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Simple and accurate prediction of settlements of stone column reinforced soil

Prévision simple et rationnelle des travaux d'amélioration des sols par des colonnes ballastées

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ABSTRACT

A new analytical method is proposed to analyse the behaviour of rigid foundations resting on soft soil stabilised by a large number of end bearing stone-columns. The stone column and the surrounding soil are treated in axial symmetric conditions as a "unit cell". The stone column is assumed to behave as an elasto-plastic Mohr-Coulomb material with non-associative flow rule and the soil as an elastic material. These assumptions, combined with equilibrium and kinematic conditions, lead to the simple analytical closed-form solution for the prediction of the behaviour for rigid footings resting on stone-column reinforced ground. The results of the proposed method are in excellent agreement with the results of the elasto-plastic finite element method analysis.

RÉSUMÉ

Une nouvelle méthode analytique est proposée pour étudier le comportement des fondations rigides qui reposent sur le sol mou stabilisé par des colonnes ballastées qui sont fondées sur la base du sol solide. La colonne ballastée et le sol autour d'elle sont traités dans les conditions axiales symétriques comme une cellule de base. Il est supposé que la colonne ballastée se comporte comme la matière élasto-plastique avec un critère d'élasticité Mohr-Coulomb non associé et le sol comme matière élastique. Ces suppositions, combinées avec les conditions d'équilibre et les conditions cinématiques, mènent à la solution analytique simple de forme fermée pour prévoir le comportement des fondations rigides qui reposent sur le sol renforcé par les colonnes ballastées. Les résultats de la méthode proposée sont en excellent accord avec les résultats de l'analyse élasto-plastique selon la méthode des éléments finis.

1 INTRODUCTION

Granular piles formed in soft clay soils using various techniques have been used effectively to reduce settlements and increase the rate of settlements of large raft foundations and embankments. In this paper an analysis of final settlement of widespread rigid foundation supported by large regular array of end bearing granular piles is considered. Several analytical methods are available to estimate the settlement reduction due to stone-columns. Many of them (Aboshi et al., 1979, Balaam and Booker, 1981) are based on elastic approach, considering the stone-column and the surrounding soil as elastic "unit cell" under axi-symmetric conditions. Several authors (Impe and De Beer, 1983, Balaam and Booker, 1985, Impe and Madhav, 1992) found out that the elastic analysis can highly overestimate the effect of stone-columns on settlement reduction due to the plastic deformations of stone-column. Thus, analytical methods based on rigid-plastic behaviour of the stone-column have been suggested for engineering purposes.

In this paper the elastic approach developed by Balaam and Booker (1981) is extended to take into account confined yielding of the frictional column material according to the dilatancy theory (Rowe, 1962). Column spacing, initial stresses in the ground, stone and soil material properties, column length, applied load and dilatancy of the stone column material are taken into account to obtain the closed form solution which gives rational predictions of the settlements of rigid foundations supported by stone columns.

2 METHOD OF ANALYSIS

If stone-columns are regularly distributed, a regularly shaped area around the stone-column may be considered as a "unit cell", consisting of stone-column and the surrounding soft soil in a zone of influence (Fig. 1). To simplify the analysis the zone of influence is approximated by a circle with a diameter d_e equal to 1.05*s*, 1.13*s* and 1.29*s*, for triangular, square or hexagonal patterns respectively, where *s* is the column spacing. The column spacing ratio is defined as d_e/d_c. The ratio between the area of column A_c and the area of the zone of influence A_e is represented by the replacement ratio $A_r = A_c/A_e = (d_e/d_c)^{-2}$.

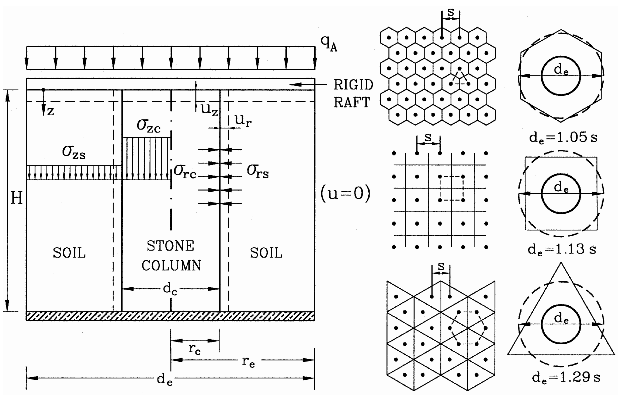

Figure 1: Basic features of the model based on regular patterns of stone-columns

The high drainage capacity of the column material ensures that it deforms under drained conditions. If the soil is considered incompressible, then the immediate settlement is negligible compared to the total final settlements (Balaam and Booker, 1985). For this reason it will not be considered in the paper.

