

# Stability of embankments founded on deep-mixing-method columns: Three-dimensional considerations

## La stabilité de remblais fondés sur profond de méthode mélangeante colonnes: Les considérations à trois dimensions

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### ABSTRACT

Two-dimensional analyses cannot capture all aspects of column behavior for deep mixing method (DMM) columns used to support embankments founded on soft soils. One difficulty is that a two-dimensional analysis cannot simultaneously provide accurate representations of both the axial stiffness and the bending stiffness of the columns. This paper compares the results of two- and three-dimensional finite difference analyses of column-supported embankments using the computer program FLAC.

### RÉSUMÉ

A deux dimensions analyse ne peut pas capturer tous aspects de comportement de colonne pour mélangeant profondément la méthode (DMM) les colonnes ont utilisé pour soutenir des remblais fondés sur les sols doux. Une difficulté est qu'une analyse à deux dimensions ne peut pas fournir simultanément le re-présentations précis de la raideur axiale et la raideur courbant des colonnes. Ce papier compare les résultats de deux- et la différence finie à trois dimensions analyse de remblais colonne-soutenus utilisant le FLAC de programme informatique.

### 1 INTRODUCTION

In Japan and Scandinavia, transportation embankments on soft ground are often supported on columns installed by deep mixing methods. This technology is also finding more frequent application in other countries, including the United States. The density and arrangement of columns at the edge of embankments is determined by considerations of slope stability. The current state of practice is that design for embankment stability is based primarily on two-dimensional limit equilibrium analyses with composite shear strengths obtained from the column strength, the strength of the soft ground, and the area replacement ratio (CDIT 2002, EuroSoilStab 2002, Broms 1999, Kivelo 1998, Wei et al. 1990). Such analyses consider a composite shearing mode of failure, but they do not consider other failure modes, such as column bending, column tilting, or extrusion of soft ground between columns or panels. Kivelo (1998) investigated the ultimate shear strength of lime-cement columns considering bending, but his approach is not currently incorporated into stability analysis procedures, mainly owing to lack of complete understanding of this phenomenon (Porbaha 2000). When slope stability is a concern for embankments founded on soil reinforced with columns, CDIT (2002) recommends that numerical analyses be performed concurrent with slope stability analyses to investigate displacements. Although no guidance has been specified, displacements from numerical analyses provide designers an indication of whether failure mechanisms other than shear failure may occur.

In the research described in this paper, numerical analyses were performed in two and three dimensions using FLAC (Fast Lagrangian Analysis of Continua) computer programs (ITASCA 2002). First, calibration analyses were performed for (1) the centrifuge experiments by Kitazume et al. (2000), (2) the centrifuge experiments by Inagaki et al. (2002), and (3) the instrumented test embankment at the intersection of US I-95 and Virginia State Route 1 in Alexandria, near Washington D.C. (Shiells et al. 2003, Stewart et al. 2004). The analysis results are in reasonably good agreement with the data from the experiments and the test embankment. The calibrated numerical

procedures were then used to investigate three-dimensional effects.

### 2 MODEL GEOMETRY

The centrifuge tests and subsequent numerical analyses performed by Inagaki et al. (2002) reveal much about the performance of DMM column-supported embankments. That work provides a baseline for comparing the two-dimensional and three-dimensional numerical analyses presented in this paper.

The centrifuge tests performed by Inagaki et al. (2002) used an acceleration of 50 g. Six centimeters of sand was placed at the bottom of the model test box. Clay slurry was poured over the sand and consolidated with a pressure of 98 kPa under an acceleration of one gravity to create the twenty-six centimeter soft clay layer. This procedure resulted in a soil that was over consolidated near the surface and normally consolidated at the bottom of the clay layer when subsequently accelerated to 50 g. Columns were installed with a 20% replacement ratio by coring two-centimeter-diameter holes in a square pattern in plan view with four-centimeter center-to-center spacing, and replacing the clay with a soil-cement mix. The embankment was placed in three separate four-centimeter lifts with a final 1 vertical on 1.6 horizontal side slope. The plan and profile schematic diagrams are included as Figure 1.

With an acceleration of 50 g, the model dimensions are multiplied by 50 to obtain prototype scale. Thus, the prototype sand layer is 3 m thick, the clay layer is 13 m thick, the column diameter is 1.0 m, and the embankment is 6 m high.

