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Measurement of bending moments in concrete

Mesure des moments de flexion dans le béton

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ABSTRACT
Inclinometers are frequently used to measure structural deflections during construction of retaining walls and piles. Attempts have been made to calculate bending moments from wall profiles measured by inclinometers but there is inconsistency in the methods and sometimes results are not always reliable. In addition, the collection and analysis of site inclinometer data can be poor. During construction of the Channel Tunnel Rail Link, UK, a section of contiguous bored pile retaining wall has been monitored using various instrumentation. Inclinometer measurements of the wall profile were scrutinized carefully and the rigorous techniques used to eliminate errors are described in this paper. Bending moments calculated from both strain gauge measurements and from the wall profile measurements have been compared and show good agreement. Complications regarding the fitting of various types of curves to inclinometer data and whether the section is cracked or uncracked are discussed and solutions suggested.

RÉSUMÉ
Des inclinomètres sont fréquemment utilisés pour mesurer les débattements structuraux pendant la construction des murs et des piles de soutènement. Des tentatives ont été faites pour calculer les moments de flexion des profils de mur mesurés par des inclinomètres mais il y a contradiction dans les méthodes et parfois les résultats ne sont pas toujours fiables. En outre, la collecte et l'analyse des données d'inclinomètre sur site peuvent être pauvres. Pendant la construction de la liaison ferroviaire du tunnel sous la Manche, au RU, une section de mur de soutènement alésé contigu de pile a été surveillée en utilisant divers instruments. Des mesures d'inclinomètre du profil de mur ont été contrôlées soigneusement et les techniques rigoureuses employées pour éliminer les erreurs sont décrites dans cet article. Les moments de flexion calculés à partir des mesures de jauge de contrainte et des mesures du profil de mur ont été comparées et montrent un bon accord. Des complications concernant le fit de divers types de courbes aux données d'inclinomètre et si la section est fissurée ou non sont discutées et des solutions sont suggérées.

1 INTRODUCTION

Bending moments are frequently calculated from strains measured using vibrating-wire strain gauges embedded in concrete retaining walls (e.g. Tedd *et al*., 1984) and piles (e.g. pile stabilized slopes: Smethurst, 2003). Bending moments can also be estimated from inclinometer measurements (see Saxena, 1975 and Soares, 1983) by fitting a curve (e.g. a polynomial) to the measured wall profile. The second derivative of the equation of this curve gives an expression for the curvature of the wall. The bending moment is the product of the curvature and the wall's flexural rigidity.

In the interpretation of bending strain data uncracked concrete behaviour is usually assumed. However if the cracking moment of the wall is exceeded and cracks are initiated, the calculated bending moments may significantly overestimate actual values. This together with a poor understanding of the interpretation of inclinometer measurements generally lead to a poor fit between bending moments calculated by these two methods. In the past, little if any attempt has been made to determine whether the cracking moment of monitored walls or piles has been exceeded, or indeed whether the strain gauge measurements indicate that cracking has occurred.

Throughout construction of a propped contiguous bored pile retaining wall in Ashford, Kent, UK, which forms part of the Channel Tunnel Rail Link (CTRL), structural loads and soil and water pressures around a section of the wall have been monitored. Wall bending moments measured using embedded vibrating-wire strain gauges have been compared with those estimated by analysis of the wall profile found from inclinometer measurements. There is close agreement for values falling below the wall cracking moment. Further studies investigate how cracking can be recognised, reveal the impact of cracking on the wall shape and show how bending moments in cracked walls can be estimated from inclinometer readings.

2 CTRL, ASHFORD

The instrumented section consists of a propped contiguous bored pile retaining wall formed from 21 m long, 1.05 m diameter piles spaced at 1.35 m centres and founded in overconsolidated clay. The excavation sequence was as follows. After reinforced concrete (RC) props had been constructed at the top of the wall, 6 m of material was removed from in front of it (excavation Phase 1). Temporary props were installed at the excavation level and subsequently a further 3.2 m of material was removed (excavation Phase 2). After construction of a base slab the temporary props were removed (see Figure 1). An inclinometer tube was installed in pile Z and piles X and Y were instrumented with strain gauges. The strain gauge positions are shown in Figure 1. The sequence of construction with the number of days counted from the beginning of excavation Phase 1 is listed.

