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# Reliability of API and ISO Guidelines for Bearing Capacity of Offshore Shallow Foundations

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**Abstract.** The safe bearing capacity for offshore shallow foundations has been traditionally assessed using working stress design (WSD) methods (e.g. the API RP 2GEO guideline). Other codes of practice such as the ISO standard strive to provide designs achieving a desired target reliability level in the form of the Load and Resistance Factor Design (LRFD) approach. This study compares the levels of safety achieved for offshore shallow foundations. Calculations are made for one foundation on soft clay and one on medium dense sand, using the API RP 2GEO, API RP 2GEO-LRFD and ISO 19901-4 design guidelines. Three probabilistic models were used, the first-order, second moment (FOSM) approximation, the first order reliability method (FORM) and the Monte Carlo simulation (MC) approach, to do the reliability assessment. The results showed that the reliability level achieved with current practice varies with the design methods. The FORM and MC models yielded consistent results, while the FOSM model yielded inconsistent results when the performance function was non-linear.

**Keywords.** reliability, shallow foundation, offshore design codes, bearing capacity, LRFD

## 1. Introduction

The bearing capacity of a shallow foundation is usually evaluated with a working stress design (WSD) format with a lump Factor of Safety (FoS). The lump FoS accounts for natural variability of soil properties, measurement errors, statistical uncertainty, analytical model uncertainty and foundation load variation. In the last several decades, load and resistance factor design (LRFD) has received increasing attention in the geotechnical design of shallow foundations as reflected in several new codes of practice (e.g. ISO 19901-4). The LRFD approach attempts to separate to some extent the different sources of uncertainty. The load factor accounts for the uncertainty in the loads, whereas the resistance factor (or material factor) takes into account the uncertainties related to soil properties, testing and calculation models. With such a formulation, the LRFD approach enables an improved consideration of the uncertainties.

The recently developed API RP 2GEO (2011) for geotechnical designs retains the traditional WSD. An LRFD version of RP 2GEO was also developed to align the guideline with the ISO standard 19901-4 (2003) for shallow foundations. To assess whether or not the guidelines and standards provide a consistent level of reliability, three probabilistic models, the first-order, second moment (FOSM) approximation, the first order reliability method (FORM) and the Monte Carlo simulation (MC), were used. Two shallow foundations were studied with different load combinations, one on a soft clay, the other on a medium dense sand.

## 2. Ultimate Bearing Capacity

### 2.1. API RP 2GEO Guideline

#### 2.1.1. Undrained Bearing Capacity

With the API RP 2GEO guideline, the undrained bearing capacity for a shallow foundation on a clay with shear strength increasing linearly with depth is:

$$Q_d = F\left(N_c s_{u,0} + kB'/4\right)K_c A' \quad (1)$$

where $F$ is a factor function of $kB'/s_{u,0}$; $k$ is the rate of increase of undrained shear strength with depth; $s_{u,0}$ is the undrained shear strength of the soil at the foundation base level; $N_c = 5.14$; $B'$ is the minimum effective lateral foundation dimension; $A'$ is the effective area of the foundation depending on the load eccentricity; $K_c$ is a factor to account for load inclination, footing shape, depth of embedment, inclination of base, and inclination of the seafloor.

### 2.1.2. Drained Bearing Capacity

The API RP 2GEO drained bearing capacity for shallow foundation is evaluated from:

$$Q_d = \left\{ p_0^{'} \left( N_q - 1 \right) K_q + 0.5\gamma^{'} B^{'} N_\gamma K_\gamma \right\} A^{'} \qquad (2)$$

where $p_0'$ is the vertical effective overburden stress at base level; $N_q = exp(\pi \ tan\phi') \ tan^2(45°+\phi'/2)$; $N_\gamma = 1.5 \ (N_q-1) \ tan\phi'$; $K_q$, $K_\gamma$ are the factors to account for load inclination, footing shape, depth of embedment, inclination of base, and inclination of the seafloor; $\gamma'$ is the submerged unit weight of soil.

## 2.2. API RP 2GEO-LRFD Guideline

API RP 2GEO-LRFD is a hybrid of the API RP 2GEO using the load and resistance factors from API 2A-LRFD. The formulations to calculate the undrained and drained bearing capacity are identical to those of API RP 2GEO. The factored capacity is resistance factor $\phi$ times calculated capacity above.