2.1 *Elastic solution*

The analytical solution for the elastic response of the "unit cell" was presented by Balaam and Booker (1985). The relation between the applied load q_A and the elastic vertical strain ε_z^{el} is given by the expression

$$q_A = \left[(\lambda_c + 2G_c)A_r + (\lambda_s + 2G_s)(1-A_r) - 2A_r(\lambda_c - \lambda_s)F\right]\varepsilon_z^{el} \quad (1)$$

where F is a constant defined as

$$F = \frac{(\lambda_c - \lambda_s)(1-A_r)}{2\left[A_r(\lambda_s + G_s - \lambda_c - G_c) + (\lambda_c + G_c + G_s)\right]} \quad (2)$$

Lamé's parameters are defined as

$$\lambda = \frac{\nu\, E}{(1-2\nu)(1+\nu)}, \; G = \frac{E}{2(1+\nu)} \quad (3)$$

where E_c, E_s and ν_c, ν_s are Young's modulus and Poisson's ratios for the stone column and for the clay, respectively. The vertical and radial stress in the column, $\Delta\sigma_{zc}$ and $\Delta\sigma_{rc}$, which are of further interest, are

$$\Delta\sigma_{zc} = \left[\lambda_c + 2G_c - 2\lambda_c F)F\right]\varepsilon_z^{el} \quad (4)$$

$$\Delta\sigma_{rc} = \left[\lambda_c - 2(\lambda_c + G_c)F\right]\varepsilon_z^{el} \quad (5)$$

Finaly, the settlement reduction factor β^{el}, which is usually used as a measure of the improvement of the ground, can be defined as the ratio of elastic settlements of the treated to untreated ground under widespread load

$$\beta^{el} = \frac{E_{oed}}{(\lambda_c + 2G_c)A_r + (\lambda_s + 2G_s)(1-A_r) - 2A_r(\lambda_c - \lambda_s)F} \quad (6)$$

where E_{oed} is the edometric modulus of the soil.

From the elastic response, it can be found that there may be significant yielding of the stone column due to high stress ratio, but little yield in the clay. According to Balaam and Booker (1985), the problem can be idealized by assuming that the stone column is in triaxial state and probably yielding, that there is no shear stress at the stone-soil interface and the soil remains in the elastic state.

2.2 *Elasto-plastic solution*

Since it has been assumed that no yielding occurs in the clay, the response of the clay will be elastic throughout the range of applied load. It is assumed that the column material is perfectly elastic-plastic material satisfying the Mohr-Coulomb yield criterion with constant dilatancy angle ψ.

Consider a thin horizontal slice of the "unit cell" at a selected depth z before any load is applied on the ground surface. Let the effective vertical stress in the column at the depth z be equal to $\sigma_{cs} = \gamma_c' z$ and let the vertical effective stress in the soil be equal to $\sigma_{vs} = \gamma'_s z$, where γ_c' and γ_s' are the effective unit weights for the column material and the soil, respectively. Assume that the radial stress at the soil-column interface is equal to $\sigma_{rc} = \sigma_{rs} = K_{ini}\,\gamma_s' z$, where K_{ini} is the lateral earth pressure coefficient. The yield condition for the stone column under applied load, the initial response of which will be elastic, is given by the expression

$$\frac{\gamma_c' z + \Delta\sigma_{zc}}{K_{ini}\,\gamma_s' z + \Delta\sigma_{rc}} = \frac{1+\sin\varphi_c}{1-\sin\varphi_c} = K_{pc} \quad (7)$$