3 INCLINOMETER SURVEYING

Inclinometers have been used in numerous construction projects over at least the past 30 years to monitor ground and structural movements, often as part of observational methods of construction (Peck, 1969), e.g. The World Trade Centre (Saxena, 1975); Limehouse Link (Glass and Powderham, 1994); Channel Tunnel Rail Link (Loveridge, 2000) and the Heathrow Airside Road

Tunnel (Hitchcock, 2003). To find the wall profile a probe is lowered manually to the lowest point within a tube. As the probe is pulled back up, readings of the tilt angle are taken at 0.5 m intervals in directions perpendicular and parallel to the wall. The inclinometer probe follows longitudinal grooves in the casing to ensure that it does not spin as it moves. Base readings are taken before excavation (or before whatever changes being monitored commence) and subsequent readings reveal changes that occur due to construction activities, land movements, etc.

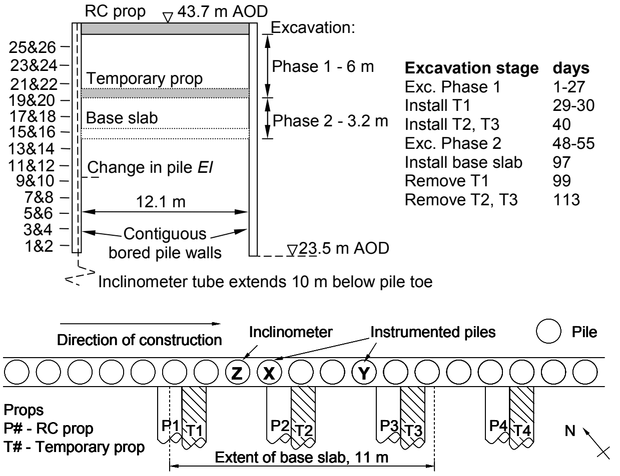


Figure 1 – Elevation and plan showing construction sequence and pile and prop locations

To measure the absolute movement of the structure or ground mass being monitored, either the tube must be extended below the item into ground known to be stable, or the top of the tube must be surveyed at each reading. At the monitoring site at the CTRL the inclinometer tube extends 10 m into the ground below the pile toe (as shown in Figure 1) so that fixity can be established and the absolute movements of the wall determined. Further information regarding the use of inclinometers can be found in Dunnicliff (1993).

Mikkelsen (2002) describes the types of errors that may occur when using an inclinometer probe. In basic terms these errors can be minimized by ensuring that the same probe is used for all readings in a borehole; the same user takes the readings every time; the user is technically competent; the person who collects the data processes it; and the inclinometer probe is regularly calibrated. These measures were taken by the monitoring team on the CTRL, who took inclinometer readings across the site at Ashford (the data were mainly used for implementing the Observational Method during construction).

For each inclinometer profile two sets of data are collected; for the second set the inclinometer probe is rotated through 180°. The difference between individual pairs of readings at a given level is called the face error or *checksum*. This is equal to twice the zero offset, or *bias*. The face errors are displayed on the readout unit during data collection so that the user can check that these errors are approximately constant throughout the survey, and therefore have no overall effect on the final inclinometer measurements. However, operator technique, instrument performance and casing conditions (such as variability at joints or dirt) can affect the face errors. In the latter case the error should exist in all profile readings and can therefore be identified.

If the face errors are randomly inconsistent, analysis of the individual readings can reveal where the errors occur and the data can be altered accordingly. Figure 2a shows the face errors from an inclinometer profile taken on Day 99 (before temporary prop removal). Some unusually large face errors are recorded at various locations in the inclinometer tube. Table 1 shows the Face A and Face B inclination readings at 41.593 m AOD and the calculated face errors for the data sets taken on Day 99. At this reading location only the Face A reading was inconsistent. Having identified the locations of these large face errors, comparison of the readings recorded at these locations with those taken on the preceding and following days allows the rogue readings to be identified and adjusted (amendments are shown in brackets).

