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Finite element modeling of stone columns – a case history

Modèle d'élément fini pour les colonnes de pierre - un cas d'histoire

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ABSTRACT
Soil reinforced by means of stone columns can be analyzed by two dimensional finite element method. This paper introduces a conversion scheme for stone column unit cell from axisymmetric model into equivalent plane strain model. A stone column reinforced embankment system in Malaysia is back analyzed and results are compared against the field measurement data. The findings show good agreement between the FEM results and field measurements.

RÉSUMÉ
Le sol renforcé au moyen de colonnes de pierre peut être analysé par la méthode d'éléments finis a deux dimensions. Cet article introduit un plan de conversion du cellule unitaire de colonne de pierre du modele axisymmetrique au modele à déformation plane equivalent. Un système de barrage renforcée par des colonnes de pierre en Malaisie est analysée et les résultats sont comparés contre le données des mesures de champ. Les conclusions montrent le bon accord entre les résulte des analyses d'éléments finis et les mesures de champ.

1 INTRODUCTION

Stone columns are widely used to improve the soft foundation soil. Their core purpose is to increase the bearing capacity and to accelerate the consolidation rate of the foundation soil. This paper deals with the finite element modeling of stone column under the embankment system. The objective of the paper is to illustrate that stone column reinforced foundation can be modeled by two dimensional finite element method.

Axisymmetric unit cell model can be used for the analysis of a single stone column in FEM. However, to simulate the stone column treated foundation, equivalent plane strain unit cell must be idealized for two dimensional FEM analysis. Since three-dimensional numerical analyses are complex and time consuming, it is desirable to model by two dimensional procedures. The proposed conversion scheme is based on simplified analytical method for stone column reinforced foundation by Han & Ye (2001) and considerations of composite stiffness of the unit cell and area replacement ratio. Two dimensional FEM model of stone column can be formulated incorporating the parameters derived from the conversion scheme.

2 THE STONE COLUMN UNIT CELL

Based on Barron's drain well theory and consolidation in radial direction, the stone column model can be idealized as an axisymmetric unit cell model. The stone column unit cell consists of stone column in the center of the cell surrounded by soil until the end of the circular influence zone. It is assumed that the stone column unit cell obey the equal vertical strain theory when the load is applied. The effects of smear and well resistance are beyond the scope of this study. Han & Ye (2001) reported the simplified analytical equation of stone column performance considering the reduction rate of cylindrical soil volume due to the dissipation of pore water pressure in a unit cell as :

$$c_r'\left(\frac{1}{r}\frac{\partial u}{\partial r}+\frac{\partial^2 u}{\partial r^2}\right)+c_v'\frac{\partial^2 u}{\partial z^2}=\frac{\partial \bar{u}}{\partial t} \quad (1)$$

Where $c_r'=(k_r/\gamma_w)[m_{v,c}(1-a_s)+m_{v,s}a_s]/[m_{v,s}m_{v,c}(1-a_s)]$, a modified composite coefficient of consolidation in radial direction and $c_v'=(k_v/\gamma_w)[m_{v,c}(1-a_s)+m_{v,s}a_s]/[m_{v,s}m_{v,c}(1-a_s)]$, a modified composite coefficient of consolidation in vertical direction. The area replacement ratio of stone column and surrounding soil can be expressed as $a_s=A_{col}/(A_{col}+A_{soil})$. The coefficient of volume compressibility in the vertical direction of the stone and surrounding soil can be computed by

$$m_v=\frac{(1+\nu)(1-2\nu)}{E(1-\nu)} \quad (2)$$

Where, E= Elastic Modulus and ν = Poisson ratio

The important function of stone column is to remove the excess pore water pressure from the surrounding soil through its column drainage. When the load is applied, dominant radial flow occurs towards the stone column which is more significant with the consideration of equal vertical strain condition. Therefore, it is reasonable that consolidation due to radial flow is more of a controlling factor with stone columns. The average rate of consolidation in the radial direction can be given by:

