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Settlement estimation of soils reinforced by columns using a poroelastic model

Estimation du tassement des sols renforcés par colonnes à l'aide d'un modèle poro-élastique

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ABSTRACT
A significant part of the consolidation of soft clay fitted with vertical drains (or stone columns) occurs during a step by step embankment loading. In order to schedule the loading program, it is necessary to predict the settlement evolution during the construction process. A unit cell model, made up with one column surrounded by saturated soft clay overlaid by drained layer, is considered with oedometric conditions. An analytical poroelastic solution is derived that provides, in addition to the degree of radial consolidation and excess pore water pressure dissipation, the evolution with time of the reinforced soil settlement. The latter is predicted by introducing the concept of "equivalent membrane" assuming a uniform excess pore water pressure in the soft clay. The Barron's factor time of radial consolidation is also adopted that provides the determination of the permeability of this "equivalent membrane". The results of the proposed method are illustrated as a function of the substitution factor.

RÉSUMÉ
En ayant recours à un modèle de comportement poro-élastique linéaire, on résout le problème de consolidation radiale d'une argile molle renforcée par colonnes et soumise à une histoire de chargement donnée. Cette résolution est faite en utilisant l'hypothèse de la membrane équivalente. Outre le calcul des surpressions interstitielles, cette résolution, conduite analytiquement, permet le calcul du tassement durant la consolidation de l'argile molle. Ce dernier point constitue un apport original par rapport à la résolution faite par la théorie de Barron. La validité du modèle poro-élastique proposé a été confirmée suite à l'analyse d'un cas pratique.

1 INTRODUCTION

In this paper the settlement evolution of rigid foundations resting on reinforced soft clay by a group of columns is studied. Assuming a linear elastic behaviour for the reinforced soil constituents allows to predict the settlement of the reinforced soil by using either the unit cell model (Balaam et al, 1981), or the group of columns model (Bouassida et al, 2003), but provides no information regarding the settlement evolution due to the drainage, and subsequent consolidation, of the saturated soft clay by the reinforcing columns. In such a situation the consolidation time of reinforced soil is highly reduced because of little radial drainage distance between columns and high permeability of their constitutive material such as gravel, ballast, etc. Nevertheless, the consolidation process taking place in soft clay depends strongly on the history of loading exerted on the reinforced soil. Based on the unit cell model, Barron's contribution (1947) was first proposed to predict the consolidation evolution with time resulting from the presence of a vertical drain within the soft soil cylindrical mass. Using the unit cell model, other analytical contributions were carried out by Yoshunki and Nakanodo (1975), Hansbo (1981), etc. Since then two-dimensional analyses by finite element method have been investigated to simulate the consolidation due to vertical drain in soft soil (Hird et al., 1992). In all these previous studies the excess pore pressure evolution due to instantaneous loading was only aimed to predict the soft soil improvement caused by vertical drain. However, practitioners often face both settlement reduction and settlement evolution taking place in soils improved by vertical drains. Such a situation frequently occurs during a step by step embankment construction on soft clays reinforced by stone columns. In the present contribution, where the unit cell model is used, an analytical procedure is performed in order to predict both settlement and excess pore pressure evolution occurring in soft clay improved by vertical drains due to a given history of loading. For this purpose a poroelastic modelling is considered for soft clay assumed to be fully saturated, while the column material is perfectly drained.

2 FROM THE INITIAL TO THE AUXILIARY PROBLEM

End bearing columns, all located under the rigid foundation, are resting on an impervious substratum. The reinforced soil mass is overlaid by a thin perfectly pervious sand layer. Assuming that the foundation's horizontal extension L is large enough with respect to the spacing s between two adjacent columns, the unit cell model can be adopted with oedometric conditions.