## 2.3. ISO 19901-4 Standard

### 2.3.1. Undrained Bearing Capacity

With the ISO 19901-4 standard, the undrained bearing capacity for a foundation on a clay with shear strength increasing linearly with depth is:

$$Q_d = \left\{ F \left( N_c s_{u,0} + kB^{'}/4 \right) K_c / \gamma_m + p_0^{'} \right\} A^{'} \qquad (3)$$

where $\gamma_m$ is the material factor. (the other parameters are identical as in API RP 2GEO).

### 2.3.2. Drained Bearing Capacity

The ISO 19901-4 drained bearing capacity for shallow foundation is evaluated from:

$$Q_d = \left\{ \left( p_0^{'} + a \right) N_q K_q + 0.5\gamma^{'} B^{'} N_\gamma K_\gamma - a \right\} A^{'} \qquad (4)$$

where $N_q = exp(\pi \ tan\phi'/\gamma_m) \ tan^2(45°+0.5arctan \ (tan\phi'/\gamma_m))$; $N_\gamma=1.5 \ (N_q-1) \ (tan\phi'/\gamma_m)$; $a$ is the soil attraction and $a=c' \ cot\phi'$, $c'$ is the cohesion intercept in terms of effective stresses.

## 2.4. Required Safety Factors

Table 1 lists the required safety factor, resistance factor and material coefficient for bearing capacity by the three guidelines.

**Table 1.** Design check factors for three guidelines

| Guidelines | Safety factors | Value |
|---|---|---|
| API RP 2GEO | Global *FS* | 2.0 |
| API RP 2GEO-LRFD | $\phi$ (on capacity) | 0.67 |
| ISO 19901-4 | $\gamma_m$ (on soil property) | 1.5 (undrained)<br>1.25 (drained) |

# 3. Probabilistic Methods

## 3.1. First Order Second Moment (FOSM)

As a practical approximation, the safety factor *SF* (the ratio of foundation capacity to the load) can be modelled as a lognormal variable. The probability of foundation failure can then be formulated as follows:

$$p_f = p\left(\ln\left(SF\right) < \ln 1\right) = \Phi\left(-\ln\left(\mu_{SF}\right)/\delta_{SF}\right) \qquad (5)$$

where $\Phi(.)$ is the cumulative standard normal function, and $\mu_{SF}$ and $\delta_{SF}$ are the mean value and coefficient of variation of the safety factor, respectively. For a function of multiple random variables, the mean and variance of safety factor can be approximated by:

$$\mu_{SF} = g\left(E\left(x_1\right), E\left(x_2\right), ..., E\left(x_n\right)\right) \qquad (6)$$

$$\sigma_{SF}^2 = \sum_{i=1}^{n} \left[ \left( \partial g / \partial x_i \right)^2 \sigma_{x_i}^2 \right] \qquad (7)$$

where $n$ denotes the number of random variables $x_i$.

The finite difference approximation of the derivatives, e.g. $\partial g / \partial x_1$, can be approximated by (US Army Corps of Engineers, 1997):

$$\frac{\partial g}{\partial x_1} = \frac{g(\mu_1 + \sigma_1, \ldots, \mu_n) - g(\mu_1 - \sigma_1, \ldots, \mu_n)}{2\sigma_1} \tag{8}$$

where $\mu_1$ and $\sigma_1$ are the mean and standard deviation of $x_1$ respectively.

### 3.2. First Order Reliability Method (FORM)

This method, proposed by Hasofer and Lind (1974), calculates the reliability index β from:

$$\beta = \min_{x \in F} \sqrt{\left[\frac{x_i - \mu_i}{\sigma_i}\right]^T [R]^{-1} \left[\frac{x_i - \mu_i}{\sigma_i}\right]} \tag{9}$$

where $\mu_i$ and $\sigma_i$ are the mean and standard deviation of $x_i$ respectively; $R$ is the correlation matrix; $F$ is the failure domain, i.e. where $g(x) = 0$.

### 3.3. Monte Carlo Simulation (MC)

Monte Carlo simulations were done to validate the results obtained by the FOSM and FORM analyses. Each simulation generated 5,000,000 sets of random numbers.

## 4. Realistic Design Examples

The design examples investigated in this study were similar to those analysed by Gilbert (2013).

### 4.1. Case 1- Well manifold with vertical load on normally consolidated highly plastic clay

The loads and clay characteristics are shown in Figure 1. The vertical load is due to the weight of the manifold and jumpers. Maximum load occurs during the first year. The undrained shear strength, $s_u$, was characterized primarily with miniature vane shear strength tests on samples from borings, jumbo piston cores and box cores and with Halibut remote vane shear tests.