$$U_r=1-\exp^{-[8/F(N)]T_r'} \quad (3)$$

Where,

$$F(N)=[N^2/(N^2-1)]\ln(N)-(3N^2-1)/(4N^2)$$

$$T_r'=c_r' t/d_e^2$$

3 FEM ANALYSIS OF STONE COLUMN

3.1 *Conversion of axisymmetric model into equivalent plane strain model*

In order to model the stone column treated ground in two dimensional numerical program, the relevant plane strain parameters are to be provided. Thus, the conventional axisymmetric stone column unit cell must be transformed into equivalent plane strain unit cell. The most significant nature of stone column unit cell is its composite material attribute. In reality, the composite stiffness of the stone column treated ground will be the same for both axisymmetric and plane strain cases. Therefore, the assumption of the same composite stiffness for both types of model is valid.

The stone column and the surrounding soil have different material properties and stiffness. The area replacement ratio of stone column in the plane strain model will differ from that in the axisymmetric model by assuming the diameter of the stone column and the width of the stone wall in the plane strain condition are considered the same. In the case of axisymmetric model, we need to take the geometry dependent constants for stone column arrangement into consideration. Considering the construction nature of stone columns and simplicity for calculation, the stone wall thickness of the plane strain model can be considered the same as the stone column diameter. Thus, in transforming a stone column unit cell from axisymmetric model into plane strain model, it is obvious that the area replacement of stone column in a unit cell differs from axisymmetric model to plane strain model as can be seen in Figure 1.

3.2 *Stiffness Matching Procedure*

The composite stiffness of the axisymmetric unit cell is calculated by using the stiffness of the stone column and stiffness of the surrounding soil with respective area replacement ratio. The following relation is applied.

$$E_{composite} = E_{col,axi} * a_{s,axi} + E_{soil,axi} * (1 - a_{s,axi}) \quad (4)$$

Where, $a_s = A_{col} / (A_{col} + A_{soil})$; area replacement ratio,
$E_{composite}$; stiffness of the composite material in the unit cell,
$E_{col,axi}$; Stiffness of axisymmetric stone column material,
$E_{soil,axi}$; Stiffness of the surrounding soil in the axisymmetric unit cell.

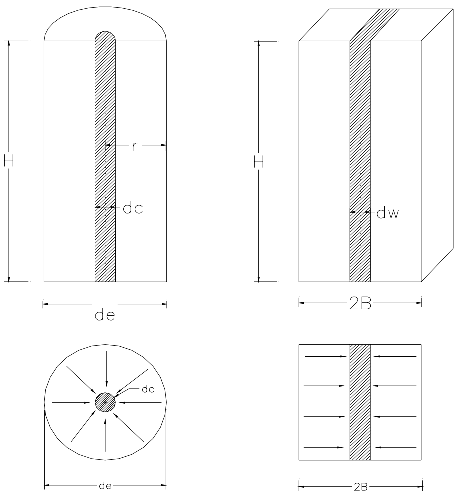

Figure 1. Conversion from Axisymmetric Unit Cell into Equivalent Plane Strain.

The stiffness of the composite ground treated with stone columns can well be represented by the obtained composite stiffness. In order to convert the condition of axisymmetry into plane strain, the composite stiffness obtained for actual axisymmetric condition must be applied for the equivalent plane strain condition. In addition, the area replacement ratio for the plane strain model must be computed again using plane strain unit cell geometry.

For the proposed stiffness matching scheme, the composite stiffness for axisymmetric model and plane strain model must be equal. With the increase in area of stone in a plane strain unit cell model, the stress conditions in the stone wall and the surrounding soil have changed. It is therefore necessary to adjust the stiffness of the surrounding soil and stonewall in the plane strain unit cell. The plane strain stone column unit cell was idealized using the same stiffness value for soil as in the axisymmetric model (i.e. $E_{soil,axi} = E_{soil,pl}$) while adjusting the equivalent stone wall stiffness by using following equation;

$$E_{composite} = E_{col,pl} * a_{s,pl} + E_{soil,pl} * (1 - a_{s,pl}) \quad (5)$$

Where, $a_s = A_{col} / (A_{col} + A_{soil})$; area replacement ratio, $E_{composite}$; stiffness of the composite material in the unit cell, $E_{col,pl}$; Stiffness of axisymmetric stone wall material, $E_{soil,pl}$; Stiffness of the surrounding soil in the plane strain unit cell.