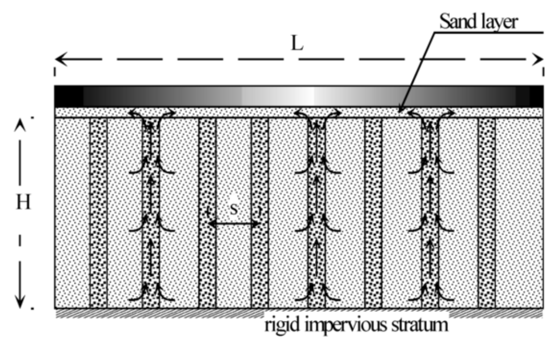

Figure 1. Foundation supported by reinforced soil

Assuming for instance that the columns are distributed throughout the soil following a square pattern, the unit cell is a parallepipedic volume comprising one column of radius a as sketched in fig. 2a. For the sake of simplicity, the domain of influence is assumed to have a circular shape of external radius b (fig.2b), so that the global improved area ratio (ratio between the sum of the total columns section and the rigid area foundation area) is identified with the substitution factor resulting from the unit cell model, then:

$$\eta = \frac{\pi a^2}{s^2} = \frac{\pi a^2}{\pi b^2} = \frac{a^2}{b^2} \tag{1}$$

An initial increase of excess pore pressure in the soft clay is instantaneously generated by the application of the loading. Due to the high permeability of the column material, radial drainage takes place in the soft clay horizontally towards the column, then the flow continues vertically in the column towards the horizontal drainage layer.

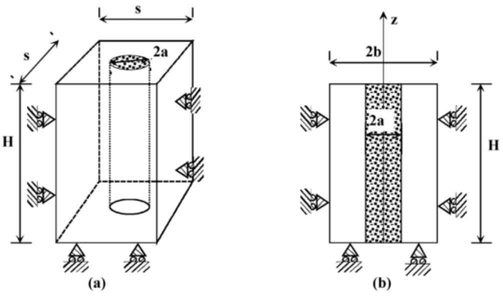

Figure 2. (a) Reinforced unit cell, (b) Auxiliary problem.

3 EQUATIONS OF LINEAR POROELASTICITY

A brief recall of equations governing the linear poroelastic behaviour of saturated soils whose constituents (grain matrix and water) are assumed incompressible, is first made (Coussy, 1999; Dormieux & Bourgeois, 2002).

3.1 *The first state equation*

This state equation expresses that the volume variation of the saturated soil $tr(\underline{\underline{\varepsilon}})$ is only due to the water exchange between the soil itself and the outside, then:

$$tr(\underline{\underline{\varepsilon}}) = \frac{m}{\rho_w} \tag{2}$$

where m is the supply of water per unit volume of soil, and ρ_w is the specific mass of water.

3.2 *The isotropic linear porous elastic behavior:*

In the case of isotropic linear behavior, the initial state being natural ($\underline{\underline{\sigma}} = 0$, $p_0 = 0$), the soil strains $\underline{\underline{\varepsilon}}$ are governed by the classical Terzaghi effective stress $\underline{\underline{\sigma}}'$, such that:

$$\underline{\underline{\sigma}}' = \lambda' tr(\underline{\underline{\varepsilon}})\underline{\underline{I}} + 2\mu'\underline{\underline{\varepsilon}} \tag{3}$$

where λ' and μ' are the drained coefficients of Lamé of the soil skeleton. $\underline{\underline{I}}$ denotes the identity tensor.

3.3 *Darcy's law:*

According to Darcy's law a linear relationship is established between the vector of fluid mass transfer $\underline{w}$ and the gradient of the excess pore water pressure u:

$$\frac{\underline{w}}{\rho_w} = -k \; grad\left(\frac{u}{\rho_w g}\right) \tag{4}$$

in which the excess pore-water pressure is defined by: $u = p - p_0$, where p is the pore-water pressure and p_0 is the hydrostatic pore pressure. g is the gravity and k is the coefficient of permeability of the soil.