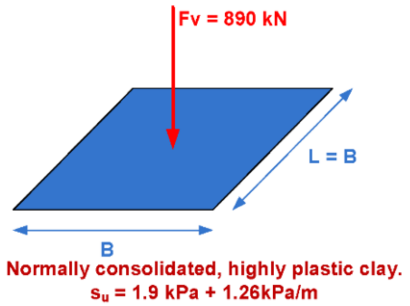


**Figure 1.** Case 1– Well manifold with vertical load on a normally consolidated highly plastic marine clay.

### 4.2. Case 2- Subsea isolation valve with inclined load on medium dense sand

The loads and sand characteristics for this case are shown in Figure 2. The vertical load is due to the weight of the valve. The horizontal load is a short-term, extreme load due to winds, waves and currents. Maximum environmental load can occur at any time during the 30-yr design life. The strength of the medium sand was characterized using driven sampler blow counts from one boring and one cone penetration test.

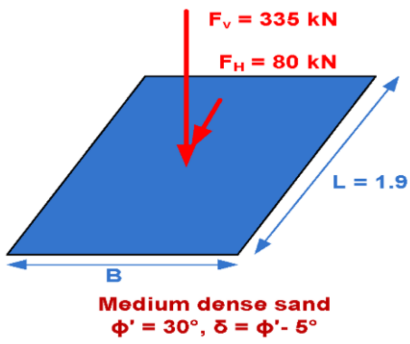


**Figure 2.** Case 2– Subsea isolation valve with inclined load on medium dense sand.

## 5. Input Parameters

The limit state function was taken from Eq.(5):

$$g = M * SF - 1 \tag{10}$$

where $M$ is model uncertainty, and is often formulated as:

$$M = \frac{Observed\ foundation\, capacity}{Predicted\ capacity} \tag{11}$$

Tables 2 and 3 list the statistics for the random variables in the bearing capacity analyses.

**Table 2.** Input parameters for Case 1 reliability analyses

| Random variable | Bias* | COV | Distribution |
|---|---|---|---|
| Vertical load $V$ | 1.0 | 0.05 | Lognormal |
| Undrained shear strength $s_u$ | 1.1 | 0.15 | Lognormal |
| Bearing capacity model $M$ | 1.1 | 0.15 | Lognormal |

* Bias is defined as ratio of actual value to mean value

**Table 3.** Input parameters for Case 2 reliability analyses

| Random variable | Bias | COV | Distribution |
|---|---|---|---|
| Vertical load $V$ | 1.0 | 0.05 | Lognormal |
| Horizontal load $H$ | 0.9 | 0.15 | Lognormal |
| $tan\phi'$ | 1.2 | 0.05 | Lognormal |
| $M_{tan\phi'}$ * | 1.13 | 0.14 | Lognormal |
| $M_{tan\phi'}$ ** | 0.99 | 0.14 | Lognormal |

* For API RP 2GEO ** For ISO 19901-4

### *5.1. Uncertainty in load*

The vertical load due to the self-weight of structures is generally relatively well known within the specified material tolerances. A COV of 0.05 was used for the vertical load.

The uncertainty in the horizontal load due to environmental loads, including extreme storm loading, is more complex than for the dead load. The live loads used in design are usually based on the maximum (extreme) live load experienced by the structure over the structure's lifetime. A bias of 0.85 and COV of 0.15 was assumed for the lifetime extreme live load in the present analyses.

The lognormal distribution is a good distribution for modelling variable loads with large coefficients of variation because of the heavy tail in the positive direction and no negative load values. The variations in the vertical and horizontal loads were assumed to be independent in Case 2.

### *5.2. Uncertainty in soil properties*

A bias of 1.1 and a COV of 0.15 were assumed for $s_u$ in clay and the COV of 0.05 for $tan\phi'$ was assumed. The assumed value of 30° is a rather conservative estimate for a medium dense sand. Therefore, a bias of 1.2 was used for the tangent of the friction angle.

Lacasse and Nadim (1996), and others, suggested that both normal and lognormal distributions can be used for describing the undrained shear strength and friction angle. To avoid negative values, lognormal distributions were assumed for both $s_u$ and $tan\phi'$.

### *5.3. Model uncertainty*

Several studies have been conducted to quantify the model uncertainty in the undrained bearing capacity of a shallow foundation. Nadim and Lacasse (1992) used a mean of 1.0 and a COV of 0.1 to account for model uncertainty in the bearing capacity of spudcan foundations for a jack-up structure under vertical loading. This model uncertainty was based on comparisons of observed and predicted spudcan penetrations from the literature. Forrest and Orr (2011) used a mean bias of 1.0 and a range of COV values between 0 and 0.2 for the model uncertainty in the undrained bearing capacity of footings under a variety of loading conditions. A relatively large COV of 0.15 and a bias of 1.1 are used for the present analyses.