By taking $\Delta\sigma_{zc}$ and $\Delta\sigma_{rc}$ as elastic stress increments caused by the applied load q_A, the equation (7) can be rearranged to get the yield load q^y for the selected depth z

$$q^y = q^y(z) = C_0\; \gamma_s' z \quad (8)$$

The constant C_0 depends on the soil and the column material parameters and is defined as

$$C_0 = (K_{pc}K_{ini} - \mu)\cdot \frac{\left[(\lambda_c + 2G_c)A_r + (\lambda_s + 2G_s)(1-A_r) - 2A_r(\lambda_c - \lambda_s)F\right]}{2G_c(1+F\,K_{pc}) - \lambda_c(1-2F)(1-K_{pc})} \quad (9)$$

where μ represents the ratio of the effective column unit weight to the soil unit weight $\mu = \gamma_c'/\gamma_s'$. The elastic vertical deformation ε_z^y under yield load q^y at the selected depth z can be obtained as

$$\varepsilon_z^y = \varepsilon_z^y(z) = \frac{(K_{pc}K_{ini} - \mu)\gamma_s' z}{2G_c(1+F\,K_{pc}) - \lambda_c(1-2F)(1-K_{pc})} \quad (10)$$

If the applied load q_A is greater than the yield load q^y for the selected depth z, then the column will yield. Once yielding, the yield criterion must also be satisfied for the column vertical and radial stress increments, $\Delta\sigma_{zc}^p$ and $\Delta\sigma_{rc}^p$, caused by the load difference $q^p = q_A - q^y$, so

$$\frac{\Delta\sigma_{zc}^p}{\Delta\sigma_{rc}^p} = \frac{1+\sin\varphi_c}{1-\sin\varphi_c} = K_{pc} \quad (11)$$

Index *p* denotes the share of stresses and strains, which are caused by load q^p. The ratio between vertical and horizontal stresses in the yielding column is completely defined by the strength of the column material (Eq. 11). Similarly, the ratio between the contained plastic volumetric strain ε_v^p and plastic vertical strain ε_z^p due to dilation of the column are related to the dilatancy angle (Schanz and Vermeer, 1996)

$$\sin\psi = -\frac{\varepsilon_v^p}{2\varepsilon_z^p - \varepsilon_v^p} \quad (12)$$

where plastic volumetric strain of the column ε_v^p is defined as

$$\varepsilon_v^p = \varepsilon_z^p + 2\varepsilon_r^p \quad (13)$$

The soil surrounding the stone-column can be analysed as an elastic cylinder using equations relating vertical and radial strains, ε_z^p and ε_r^p at the soil-column interface with the vertical stress in the soil and the radial interface stress, σ_{zs}^p and σ_{rs}^p (Poulus and Davis, 1974)

$$\varepsilon_z^p = \frac{1}{E_{oed}}\left[\frac{C_2\sigma_{zs}^p - C_1\sigma_{rs}^p}{C_3}\right] \quad (14)$$

$$\varepsilon_r^p = \frac{1}{E_{oed}}\left[\frac{\sigma_{rs}^p - k_0\sigma_{zs}^p}{C_3}\right] \quad (15)$$

where E_{oed} is the oedometric modulus of the soil and C_1, C_2 and C_3 are constants defined as

$$C_1 = \frac{2k_0A_r}{1-A_r}, \quad C_2 = \frac{1-2\nu_s + A_r}{(1-A_r)(1-\nu_s)}, \quad C_3 = C_2 - k_0\,C_1 \quad (16)$$

where $k_0 = \nu_s/(1-\nu_s)$.

The vertical stresses in the column and the soil caused by the vertical load $q^p = q_A - q^y$ must satisfy the equilibrium

$$q^p = \sigma_{zc}^p A_r + \sigma_{zs}^p(1-A_r) \quad (17)$$

The stresses at the soil–column interface must be equal, thus $\sigma_{rc}^p = \sigma_{rs}^p = \sigma_r^p$. Equations (11), (12), (14), (15) and (17) represent a set of five equations for five unknowns: vertical stresses in the column σ_{zc}^p and in the soil σ_{zs}^p, radial stress at the soil-column interface σ_r^p, vertical strain ε_z^p and interface strain ε_r^p. This set of equations can be solved to obtain simple analytical closed-form solutions for strains and stresses caused by vertical load $q^p = q_A - q^y$. The expression for the vertical strain is