The original and amended profiles are plotted in Figure 2a. The other data points highlighted in this plot were similarly analyzed and altered. For this profile adjusting the data in this way ‘moved’ the top of the wall by 1.5 mm. This is about 8 % of the overall movement at the top of the wall. All the inclinometer data were adjusted in this way. Data detailing the amended wall profiles at significant construction events are shown in Figure 2b.

Table 1 – Data analysis and correction using face errors 41.593 m AOD

Day	84	98	99	103	110
Face A	4.1	4.1	4.6 (4.1)	4.0	4.2
Face B	-2.4	-2.4	-2.5	-2.4	-2.3
Face error = (A+B)/2	0.85	0.85	1.05 (0.8)	0.80	0.95

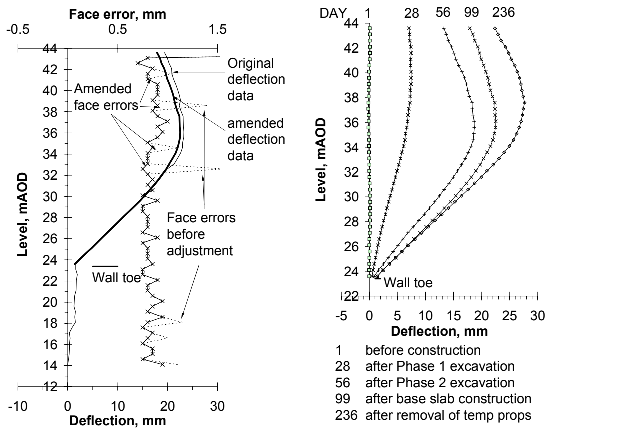


Figure 2 – (a) Analysis of inclinometer face errors (b) Wall profiles measured with the inclinometer during excavation in front of the wall

4 BENDING MOMENTS FROM INCLINOMETER DATA

To estimate pile bending moments from inclinometer readings, it is necessary to differentiate twice an equation representing the deflected shape of the pile (measured with the inclinometer). To find an equation for the wall’s deflected profile, a curve (e.g. a polynomial) must be fitted to the deflection data.

Ooi and Ramsey (2003) compared twelve methods for calculating bending moments from inclinometer data including piecewise fitting of quadratic and cubic curves, and global polynomial curve fitting. They noted that if using global polynomial curve fitting, the lowest possible degree of polynomial that will “adequately” describe the data should ideally be used. This will allow the main changes in wall shape to be modelled without including small local errors in the inclinometer data. In general Ooi and Ramsey concluded that piecewise fitting of cubic curves to windows of five points yielded “middle” values of curvature and that when maximum curvature values found using this method were compared to strain gauge data collected from laterally loaded piles the values were in good agreement. For inclinometer data collected at the CTRL site global polynomial fitting has been found to give close correlation with bending moments calculated from strain gauge readings.

The second derivative of the equation representing the deflected wall profile yields an expression for its curvature, κ. The bending moment, M, is calculated from the product of the curvature and the flexural rigidity of the pile, EI, where E is Young’s modulus and I is the second moment of area. The bending moment is defined as positive when the excavated side of the wall is in tension.

A 5th order polynomial is most suitable for representing the deflected shape of a wall which has a linear horizontal pressure distribution acting on it (the pressure distribution is given by the 4th derivative of the chosen polynomial). In reality the pressure distribution is unlikely to be perfectly linear, and therefore both 5th and 6th order polynomial equations were fitted to the deflected wall profile (determined by the inclinometer readings) before and after two construction stages at which cracking appears to have occurred (described later). The correlation coefficients (r^2) for the curves were all greater than 0.997, which superficially indicates a high level of accuracy for the curve fit. Comparisons between the bending moments calculated from the inclinometer and strain gauge measurements are made later.

