be evaluated using fractal theory (Mandelbrot, 1977; Vallejo, 1995, 1996, 2001; Yeggoni et al., 1996).

2 FRACTALS AND THE CONCEPT OF THE FRACTAL DIMENSION

The shape of forms in nature is usually analyzed using Euclidean geometry. According to this kind of geometry, straight lines are perfectly straight lines and curves are arcs of perfect circles. However such perfection is seldom found in natural forms (Mandelbrot, 1977). Most of the time, the shapes of natural forms are irregular. Fractals are a relatively new mathematical concept to describe the geometry of irregularly shaped objects in terms of a fractional number (the fractal dimension) rather than an integer. In this study fractals are used to evaluate changes in the size distribution in a granular material subjected to varying crushing levels.

2.1 *Fractal Dimension of the Grain Size Distribution: Fragmentation Measurement*

Grain size distribution of naturally occurring soils has been found by Tyler and Wheatcraft (1992) and Hyslip and Vallejo(1997) to be fractal. Tyler and Wheatcraft (1992) have developed a relationship that uses the results of a standard sieve analysis to calculate the fragmentation fractal dimension, D_F, of the size ditribution of natural soils. This relationship is:

$$\frac{M(R<r)}{M_T}=\left(\frac{r}{r_L}\right)^{3-D_F} \qquad (1)$$

where M(R<r) is the cumulative mass (weight) of particles with size R smaller (finer) than a given comparative size r; M_T is the total mass (weight) of particles; r is the sieve size opening; r_L is the maximum particle size as defined by the largest sieve size opening used in the sieve analysis; and D_F is the fragmentation fractal dimension. The results of a sieve analysis tests using Equation. (1) can be plotted on log-log paper. The slope, m , of the best fitting line through data obtained using Equation (1) and the fragmentation fractal dimension, D_F , are related as follows:

$$D_F= 3 – m \qquad (2)$$

Equations. (1) and (2) are used to obtain the fractal dimension of the size distribution in a granular material subjected to crushing. The crushing is the result of compressive and shear loads induced in the ring shear apparatus. Changes in the degree of crushing of the granular material are reflected in the value of the fragmentation fractal dimension, D_F , of the size distributions. Higher values of D_F represent higher levels of crushing. For natural soils the value of D_F has been found to be greater than 1.4; soils being non fractal if D_F is smaller than 1.

3 LABORATORY TESTS AND RELATED FRACTAL DIMENSION

An oven dried sand with a specific gravity equal to 2.6 containing grains with a diameter that passed sieve No. 10 (2 mm) and grains that were retained in No. 16 sieve (1.18 mm) was subjected to fragmentation in a Bromhead's ring shear apparatus.

The ring shear tests were carried out to investigate crushing of the sand as a result of sustained normal and shearing stresses. The sand was subjected first to one normal constant stress after which shearing was induced in the sample for the completion of one 360^0 rotation. After this rotation was completed, the normal stress was increased and the sample was sheared again for another 360^0 rotation. These rotations were carried out for various normal stresses that vary between 15 and 1,374.3 kPa. The combination of the normal and shear stresses caused some of the sand grains to crush. The crushing of some of the grains caused the original size distribution to change from a uniform sand to a well graded or fractal sand. The grain size distribution of the particles after some of the fifteen 360^0 rotations is shown in Fig. 1. The fractal dimension of the grain size distribution was calculated using Eqs. (1) and (2) and the results shown in Figs. 2 and 3. Fig.2 shows a typical example. For other pressures the results were similar and R^2 varied between 0.9155 and 0.9740.