For drained bearing capacity, the model uncertainty, $M$ was formulated as a multiplier on the $\tan\phi'$ term in the calculation method. Figures 3 and 4 present the results of lognormal distribution fit through the left-hand tail (i.e. percentiles less than 30%) of the cumulative frequency distribution of model uncertainty factor $M$ for the API and ISO methods based on a database of field load tests for footings on coarse-grained materials (Akbas, 2007, Akbas and Kulhawy, 2009, Lai 2013). A mean ($b_M$) of 1.13 and a COV ($\Omega_M$) of 0.14 in $M$ were obtained for the API RP 2GEO method. A bias of 0.99 and a COV of 0.14 for $M$ were obtained for the ISO 19901-4 method.

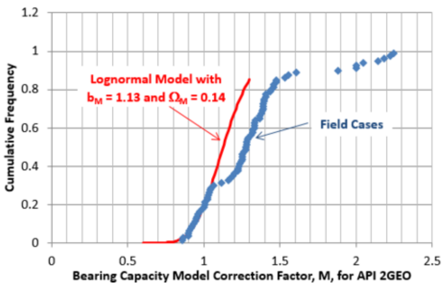


**Figure 3.** Cumulative frequency for the API RP 2GEO model correction factor from field load tests, drained capacity of shallow foundations.

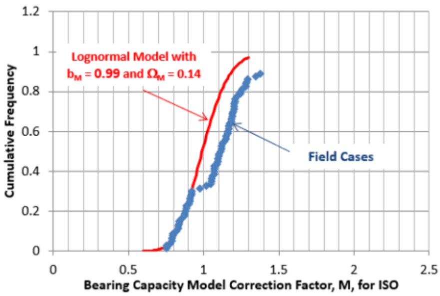


**Figure 4.** Cumulative frequency for the ISO model correction factor from field load tests, drained capacity of shallow foundations.

## 6. Resuts of Reliability Analyses

The reliability analyses compared the probability of failure obtained with the FOSM, FORM and MC approaches at the prescribed design check factors (listed in Table 1). The results are presented in Figures 5 to 10. The graphs show the calculated probability failure (horizontal axis) for different values of the safety parameter (vertical axis). The prescribed safety parameter is indicated by a horizontal line in the graph.

### *6.1. Undrained Bearing Capacity Failure*

Figures 5 to 7 show how probability of failure varies with design the safety parameter for Case 1 using the three guidelines and the three probabilistic methods. The probability of failure for the three guidelines with the three reliability methods ae very close. This is due to the limit state function being quite linear.

The probabilities of failure for the API RP 2GEO guideline with a factor of safety of 2, the API RP 2GEO-LRFD guideline with a resistance factor of 0.67 and the ISO 19901-4 standard with a material coefficient of 1.5 are $4.1\times10^{-6}$, $9.0\times10^{-6}$ and $8.0\times10^{-6}$, respectively (FORM-results).