3.4 *Water mass balance equation*

It expresses the connection between the rate of the unit supply of water and the vector of fluid mass transfer $\underline{w}$, in the form:

$$\dot{m} + div\underline{w} = 0 \tag{5}$$

3.5 *Diffusion equation:*

Substituting Eqs (2) and (4) into Eq (5), it comes:

$$\rho_w g\, tr(\underline{\underline{\dot{\varepsilon}}}) = k \; \Delta u \tag{6}$$

where Δ denotes Laplace operator.

4 STATEMENT OF THE AUXILIARY PROBLEM

Due to the geometry of column, cylindrical coordinates *(r, θ, z)* are considered, in order to state mechanical and hydraulic boundary conditions of the composite cell (figure 3).
Mechanical boundary conditions write as follows.

The bottom cross section ($z = 0$) is in smooth contact with a rigid and fixed plate:

$$T_r = T_\theta = 0 \qquad \xi_z = 0 \tag{7}$$

The top cross section ($z = H$) is in smooth contact with a rigid plate subject to a downward uniform vertical displacement d as function of time t:

$$T_r = T_\theta = 0 \qquad \xi_z = -d(t) \tag{8}$$

The lateral surface ($r = b$) is in smooth contact with fixed border.

$$T_z = T_\theta = 0 \qquad \xi_r = 0 \tag{9}$$

Hydraulic conditions take into account the high permeability of column material and sand layer with respect to that of the surrounding soft clay:

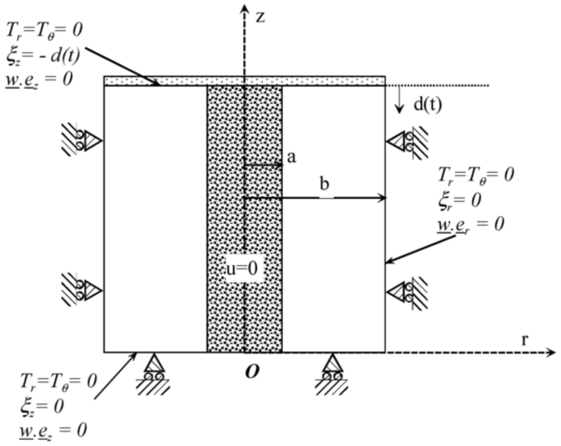

Figure 3. Boundary conditions of the auxiliary problem

$$u(0\le r\le a, 0\le z<H)=0 \tag{10a}$$

The excess pore-water pressure at the top horizontal boundary of the soft clay ($z=H$) is zero:

$$u(0\le r\le a, z=H)=0 \tag{10b}$$

The top and bottom horizontal boundaries of the soft clay ($z=0$, H), as well as the lateral surface ($r=b$) are impervious:

$$\underline{w}(a\le r\le b, z=H)\underline{e}_z=0 \tag{11a}$$

$$\underline{w}(0\le r\le b, z=0)\underline{e}_z=0 \tag{11d}$$

$$\underline{w}(r=b)\underline{e}_r=0 \tag{11c}$$

5 SOLUTION OF POROELASTIC PROBLEM

It is carried out under the following assumptions:

The water flow occurs in radial direction:

$$\underline{w}=w(r,t)\ \underline{e}_r \tag{12}$$

For the sake of simplicity, an assumption called "equivalent membrane" hypothesis is done, according to which the soft clay permeability is infinite, such that the excess pore water pressure remains homogenous (figure 4):

$$a<r\le b, \quad u(r,t)=u_s(t) \tag{13}$$

As a result of this simplifying assumption, the radial water flow towards the column will be controlled through this equivalent membrane, whose global permeability is denoted by K, its dimension being the inverse of time (s^{-1}). Then the water flow is expressed by:

$$\frac{w}{\rho_w}(a,t)=-K\frac{u_s(t)}{\gamma_w} \tag{14}$$

The solution of the poroelastic problem, taking into account the above conditions, reduces to that of an elastic problem relative to the composite cell subjected to a uniform displacement $d(t)$ at the top surface and an isotropic state of stress within the soft clay, corresponding to the excess pore pressure in the clay. This leads to the following differential equation:

$$abD_u(a)\frac{\dot{u}_s}{E_s}-K\frac{a}{\gamma\omega}u_s=\left[\frac{a^2-b^2}{2}-abD_d(a)\right]\frac{\dot{d}}{H} \tag{15}$$

where the expressions $D_u(a)$ and $D_d(a)$ are detailed in Guetif (2004).