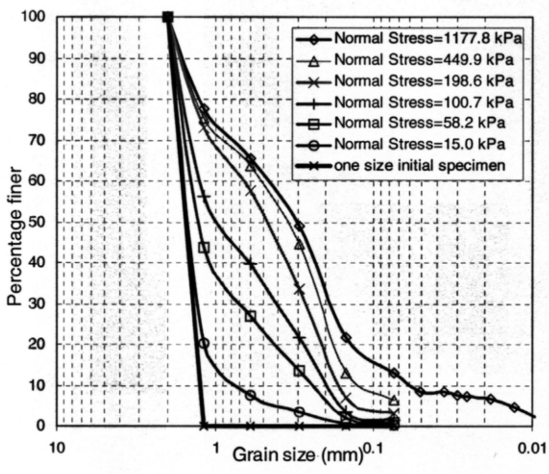
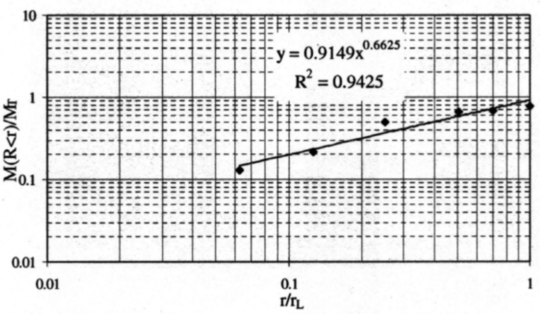


Figure 1. Grain size distribution of sand crushed in the ring shear apparatus.



$D_F = 2.3375$ ($D_F = 3\text{-}0.6625$)

Figure 2. Typical plot of sieve analysis results to obtain D_F at a normal stress = 1374.3 kPa.

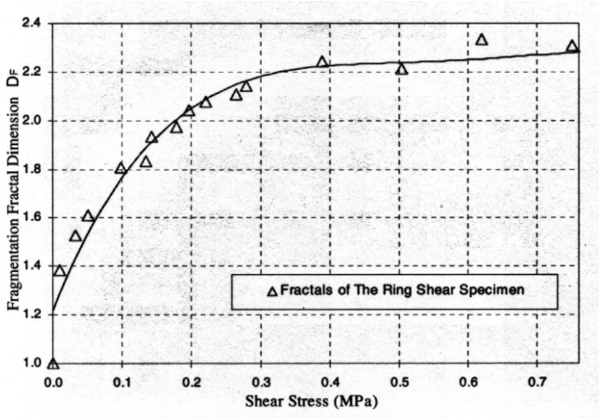


Figure 3. Fragmentation fractal dimension values for sand subjected to crushing in ring shear test.

An analysis of Fig. 3 (obtained using a regression analysis) indicates that the fragmentation fractal dimension, D_F, changed gradually from a value of 1.4 to a value of 2.3. This change in the fractal dimension represents a sand that is gradually being crushed or fragmented in the ring shear test.

3.1 *Effects of Sand fragmentation on the Hydraulic Conductivity*

There are a number of empirical correlations relating grain size with hydraulic conductivity. One of the most widely used correlations relating hydraulic conductivity and grain size is that of Hanzen (1911) formula :

$$K = 100\,(D_{10})^2 \quad (3)$$

in which K is the hydraulic conductivity (cm/s), and D_{10} = grain diameter (cm) corresponding to 10% of the material being smaller by weight (also called the effective grain size). Eq. (3) was used in conjunction with the grain size distribution curves shown in Fig. 1 to evaluate the hydraulic conductivity during the crushing of the sand in the ring shear apparatus. The results of this analysis is shown in Fig. 4.

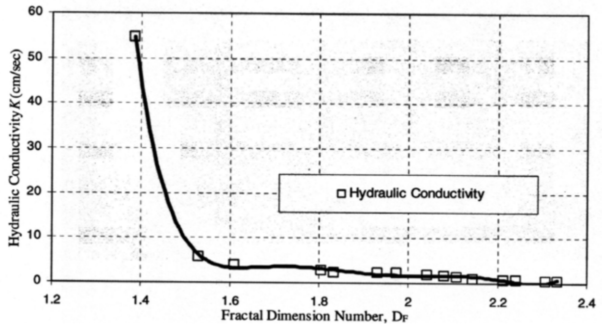


Figure 4. Relationship between hydraulic conductivity and fragmentation fractal dimension D_F.

An analysis of Fig. 4 shows the hydraulic conductivity, K, decreases as the value of the fragmentation fractal dimension, D_F, increases. At the beginning of the ring shear test, the normal stress is low, the sample is loose, and the profile of the sand particles is rough. These conditions make that the hydraulic conductivity of the sand be large. As the normal and shear stresses acting on the sample increases, the sand experience crushing. The crushed material fills the void spaces located within the grains that have not crushed. This results in an overall decrease in the hydraulic conductivity of the sand used in the experiments..