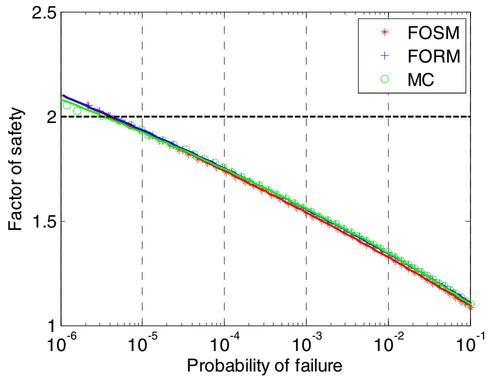


**Figure 5.** Probability of bearing capacity failure for API RP GEO factor of safety for Case 1.

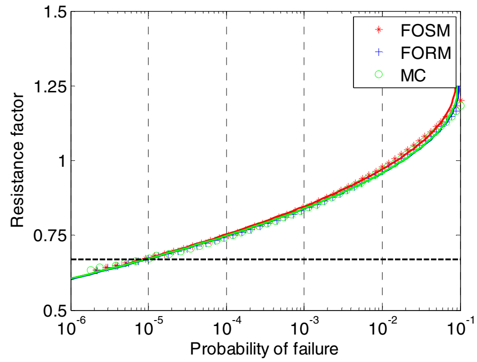


**Figure 6.** Probability of bearing capacity failure for API RP GEO-LRFD resistance factor for Case 1.

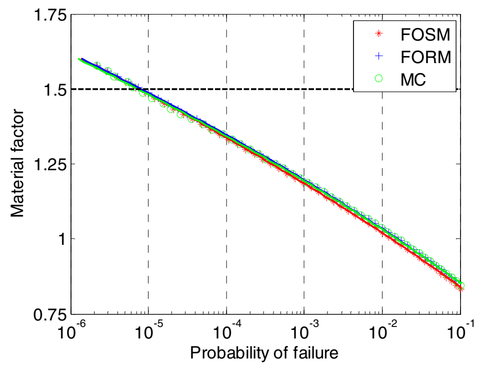


**Figure 7.** Probability of bearing capacity failure for ISO 19901-4 material factor for Case 1.

### *6.2. Drained Bearing Capacity Failure*

Figures 8 to 10 show the probability of failure over 30-yr design life for Case 2. The FORM and

MC results are very similar, even if the limit state function is very nonlinear. The FOSM results, however, differ significantly from the FORM and MC results. The assumed linearized limit state function around its mean point in the FOSM formulation is the explanation for the difference.

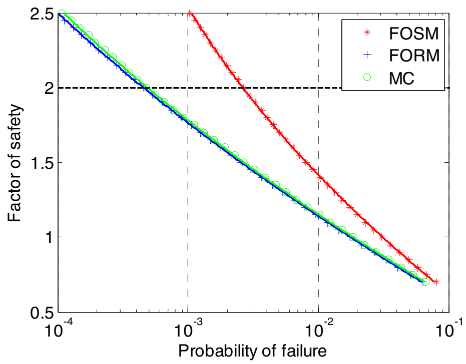


**Figure 8.** Probability of bearing capacity failure for API RP GEO factor of safety for Case 2.

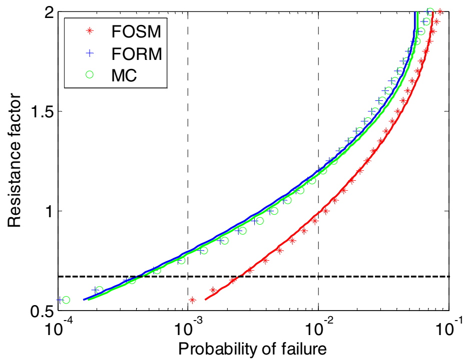


**Figure 9.** Probability of bearing capacity failure for API RP GEO-LRFD resistance factor for Case 2.

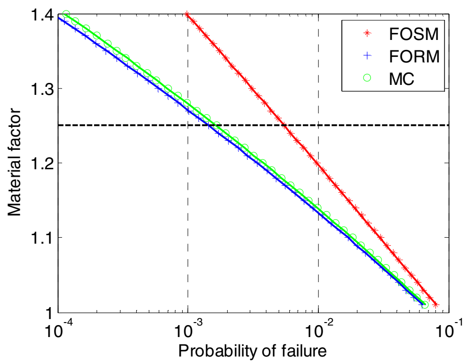


**Figure 10.** Probability of bearing capacity failure for ISO 19901-4 material factor for Case 2.

The probability of failure for the API RP 2GEO guideline with a factor of safety of 2, the API RP 2GEO-LRFD guideline with a resistance factor of 0.67, the ISO 19901-4 standard with a material coefficient of 1.25 are $4.6\times10^{-4}$, $4.1\times10^{-4}$ and $1.4\times10^{-3}$, respectively(again FORM results). The corresponding probability of failure for the API RP 2GEO, API RP 2GEO-LRFD and ISO 19901-4 guidelines obtained with FOSM approximation are $2.7\times10^{-3}$, $2.6\times10^{-3}$ and $5.5\times10^{-3}$, respectively. The degree of divergence between the FOSM and FORM-MC results varies with the size of the safety parameter used as reference.

## 7. Conclusions

This study illustrated the bearing capacity of a shallow foundations founded on soft clay and on medium dense sand with deterministic and probabilistic analysis methods. The following conclusions were reached:

(1) For both soils, the reliability level being achieved with current practice varies depend on the design methods.

(2) The FORM and Monte Carlo simulation approaches gave similar reliability level for both clay and sand.

(3) The FOSM approach gave a reliability level similar to that from FORM and MC for a shallow foundation on clay, where the limit state function is quite linear.

(4) The FOSM approximation overestimated the probability of failure for sand by a factor of about 4 to 6, and would therefore result in different foundation size for the same reliability level.

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