3.3 *Permeability Matching Procedure*

Considering the equality of the average degree of consolidation in the axisymmetric and equivalent plane strain unit cells, Hird et al.,(1992) suggests

$$\overline{U}_{hpl} = \overline{U}_{hax} \quad (6)$$

In the proposed matching procedure, the axisymmetric and plane strain unit cells are considered ideally representing the same area of treated ground. Therefore, the required equivalent permeability for the stone column plane strain unit cell can be computed by Eqn (3) and (6). Thus, the relation becomes:

$$\frac{T'_{hpl}}{F(N)_{pl}} = \frac{T'_{hax}}{F(N)_{ax}} \quad (7)$$

By considering the field nature of same representing geometry and the same degree of consolidation at every time, Eqn (8) can be further simplified to:

$$\frac{c'_{hpl}}{F(N)_{pl}} = \frac{c'_{hax}}{F(N)_{ax}} \quad (8)$$

By employing the proposed stiffness matching scheme for the respective coefficients of volume compressibility for stone column and surrounding soil as well as different area replacement ratio for axisymmetric and equivalent plane strain unit cells, the permeability relation for the axisymmetric to the equivalent plane strain can be estimated as:

$$\frac{k_{hpl}}{k_{hax}} = \frac{F(N)_{pl}}{F(N)_{ax}} \left[\frac{m_{v,s} m_{v,c} (1 - a_s)}{m_{v,c}(1 - a_s) + m_{v,s} a_s} \right]_{pl} \left[\frac{m_{v,c}(1 - a_s) + m_{v,s} a_s}{m_{v,s} m_{v,c} (1 - a_s)} \right]_{ax} \frac{B^2}{R^2} \quad (9)$$

Where,
$F(N) = [N^2/(N^2-1)]\ln(N) - (3N^2-1)/(4N^2)$ with $N = \frac{d_e}{d_c}$ for axisymmetric case and $N = \frac{2B}{d_w}$ for plane strain case, respectively.

3.4 *Introduction of area reduction ratio for stone column*

The behavior of soil is sometime non-linear in nature, especially for the soft soils. Therefore, the proposed matching scheme for conversion of axisymmetric model into equivalent plane strain model needs to be validated for such non-linear soil models. In doing so, the advanced constitutive soil model can be used for the finite element analyses of both axisymmetric and plane strain unit cells and compare the results. Since the advanced soil models consider the non-linear behavior of soil and the stress-dependent behavior of soil, the utilization of advanced constitutive models needs to consider the adjustment of the soil stiffness in plane strain condition from the axisymmetric condition.

The area reduction ratio is considered to have impact on the determination of stiffness of soft soil in the plane strain model. The relation of the area reduction ratio for the surrounding soil due to the conversion of axisymmetric into plane strain model can be considered with respect to area replacement ratios as:

$$A_{red} = \frac{(1-a_s)_{pl}}{(1-a_s)_{ax}} \quad (10)$$

Incorporating the area reduction ratio for plane strain unit cell (A_{red}), the stiffness of soft soil in the plane strain model is modified for the proposed stiffness matching procedure as:

$$E_{soil,pl} = A_{red} * E_{soil,axi} \quad (11)$$

Where, A_{red} = Area reduction ratio due to conversion of axisymmetric model into plane strain model

3.5 *FEM Analysis of the unit cells*

The FEM models of stone column unit cells in axisymmetric and plane strain conditions were created using the 15-node triangular elements. A rigid plate element is utilized to comply with the equal vertical strain theory.

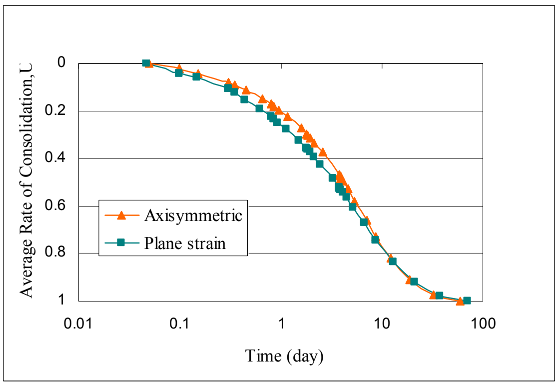

Figure 2. Comparison of average rate of consolidation.