The geometry shown in Figure 1 was discretized into 5913 zones for the two-dimensional plane strain analyses. Five rows of deep mixed columns in the improvement zone were represented by vertical strips, with the center-to-center spacing equal to the spacing of the rows of deep mixed columns installed in the centrifuge model. Two separate analyses were performed, one with narrow strip widths and one with wide strip widths. The width of narrow strips was chosen to result in the same area replacement ratio of 20% that corresponds to the column diameter

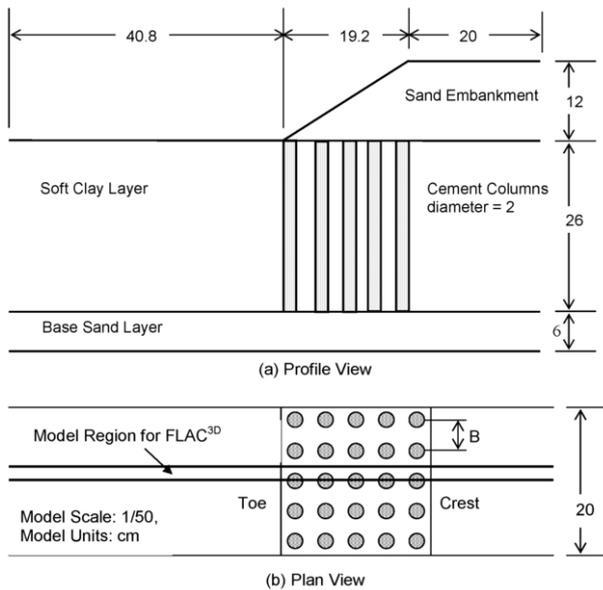


Figure 1. Inagaki et al. (2002) Centrifuge Model Schematic

and spacing used in the centrifuge model. The width of wide strips was chosen to result in the same moment of inertia as the circular columns. The column diameter and strip widths are summarized in Table 1. In three dimensions, symmetry was used to analyze only the portion indicated in the plan view in Figure 1. For the three dimensional analysis, the geometry was divided into 52,560 zones.

Table 1: DMM Element Widths

Element	Width (m)	
	Model Scale	Prototype Scale
3D Column	0.020	1.00
2D Narrow Strip	0.008	0.40
2D Wide Strip	0.013	0.67

A second set of analyses was performed with five additional columns to extend the treatment zone from the toe of the embankment to the centerline boundary of the model on the right-hand side, in order to support the full embankment, as would be done in practice. The ten columns in this analysis were at the same geometry and spacing as the five columns in the first analysis. This geometry was discretized into 6243 zones for the two-dimensional plane strain analyses and 65,160 zones for the three-dimensional analyses. Centrifuge model test results from Inagaki et al. (2002) are available for the five-column case, but not for the ten-column case.

### 3 MODEL PARAMETERS

Analyses were performed using model parameter values provided in Inagaki et al. (2002). The embankment, columns, and sand layer were modeled as elastic materials. The Sekiguchi/Ohta model used for the clay layer was replaced with the Modified Cam-Clay model which is programmed into FLAC, and consequently results in much less computation time than a user programmed version of the Sekiguchi/Ohta model. Even so, the water-soil coupled analyses of this model took several days using FLAC<sup>2D</sup>, and a three-dimensional analysis would take many times longer.

In the case where DMM columns are used with low permeability clay, and embankments are placed rapidly, it is assumed that lateral deflections are controlled by undrained response. Rather than using water-soil coupled analyses, an undrained

analysis with carefully selected modulus values for the clay layer was found to produce accurate calculations of lateral deflections for the cases investigated. Undrained soil modulus values for the clay layer were obtained using the Modified Cam-Clay model, with the parameter values provided by Inagaki et al. (2002). It was found that the shear modulus increased approximately linearly with depth and ranged from 37 kPa at the ground surface to 1275 kPa at the bottom of the clay layer. The cohesion value of the clay layer was based on the preconsolidation pressure. Soil parameter values used in these analyses are shown in Table 2.

Table 2: Soil parameter values for undrained analyses.

Material	$\gamma$ ( $\text{kg/m}^3$ )	$c$ (kPa)	$\phi$ (degrees)	$E$ (kPa)	$\nu$
Embankment	1878	elastic		1000	0.3
Clay layer	1714	23	0	*	0.49
Base sand layer	1608	elastic		21000	0.3
Columns	1714	elastic		60000	0.49

\* Modulus values increase with depth as described in text.

### 4 RESULTING DISPLACEMENTS

For the five-column case, results for the two-dimensional analyses using both narrow strips and wide strips, as well as results from the three-dimensional analysis all closely match the measured column displacements from Inagaki et al. (2002). This can be seen in a plot of displacement versus depth for the first column located under the toe, as shown in Figure 2. Because there is no columnar support under the center portion of the embankment in this analysis, the embankment load produces large lateral pressures in the clay and large lateral deflections of the columns. System behavior is largely influenced by the shear modulus of the clay, and there is little difference between deflections from narrow strips, wide strips, or the three-dimensional analysis.