5 BENDING MOMENTS FROM STRAIN GAUGES

Bending moments in concrete retaining walls are calculated from strain measurements taken using embedment strain gauges placed in pairs at intervals up the concrete section. For an uncracked section of wall, the bending moment in the wall, M, is calculated from the longitudinal strains ε_1 and ε_2 measured by the strain gauges at the back and front of the wall respectively using Equation 1, where y is the distance from the gauge to the neutral axis.

$$M = \frac{EI(\varepsilon_1 - \varepsilon_2)}{2y}. \qquad (1)$$

At Ashford the lower section of the piles (incorporating gauges 1-10) has smaller reinforcement bars and therefore a lower flexural rigidity. Hence for the bottom and top of the pile EI = 1826600 kNm^2 and 2015300 kNm^2 per pile respectively.

Uncracked behaviour is usually assumed in the calculation of bending moments from strain gauge data (Tedd *et al.*, 1984; Wood and Perrin, 1984; Hayward, 2000). However, it is important to determine whether cracking has occurred because if so and the value of EI is not adjusted accordingly, bending moments may be significantly over-estimated from gauge measurements. Details of how to calculate the cracked flexural rigidity can be found in Branson (1977).

The cracking moment of the wall, M_{cr}, is the moment corresponding to the maximum tensile stress that the wall can accommodate (the modulus of rupture of concrete, f_r). The most commonly used relationship between the modulus of rupture and cracking moment is shown in Equation 3, where f_r is given by Equation 4 (ACI, 1992), I is the second moment of area of the pile and y_t is distance from the centroid to the edge of the section. In Equation 4 f_c' is the cylinder compressive strength of concrete, and is taken to be $0.8 \times f_{cu}$, where f_{cu} is the 100 mm cube compressive strength (EC 2, 1992). On the basis of the analysis of the results from over 12000 tensile strength tests, Raphael (1984) proposed Equation 5 for the calculating of f_r.

$$M_{cr} = \frac{f_r I}{y_t}. \qquad (3)$$

$$f_r = 0.623\sqrt{f_c'} \text{ (in MPa)}. \qquad (4)$$

$$f_r = 2.3 f_c'^{2/3} \text{ (in psi)}. \qquad (5)$$

Cube tests of samples taken on site showed that the pile concrete has a characteristic strength of 60 MPa, giving M_{cr} = 490 kNm for f_r calculated from Equation 4 and M_{cr} = 657 kNm for f_r calculated from Equation 5.

If the neutral axis is in the middle of a pile section the strains at the front and back of the pile at any elevation will be equal and opposite. In reality there will be some axial load from the self-weight of the pile and the capping beam. Figure 3 shows the difference in strain measured between the gauges pairs against time for Piles X and Y. The figure shows that for most gauge pairs the difference in strain between the front and back of the pile is small. However, the values increase suddenly for gauge pairs 15&16 and 17&18 in pile X at the start of excavation Phase 2, and for 21&22 in pile Y after temporary prop removal. Further analysis of the data reveals that at these stages much larger strains were measured by gauges 16, 18 and 22 (even numbered gauges are at the front of the pile) than by their partners, gauges 15, 17 and 21 respectively. These changes are likely to be due to cracking in the piles.

Table 2 lists the wall deflections and curvatures measured by the inclinometer for the gauges at which the largest values were recorded before and after the two construction activities that appeared to cause concrete cracking. The changes in the values over these periods are included. As expected, the data show that cracks occurred at positions where the largest changes in deflection and curvature occurred, and indicates that the change may be more important than the degree of deflection and curvature in initiating cracks. However, cracks do not appear to have formed at the position of gauges 13&14 during excavation Phase 2 and at 23&24 during temporary prop removal, where the changes in curvature that occurred were at least as high as values at other locations where cracks did form. It is therefore likely that a combination of changes in deflection and curvature affects the propensity for cracking.