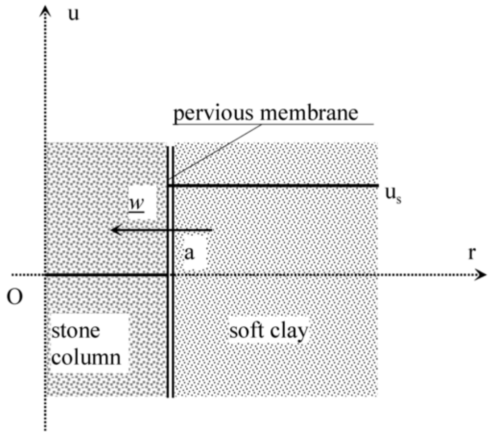

Figure 4. Equivalent membrane at the column-clay interface.

From the differential equation (15) and for a given function $d(t)$, the evolution of excess pore water pressure within the soft clay can be calculated. For the particular case when the top of the composite cell is subjected to a uniform settlement of the form $d(t)=d_0\,H(t)$, where $H(t)$ is the Heaviside function, the excess pore pressure writes:

$$u_s(t)=\left(\frac{Z}{X}\frac{d_0}{H}E_s\right)\exp\left[-\frac{t}{T_c}\right]H(t) \tag{16}$$

where the characteristic time of consolidation is expressed by:

$$T_c=-\frac{\gamma_w b D_u(a)}{K E_s} \tag{17}$$

E_s denotes the soft clay Young's modulus, while the expressions of Z and X are given in Guetif (2004).

6 IDENTIFICATION OF THE PERMEABILITY OF EQUIVALENT MEMBRANE

The coefficient of permeability K of the equivalent membrane is introduced as: $K=\frac{k}{L_k}$, where L_k is a characteristic length which is determined from the solution of the radial consolidation problem of a composite cell. For such problem (figure 3), the characteristic length will be deduced by identifying the characteristic time predicted from Eq (17) to that proposed by Barron's theory (1947), it comes :

$$L_k=\frac{k b E_s}{c_r D_u(a)\gamma_w}\left[\frac{\ln\left(\sqrt{\eta}\right)}{2(1-\eta)}+\frac{3-\eta}{8}\right] \tag{18}$$

7 SETTLEMENT PREDICTION FOR A GIVEN LOADING HISTORY

In practice, the main problem to be faced is the prediction of the settlement evolution generated by a given loading history. The load *Q(t)*, exerted by the rigid raft foundation on the reinforced soil is:

$$Q(t)=-\int_{S(z=H)} \sigma_z(H,t)ds \tag{19}$$

where σ_z is the vertical stress distribution at the top of the unit cell model. Calculations of this vertical load are detailed in (Guetif, 2004). One obtains:

$$Q(t)=\pi b^2\left\{\begin{array}{l}\left[\left(\eta(\lambda_c+2\mu_c)+(1-\eta)(\lambda_s+2\mu_s)\right)-2(\lambda_c-\lambda_s)\frac{a}{b}D_d(a)\right]\frac{d(t)}{H}\\ +\left[(1-\eta)E_s-2(\lambda_c-\lambda_s)\frac{a}{b}D_u(a)\right]\frac{u_s(t)}{E_s}\end{array}\right\}$$

(20)