The FEM simulation with Plaxis for the axisymmetric unit cell model and equivalent plane strain unit cell under linear elastic condition were carried out incorporating the stiffness matching and permeability matching schemes. However, area reduction ratio is not necessary to apply for the simulation with linear elastic models.

The comparison result from the axisymmetric model and the equivalent plane strain model on the rate of consolidation is shown in Figure. 2. The results suggest that the proposed matching scheme for plane strain model produces good representations of the actual axisymmetric model.

In order to monitor the stone column performance in advanced constitutive soil models, Soft Soil model (like Cam-Clay model) is used to represent the surrounding soft soil and simulation was performed. Taking the proposed conversion scheme incorporating the area reduction ratio for advanced constitutive soil model into account, The parameters used for Soft Soil model such as Modified Compression Index (λ^*), Modified Swelling Index (K^*) can be obtained from the following relations:

$$\lambda^* = \frac{C_c}{\ln 10(1+e)} = \frac{C_c}{2.3(1+e)} \quad (12)$$

$$K^* \approx \frac{\lambda^*}{5} \quad (13)$$

$$E_{oed} \approx \frac{2.3(1+e_0)\sigma'_y}{C_c} \quad (14)$$

$$E_{oed} = \frac{(1-\nu)E}{(1-2\nu)(1+\nu)} \quad (15)$$

Where, σ'_y = Average vertical stress in soil; C_c = Parameter of one-dimensional compression; e_0 = Initial void ratio and ν = Poisson's ratio.

The results on rate of consolidation using advanced constitutive model are compared from the axisymmetric and equivalent plane strain models. Good agreement was obtained between the two models as can be seen in Figure. 3.

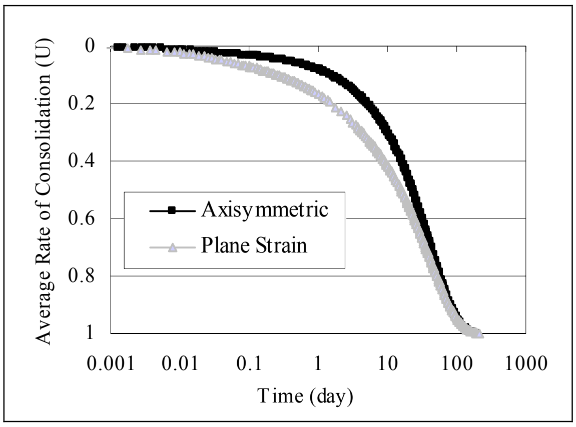

Figure 3. Comparison of average rate of consolidation using advanced constitutive soil model.

4 CASE HISTORY- STONE COLUMN WORK AT NEW PANTAI EXPRESSWAY EMBANKMENT

The test embankment, the Penchala Toll Plaza project at New Pantai Expressway, in Malaysia faced the situation in which soft clay deposits under the embankments would cause excessive settlements for heavy transportation utilities. Stone column reinforced foundation was chosen as the most appropriate method to improve the soft ground. The back analysis using FEM was carried out for the stone column improved ground for settlement.

At the test embankment section, the embankment and the stone column work are symmetrical about the centerline. Thus, only half of the embankment needs to be modeled. 15-node triangular elements are used in the finite element mesh and the mesh refinement is done for the stone column treated area for better accuracy. The clay soil is modeled as Soft Soil model and the other materials are modeled as elastic perfectly-plastic Mohr-Coulomb model. The stone columns installed under the embankment are modeled with equivalent plane strain properties.