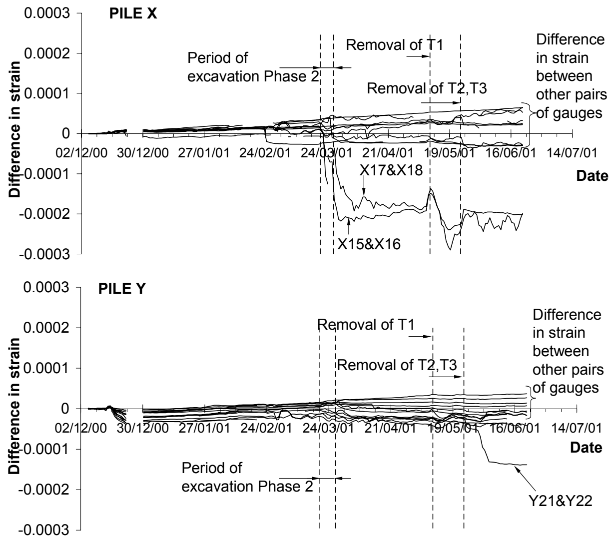


Figure 3 – Difference in strain measured in pairs of strain gauges over the period of construction

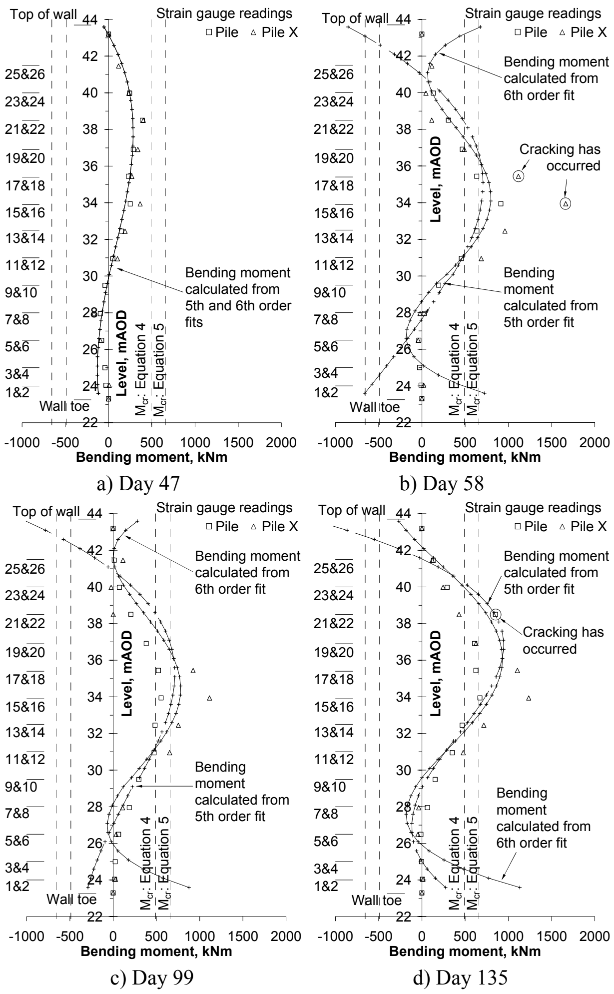
Table 2 – Changes in deflection, δ, (mm) (relative to the toe) and changes in curvature, κ, ($\times 10^3$m^{-1}) (from the 5th order polynomial curve fit) over periods of pile cracking. Positions where cracks have occurred are highlighted.