$$\varepsilon_z^p = \frac{2\, q^p}{E_{oed}\, C_4} \tag{18}$$

where constants C_4 and K_ψ are defined as follows

$$C_4 = (1 - A_r)(C_1 K_\psi + 2) + A_r K_{pc}(C_2 K_\psi + 2k_0) \tag{19}$$

$$K_\psi = \frac{1 + \sin\psi}{1 - \sin\psi} \tag{20}$$

If the area of the applied load is sufficiently large then the vertical strain of the untreated soil can be estimated as

$$\varepsilon_{z,0}^p = \frac{q^p}{E_{oed}} \tag{21}$$

Combining equation (18) with equation (21), a settlement reduction factor β^p can be calculated as

$$\beta^p = \frac{\varepsilon_z^p}{\varepsilon_{z,0}^p} = \frac{2}{C_4} \tag{22}$$

Now let us examine the behaviour of the entire “unit cell” under applied load q_A. The yield of the stone column will start at the soil surface (8) and will reach the final yield depth z^y given by the expression

$$z^y = \frac{q_A}{C_o\, \gamma_s'} \tag{23}$$

For any depth $z \leq z^y$, the applied load qA is greater than the yield load qy and the total vertical strain can is given by

$$\varepsilon_z(z) = \varepsilon_z^y(z) + \varepsilon_z^p(z) = \\ = \frac{(K_{pc} K_{ini} - \mu)\gamma_s' z}{2G_c(1 + F K_{pc}) - \lambda_c(1 - 2F)(1 - K_{pc})} + \frac{2(q_A - C_0\, \gamma_s' z)}{E_{oed}\, C_4} \tag{24}$$

At depths $z > z^y$ the soil and the column will both remain in the elastic state and the elastic vertical strain (Eq. 1) is given by

$$\varepsilon_z^{el} = \frac{q_A}{(\lambda_c + 2G_c)A_r + (\lambda_s + 2G_s)(1 - A_r) - 2A_r(\lambda_c - \lambda_s)F} \tag{25}$$

The settlement of the treated ground is obtained with the integration of vertical strains over the entire length of the column H. If the yield depth z^y is greater than the length of column H, then the total settlement is obtained as

$$u_z = \int_0^H \left(\varepsilon_z^y(z) + \varepsilon_z^p(z)\right) dz \tag{26}$$

otherwise, the total settlement is obtained as

$$u_z = \int_0^{z^y} \left(\varepsilon_z^y(z) + \varepsilon_z^p(z)\right) dz + \int_{z^y}^H \varepsilon_z^{el}\, dz \tag{27}$$

If the total settlement of the treated ground u_z is divided by the total settlement of the untreated ground, then the final settlement reduction factor β can be obtained for two distinctive cases. For the case $z^y \leq H$ we get

$$\beta = \beta^{el}\left(1 - \frac{q_A}{2C_0 H \gamma_s'}\right) + \beta^p\left(\frac{q_A}{2C_0 H \gamma_s'}\right) \tag{28}$$

and for case $z^y \leq H$

$$\beta = \beta^{el}\left(\frac{C_0 H \gamma_s'}{2q_A}\right) + \beta^p\left(1 - \frac{C_0 H \gamma_s'}{2q_A}\right) \tag{29}$$

3 VALIDITY OF THE METHOD

The method is based on several assumptions. To test the validity of the method elasto-plastic finite element analyses were made. It was assumed that the base and the foundation are perfectly rough. In the finite element analyses both materials are treated as nondilatant ($\psi = 0$) perfectly elasto-plastic materials according to the Mohr-Coulomb yield criterion.