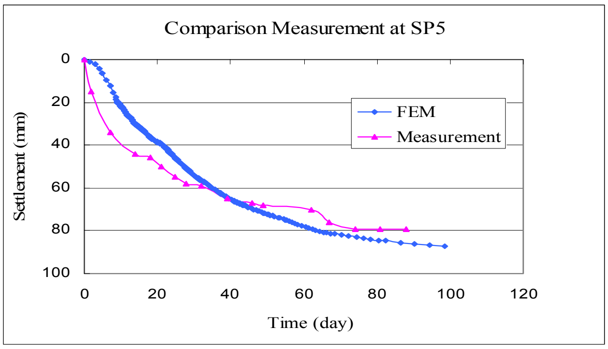

Figure 4. Settlements at the Centerline of Embankment.

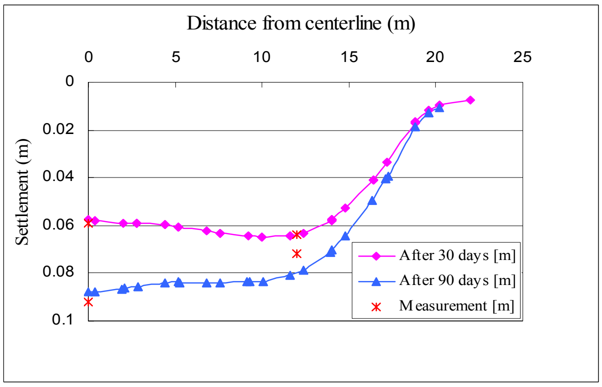

Figure 5. Surface Settlement of the Embankment.

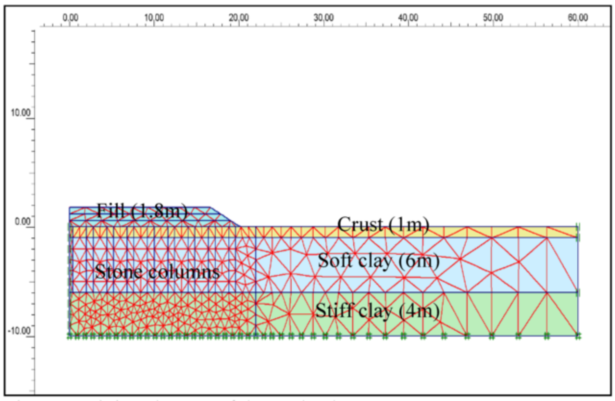

Figure 6. Finite Element of the Embankment.

The consolidation analysis was performed and the results are compared against the field measurements at the section. The field settlement measurement results were obtained from the periodic monitoring of the settlement plates under the embankment. The comparison of settlements at the centerline and the surface settlement at the section are presented in Figures. 4 and 5 respectively. The finite element mesh of the section is shown in Figure. 6. Table 1 lists the soil parameters used in the analysis of the stone column treated embankment.

Table 1. Parameters used for the Embankment Section

Parameter	Fill	Crust	Soft clay	Stiff clay	Stone Column
Material model	Mohr Coulomb	Mohr Coulomb	Soft soil	Mohr Coulomb	Mohr Coulomb
Type of behavior	Undrained	Undrained	Undrained	Undrained	Drained
γ_{unsat} (kN/m^3)	18	17	15	18	19
γ_{sat} (kN/m^3)	20	18	15	20	20
k_h (m/day)	1	0.03	3.00E-04	3.00E-04	10
k_v (m/day)	1	0.01	1.00E-04	1.00E-04	10
ν'	0.3	0.3	0.3	0.3	0.3
E_{ref} (kN/m^2)	15000	15000	-	40000	30000
c_{ref} (kN/m^2)	19	3	1	3	5
ϕ'	26	28	20	30	40
λ^*	-	-	0.12	-	-
κ^*	-	-	0.024	-	-
R_{inter}	1	1	1	1	1

5 CONCLUSIONS

The major findings that can be drawn from this study can be summarized as follows:

The stone column reinforced ground under the embankment can be analyzed by two-dimensional finite element method using the plane strain condition. In order to do this, one needs to transform the axisymmetric nature of stone column into equivalent plane strain condition. Proposed conversion scheme can be used to transform the axisymmetric stone column unit cell into equivalent plane strain unit cell. It is observed that the stiffness and area replacement ratio play a vital role in the conversion process.

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