Gauges	13&14	15&16	17&18	19&20	21&22	23&24
Elevation, m AOD	32.5	34	35.5	37	38.5	40
δ, pre exc. Phase 2	7.5	8.7	9.7	10.3	10.6	10.8
δ, post exc. Phase 2	16.9	18.6	19.1	18.8	18.1	17.3
Change in δ	9.4	*9.9*	*9.4*	8.5	7.5	6.5
δ, pre TP removal	18.3	20.2	20.7	20.4	19.5	18.6
δ, post TP removal	21.7	24.3	25.9	26.5	26.1	25.1
Change in δ	3.4	4.1	5.2	6.1	*6.6*	6.5
κ, pre exc. Phase 2	-0.065	-0.099	-0.125	-0.139	-0.139	-0.119
κ, post exc. Phase 2	-0.306	-0.344	-0.348	-0.312	-0.232	-0.102
Change in κ	-0.241	*-0.245*	*-0.223*	-0.173	-0.093	0.017
κ, pre TP removal	-0.3	-0.343	-0.353	-0.321	-0.24	-0.101
κ, post TP removal	-0.289	-0.402	-0.459	-0.446	-0.366	-0.231
Change in κ	0.011	-0.059	-0.106	-0.125	*-0.126*	-0.13

6 COMPARISON OF BENDING MOMENTS FROM INCLINOMETER AND STRAIN GAUGES

Figure 4 shows bending moments calculated from the strain gauge and inclinometer measurements at four instances; before and after excavation Phase 2 (when cracking occurred in Pile X - Figures 4a and b) and before and after temporary prop removal (when cracking occurred in Pile Y - Figures 4c and d). Data at points where cracking has been shown to have occurred (in Figure 3) are highlighted on Figures 4b and d. Bending moments calculated from 5th and 6th order polynomial curve fits to the inclinometer data are shown. The cracking moments, M_{cr} (calculated in Section 4) are indicated on the figures. *EI* values are all based on uncracked concrete properties.

In 4a and b there is close agreement between the bending moment data derived from the wall profile and the strain gauges. In Figure 4a the measured bending moments are all smaller than M_{cr}, and there is little difference between the bending moments calculated from the 5th and 6th order fits to the inclinometer data.



a) Day 47 b) Day 58

c) Day 99 d) Day 135

Figure 4 – Bending moments calculated: before (a) and after (b) excavation Phase 2 and before (c) and after (d) temporary prop removal

Figures 4b and c show that at some strain gauge locations the calculated bending moments have exceeded M_{cr}. The two highest of these are those where cracking has been observed. At the other locations the section may be cracked but not specifically at the strain gauge (the strain gauges are 140 mm long and at 1.5 m intervals). In general there is close agreement between bending moments calculated from inclinometer readings and strain gauge measurements for the middle section of the pile. However, at the pile toe and top the curves bear away from the strain gauge measurements. This is obviously a mathematical problem with the curve fitting, and is probably caused by trying to fit a polynomial curve to a wall profile which contains straight sections (the bottom of the wall is straight below approximately 27.5 m AOD – this was observed by noting that the gradient of the pile is constant below 31.5 m AOD, and that the strain gauges measure zero bending moments below about 27 m AOD). Further analysis has shown that polynomials fitted only to the curved parts of the pile (i.e. leaving out the lowest few metres) produce very similar bending moment profiles but with the top and bottom parts absent. It follows that the bending moments calculated for known straight sections of the pile can simply be ignored.

Figure 4d shows the bending moment profiles found after the base slab was installed and the temporary props were removed. The readings which relate to cracked sections are shown. Again, at the pile toe the inclinometer derived bending moments bear away from the strain gauge measurements. In Figure 4d, unlike Figure 4c, the inclinometer derived bending moments are larger than those measured using the strain gauges around the position of the maximum bending moment.

7 CONCLUSIONS

At the CTRL site at Ashford, UK, inclinometers and strain gauges have been used to derive bending moments in a propped contiguous bored pile retaining wall. Rigorous procedures used during data collection and thorough analysis and error checking of the inclinometer data produced good wall profiles. Careful consideration to the mathematical restraints to curve fitting have allowed wall bending moment profiles to be found which show good agreement with bending moment profiles found from vibrating-wire strain gauges. Identification of concreting cracking and analysis of the effect this has had on the bending moment profiles calculated from the inclinometer measurements has explained the differences in bending moment plots found by two different methods after cracking had occurred and has allowed the relationship between the different values to be realized.

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