The changes experienced by the sand under a combination of normal and shear stresses in the ring shear apparatus and the influence that these changes have on the hydraulic conductivity of the crushed sand can be best explained using Fig. 5. This figure shows what happens to a pore located within three large particles when it is gradually filled by smaller and smaller grains resulting from the gradual crushing of the larger grains. The grains shown in Fig. 5 are self-similar with respect to their sizes and represent a sand with a fractal size distribution. Fig. 5(a) shows the pore between the three grains when it is filled by one small grain. The same pore continues to be filled by smaller and smaller grains as one goes from Fig. 5(b) to Fig. 5(d). The pore space decreases gradually until it becomes completely blocked [Fig. 5(d)]. Thus, when the pore reaches the condition shown in Fig. 5(d), water can not move through the pore. Thus, the filling of the pore space by a soil with a fractal size distribution will influence the hydraulic conductivity of the pore and the sand that contains it.

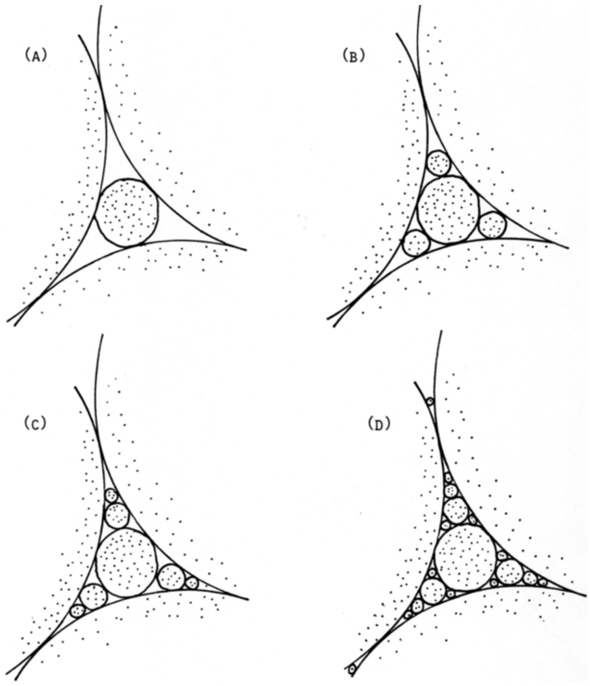


Figure 5. A pore being filled by a fractal soil.

3.2 *Effect of Sand Fragmentation on the Shear Strength*

Fig. 6 shows the relationship between the friction angle, ϕ, measured in the ring shear apparatus and the fragmentation fractal dimension, D_F for the sand used in the experiments. This figure shows that the friction angle decreases with an increase in the fractal dimension values. From the ring shear tests it was determined that an increase in fractal dimension is associated with an increase in normal and shear stresses as well as in the levels of crushing (Figs. 1 and 3). At low normal and shear stress levels, the sand particles are in a loose arrangement and their profile are rough. This roughness of the particles causes them interlock when subjected to shear. This high level of interlocking makes the friction angle of shearing resistance to be high. At high normal and shear stress levels, the roughness of the particles is somewhat eliminated due to the abrasion that takes place during the shearing of the sand. This abrasion makes the particles to change from rough to smooth, producing as a result a decrease in the friction angle of shearing resistance. This decreases in shearing resistance due to particles loosing their roughness seem to be a dominant factor over the increase in shear resistance that should take place as a result of the sand sample changing from uniform to a well graded sand (Figs. 1 and 5).

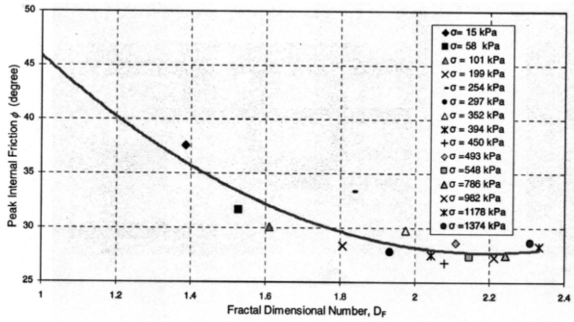


Figure 6. Relationship between friction angle, ϕ, and the fragmentation fractal dimension D_F .