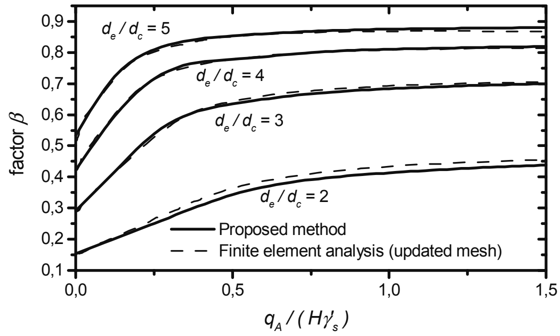

Figure 2. Comparison between the finite element analysis and the proposed method

The analyses were made for four different spacings d_e/d_c = 2, 3, 4 and 5. The column length to diameter ratio H/d_c = 10, the modulus ratio E_c/E_s = 30, the unit weight ratio μ = 1.5, the column shear angle φ_c = 40° and the soil shear angle φ_s = 20° were adopted for the analyses. Initial stresses were generated with K_0 procedure for the proposed value of lateral stress coefficient $K_0 = K_{ini}$ = 0.6. The maximum applied load in the analyses was $q_A = 0.05 E_{oed}$ and the maximum dimensionless load factor was $q_A/(H\gamma_s') = 1.5$.

Although in the proposed analytical solution the soil is considered as an elastic material and the shear stresses along the column soil interface are neglected, the settlement reduction factors β are in excellent agreement with the finite element results throughout the applied load (Fig. 2).

4 PARAMETRIC STUDY

The results show that the most important parameters affecting the settlement of the stabilized ground are the column spacing ratio d_e/d_c, the dimensionless load factor $q_A/(H\gamma_s')$, the friction angle of the stone column φ_c, the dilatancy angle ψ, the modulus ratio E_c/E_s, the initial lateral earth pressure coefficient K_{ini} and Poisson’s ratio of the soil ν_s. The effects of the most important factors on the settlement reduction are presented in Figures 3 to 7.

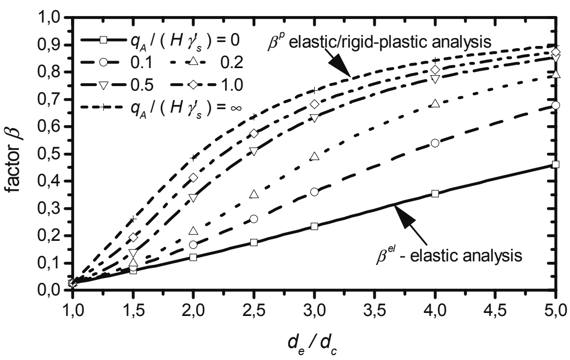

Figure 3. Effect of load factor on the settlement reduction factor β

The settlement reduction factor β against column spacing ratio d_e/d_c for different dimensionless load factor $q/(H\gamma_s')$ is shown in Figure 3. The value of modulus ratio E_c/E_s = 40, the friction angle φ_c = 40°, the dilatancy angle ψ = 0, the lateral earth pressure coefficient K_{ini} = 0.6 and the unit weight ratio μ = 1.5 were adopted for the analyses. The assumed Poisson's ratio of the column and soil was $\nu_c = \nu_s = 0.3$.

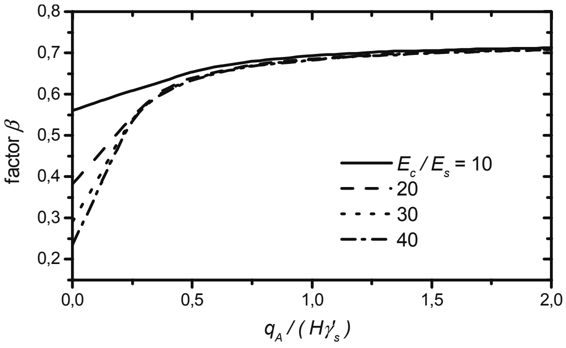

Figure 4. Effect of modulus ratio E_c/E_s on settlement reduction factor β for $d_e/d_c = 3$, $\varphi_c = 40°$, $\psi = 0°$, $K_{ini} = 0.6$, $\mu = 1.5$

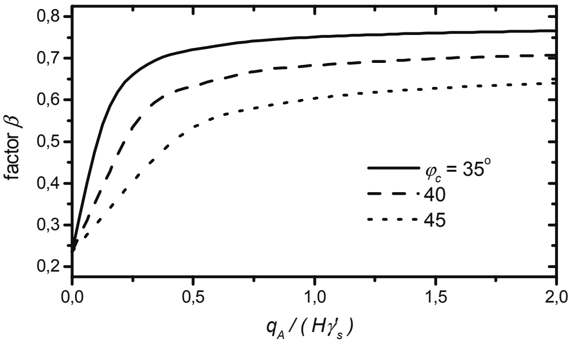

Figure 5. Effect of friction angle φ_c on the settlement reduction factor β for $d_e/d_c = 3$, $E_c/E_s = 40$, $\psi = 0°$, $\nu_c = \nu_s = 0.3$, $K_{ini} = 0.6$, $\mu = 1.5$