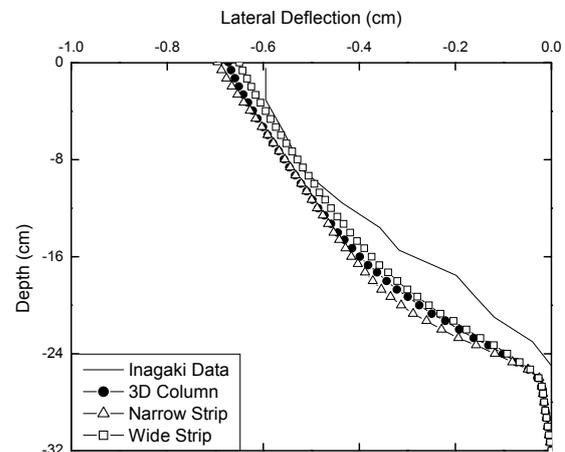


Figure 2. Deflections for analyses with five columns

A similar plot for the case with ten columns is shown in Figure 3. As can be seen in this figure, the plane strain analysis using narrow strips matches the three-dimensional analysis, but plane strain analysis using wide strips produces much smaller displacements. For the case with ten columns, the support provided by the columns is a larger part of the system stiffness than for the five-column case, and narrow strips more accurately represent system stiffness than wide strips in the two-dimensional analysis.

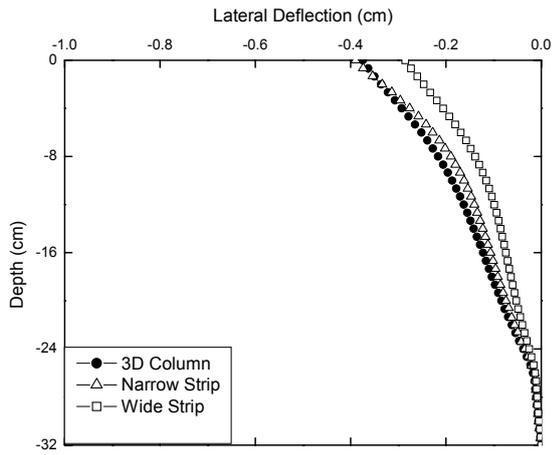


Figure 3. Deflections for analyses with ten columns

## 5 RESULTING COLUMN STRESSES

Given the brittle nature of DMM columns, especially in tension, it is critical that stresses from numerical analysis are evaluated appropriately. Proper assessment for column stresses is complicated for the two-dimensional analyses by the fact that the geometry of the column is altered to a strip that has different sectional properties than a circular column. Therefore, the stresses resulting from numerical analysis cannot be interpreted from two-dimensional analyses in the same way as stresses from three-dimensional analyses.

The vertical stresses acting on the columns can be separated into two components: the axial stress,  $\sigma_a$ , which is the average of the vertical stresses across the column at a given elevation, and the bending stress, which is the difference between the vertical stress and the average stress. The symbol  $\sigma_b$  is used here to designate the extreme fiber bending stress at the greatest distance from the neutral axis.

To convert the vertical stresses in the columns from the two-dimensional analyses to values applicable to round columns, the assumption is made that the bending moments in the two-dimensional strips are the same as in the round columns. At each elevation of interest, the bending stresses from the two-dimensional analyses are adjusted to produce this equivalency, as follows:

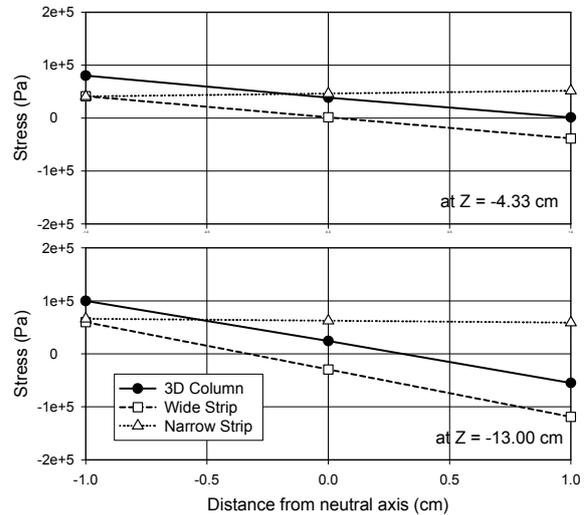
$$\sigma_{b3D} = \pm \frac{BS_{2D}}{S_{3D}} \sigma_{b2D} \quad (1)$$

where  $\sigma_{b3D}$  = the equivalent extreme fiber bending stress in a round column,  $B$  = the center-to-center spacing between rows of round columns in the direction perpendicular to the plane of the two-dimensional analyses (see Figure 1),  $S_{2D}$  = the section modulus of the two-dimensional strip, i.e., the strip width squared divided by six,  $S_{3D}$  = the section modulus of the round column,  $\sigma_{b2D}$  = the extreme fiber bending stress in the strip from the two-dimensional numerical analyses.