4 CONCLUSIONS

The fractal dimension concept from fractal theory has been presented to evaluate fragmentation of granular materials. Crushing was produced by conducting ring shear tests on a sand sample. The ring shear tests cause the size distribution of the sand to change. This change was the result of sustained normal and shear stresses during the tests. The size distribution of the sand changed from non-fractal to a fractal one. The changes in the particles size distribution in the sand had a large influence on the hydraulic conductivity and the shear strength. The hydraulic conductivity decreased as the particle size distribution changed from non-fractal to a fractal one. Similar results were obtained when the shear strength of the sand was considered. The shear strength of the sand measured by its friction angle decreased as the sand changed from non-fractal to a fractal one.

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REFERENCES

Bohac, J., Feda, J., and Kuthan, B., 2001. Modelling of grain crushing and debonding. *Proceedings of 15th Int. Conference on Soil Mech. And Geotech. Eng*., Istanbul, Turkey, Vol. 1, pp. 43-46.

Bolton, M.R., 1999. The role of micro-mechanics in soil mechanic. *Proceedings of the Int. Workshop on Soil Crushability*, Yamaguchi, Japan, pp. 58-82.

Brown, S.F., and Pappin, J.W., 1981. Analysis of pavements with granular bases. *Transportation Research Record,* NRC, Vol. 810, pp. 17-23.

Cedergren, H.R., 1994. America's pavements: world's longest bathtubs. *Civil Engineering*, September, pp. 56-58.

Coop, M.R., 1999. The influence of particle breakage and state on the behavior of sands. *Proceedings of the Int. Worshop on Soil Crushability*, Yamaguchi, Japan, pp. 19-57.

Cundall, P.A., and Strack, O.D.L., 1979. A discrete numerical model for granular Assemblie. *Geotechnique*, Vol. 29, No. 1, pp. 47-65.

Hanzen. A. 1911. Discussion of: Dams on sand foundations by A.C. Koening. *Transactions of ASCE*, New York, Vol. 73.

Hyslip, J.P., and Vallejo, L.E., 1997. Fractal analysis of the roughness and size distribution of granular materials. *Engineering Geology*, Vol. 48, No. 3-4, pp. 231- 244.

Lade, P.V., Yamamuro, J.A., and Bopp, P.A., 1996. Significance of particle crushing in granular materials. *J. of Geotechnical Eng., ASCE*, Vol. 122, No. 4, pp. 309-316.
Lee, K.L., and Farhoomand, J., 1967. Compressibility and crushing of granular soils in anisotropic triaxial compression. *Canadian Geotechnical J.*, Vol. 4, No. 1, pp. 68-86.
Mandelbrot, B.B., 1977. *Fractals: forms, chance and dimension.* San Francisco, Freeman.
McDowell,G.R., Bolton, M.D., and Robertson, D., 1996. The fractal crushing of granular materials. *Int. J. of Mechanics and Physics of Solids*, Vol. 44, No. 12, pp. 2079-2102.
Ramamurthy, T., 1968. Crushing phenomena in granular soils. *The Journal of the Indian National Society of Soil Mech. and Found. Eng.,* Vol.8, No. 1, pp. 67-86.
Raymond, G.P., 2000. Track and support for a mine company railroad. *Can. Geotech. J.*, 37:318-332.
Tyler, S.W., and Wheatcraft, S.W., 1992. Fractal scaling of soil particle-size distibution analysis and limitations. *Soil Science Society of America Journal,* 56 (2): 47-67.
Vallejo, L.E., 1995. Fractal analysis of granular materials. *Geotechnique*, 45, (1): 159- 164.
Vallejo, L.E., 1996. Fractal analysis of the fabric changes in a consolidating clay. *Engineering Geology*, 43: 281-290.
Vallejo, L.E., 2001. Fractal assessment of the surface texture of pavements. *Intern. J. of Pavement Engineering,* 2(2): 149-156.
Yeggoni, M., Button, J.W., and Zollinger, D.G., 1996. Fractals of aggregates correlated with creep in asphalt concrete. *J. of Transp. Eng*., ASCE, 122(1): 22-27.