The column spacing ratio d_e/d_c has a dominating effect on the settlement reduction. The effect of dimensionless load factor is only important for $q_A/(H\gamma'_s) < 0.5$ and for low modulus ratios E_c/E_s, as can be seen in Figure 4, where the settlement reduction factor β is plotted against dimensionless load factor $q_A/(H\gamma'_s)$ for modulus ratios E_c/E_s =10, 20, 30 and 40.

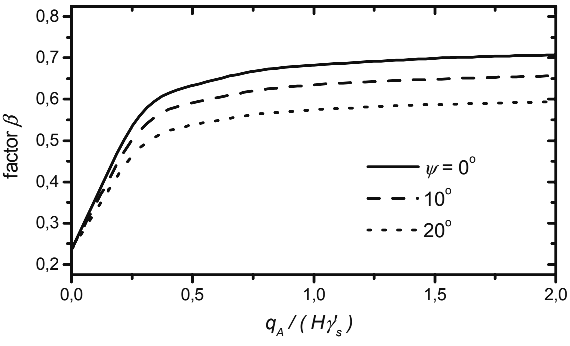

Figure 6. Effect of dilatancy on the settlement reduction factor β for $d_e/d_c = 3$, $E_c/E_s = 40$, $\varphi_c = 40°$, $\nu_c = \nu_s = 0.3$, $K_{ini} = 0.6$, $\mu = 1.5$

If the column is long and the applied load is small compared to the effective stresses at the column bottom, then most of the column will stay in the elastic state having significant effect on the settlement reduction. When the dimensionless load factor reaches $q_A/(H\gamma'_s) \geq 0.5$, then most of the column will be in the plastic state and the initial influence of modulus ratio E_c/E_s on the settlement reduction will become negligible. The effects of friction angle φ_c and dilatancy angle ψ of column material on settlement reduction are shown in Figure 5 and Figure 6. It is obvious that the stone column with high friction and dilatancy angle has greater ability to reduce settlements.

Initial lateral stresses in the "unit cell" depend greatly on the stone-column installation technique. The positive effect of high lateral stress coefficient K_{ini} on the settlement reduction factor β is shown in Figure 7.

The important feature of the method is that for a given set of soil parameters (E_s, ν_s, γ'_s) and column material parameters (E_c, ν_c, γ'_c, ψ) the settlement reduction factor β depends only on column spacing ratio d_e/d_c, on the lateral earth pressure coefficient K_{ini} and on the dimensionless factor $q_A/(H\gamma'_s)$.

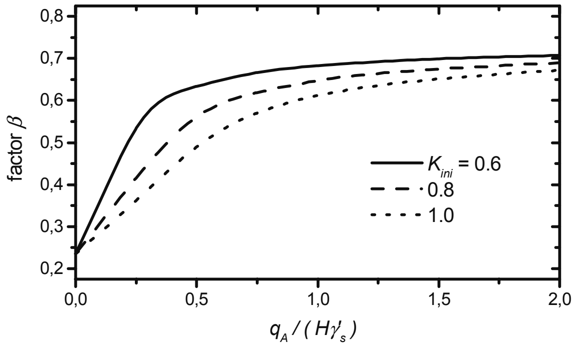

Figure 7. Effect of lateral stress coefficient K_{ini} on the reduction factor β for $d_e/d_c = 3$, $E_c/E_s = 40$, $\varphi_c = 40°$, $\nu_c = \nu_s = 0.3$, $\psi = 0$, $\mu = 1.5$

5 CONCLUSIONS

A simple but effective analytical procedure for the analysis of stone-column reinforced foundations is presented. Several factors, such as column spacing and length, initial stresses the in ground, stone and soil material properties, applied load and dilatancy of stone column material can be taken into account to obtain accurate predictions of the settlements of widespread rigid foundations supported by end-bearing stone columns. Although the method is approximate, it is in excellent agreement with elasto-plastic finite element analysis.

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