Combining Eqs. (16) and (20) finally it comes:

$$\theta\,\dot{Q}+\chi\,Q=\omega\dot{\Delta}+\psi\Delta \tag{21}$$

where Δ denotes the normalized settlement *d/H*, while the expressions of coefficients θ, χ, ω and ψ are given in Guetif (2004). Given the loading history *Q(t)*, this equation can be solved by using Laplace-Carson's transform in exactly the same way as for solving viscoelastic problems (Salençon, 1983).
In the particular case of an instantaneous loading for $t=0$:

$$Q(t)=Q_0\ H(t) \tag{22}$$

the solution in terms of normalized settlement writes:

$$\Delta(t)=Q_0\left[\frac{\theta}{\omega}\exp\left(-\frac{\psi}{\omega}t\right)+\frac{\chi}{\psi}\left(1-\exp\left(-\frac{\psi}{\omega}t\right)\right)\right]H(t) \tag{23}$$

The evolutions of settlement and degree of radial consolidation, under a prescribed load exerted by the foundation, may be calculated as functions of the substitution factor and elastic characteristics of the constituents of the composite cell model.

By the use of the poroelastic model here proposed through Eq (23), one obtains the prediction settlement variation in time of the reinforced soil due to instantaneous loading. This result is illustrated in figure 6 as a function of the substitution factor.

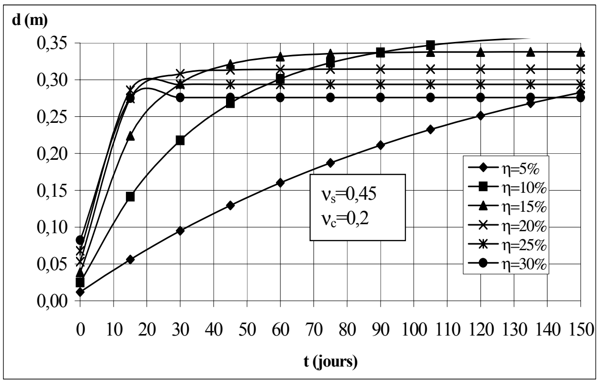

Figure 6. Settlement variation with time of reinforced soil submitted to instantaneous loading (E_s=1000kPa, E_c=10000kPa, Q=1000kN, k=10^{-8}ms^{-1}, b=1,7m, H=15m).

Furthermore, the excess pore pressure evolution $u_s(t)$ and radial consolidation degree evolution U_r (%),defined in Guetif (2004), are also predicted (figure 7).

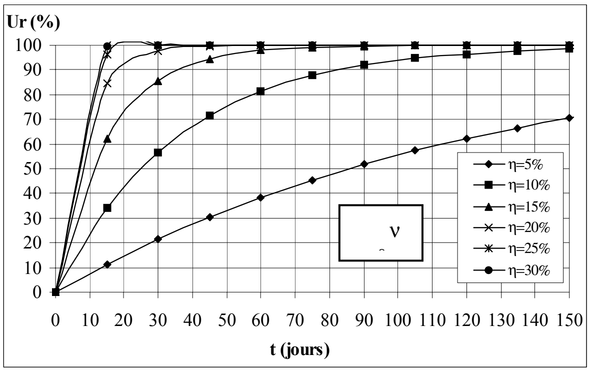

Figure 7. Radial consolidation degree variation as a function of time, of reinforced soil submitted to instantaneous loading (E_s=1000kPa, E_c=10000kPa, Q=1000kN, k=10^{-8}ms^{-1}, b=1,7m, H=15m).

8 CONCLUSION

It is important to emphasize that the poroelastic model presented here provides both the evolutions of the degree of radial consolidation and settlement for any case of loading history under which the reinforced soil is subjected. Illustration by charts have been provided for instantaneous loading the settlement evolution in time for various substitution factor values

The most important parameter affecting predictions by the proposed model remains the radial coefficient of consolidation. Because of unsatisfactory laboratory tests to determine this parameter the calibration of predicted results by the model becomes more difficult to carry out. Meanwhile more investigations with field data should be done to calibrate the proposed porous elastic model.

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