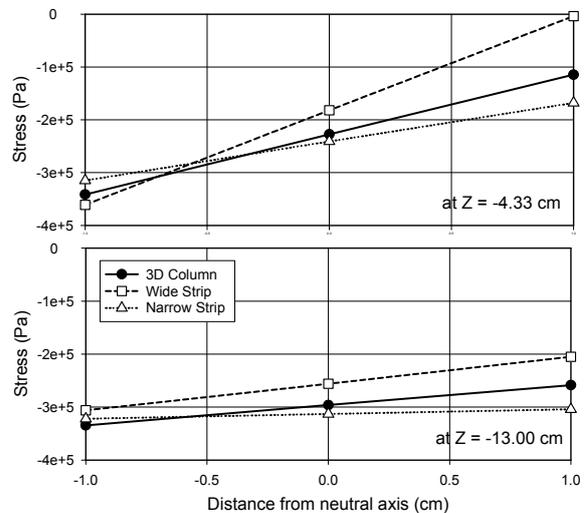
The vertical stresses from two-dimensional analyses equivalent to round columns,  $\sigma_{3D}$ , are the sum of the axial stresses,  $\sigma_a$ , and the converted bending stresses,  $\sigma_{b3D}$ , obtained from Equation 1. In the figures described below, values of  $\sigma_{3D}$  are compared with the corresponding values obtained directly from the three-dimensional analyses.

Figure 4 presents column stresses at depths of 4.33 cm and 13 cm. As with the study of lateral displacements in the previous section, the column presented in Figure 4 (a) is the first column in the five column case, located under the toe of the em-

bankment. Figure 4 (b) shows the third column of the ten column case. The stresses from the two-dimensional analysis with narrow strips, the two-dimensional analysis with wide strips, and the three-dimensional analysis are included in this figure. The slopes of the lines show that, in comparison with the results of three-dimensional analyses, the two-dimensional model with narrow strips under-predicts bending stress, while the two-dimensional model with wide strips tends to over-predict bending stresses. These trends are true for other columns, as well.



(a) First column of five columns case



(b) Third column of ten columns case

Figure 4. Vertical stresses in columns

The two-dimensional analyses with narrow strips result in approximately the same axial stress or slightly higher axial stress than the three-dimensional analyses, depending on column location underneath the embankment. Figure 5 presents axial stress from the ten column case for both the first column under the toe and the last column under the full height of the embankment. The axial stresses are plotted along the length of the columns. As can be seen in this figure, the two-dimensional analysis with narrow strips produces results that are about the same or slightly greater than the results from the three-dimensional analysis. It is also evident in this figure that the

two-dimensional analysis with wide strips can produce column axial stresses that are significantly smaller than the axial stresses from the three-dimensional analyses.

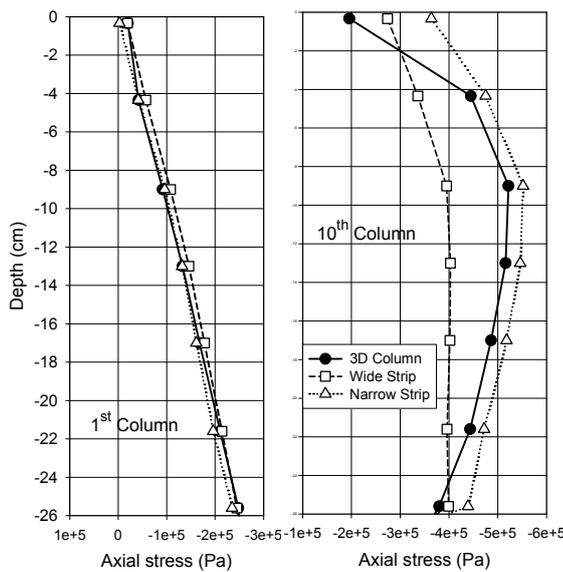


Figure 5. Axial stress of columns

## 6 CONCLUSIONS

Key findings for two- and three-dimensional analyses of embankments supported on isolated deep-mixing-method columns are that (1) lateral deflections from two-dimensional analyses are in relatively good agreement with lateral deflections from three-dimensional analyses at the same area replacement ratio, (2) two-dimensional analyses under-predict bending stresses seen in three-dimensional analyses at the same area replacement ratio, and (3) two-dimensional analyses using strip widths based on moment of inertia of the columns tend to over-predict the bending stresses and under-predict the axial stresses seen in three-dimensional analyses.

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