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Probabilistic Slope Stability Analyses Using Limit Equilibrium and Finite Element Methods

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Abstract. This paper compares the results of different probabilistic approaches and emphasizes the necessity of probabilistic analyses in slope stability studies. To do that, Limit Equilibrium Method (LEM) and Finite Element Method (FEM) are utilized and their outputs are compared in terms of probability of failure (PF), reliability index (RI), factor of safety (FS) and the failure surface. Lastly, concept of Random Finite Element Method (RFEM) is studied and effects of spatial correlation distance are investigated.

Keywords. Slope Stability, deterministic, probabilistic, failure surface, limit equilibrium method, finite element method

1. Introduction

Risk and safety assessment of dikes, earth dams, open pit mines, tailing dams, landfills and natural slopes are becoming more and more important for proper management and mitigation of natural hazards. Considering the heterogeneity and uncertainty in material properties, together with changes and variability in environmental factors, a probabilistic evaluation of slope stability becomes inevitable.

Limit Equilibrium Method (LEM) and Finite Element Method (FEM) are widely-used approaches for evaluating the stability of slopes. One of the advantages of the FEM is that it does not make any a priori assumptions about the shape or location of the critical failure surface. An extension of FEM, the Random Finite Element Method (RFEM) estimates the probability of failure of slopes while accounting for spatial variability of material properties through spatial correlation length. Therefore, it provides further benefits in slope stability evaluations.

In this study, probabilistic LEM, FEM, and RFEM are utilized for the analyses of a benchmark slope problem from the literature.

The results of this study are believed to be useful for further understanding of the probabilistic slope stability concept and effects of soil heterogeneity on slope stability with the aim of better geotechnical risk management and communication.

2. Comparison between LEM and FEM (r_u=0.2 case)

2.1. Input Parameters

A benchmark slope problem with a simple geometry is chosen from the literature for the analyses. It is a homogeneous slope with a height of 10 m and 1V:1H slope inclination. Its geotechnical parameters and slope geometry are retrieved from Bhattacharya et al. (2003) and shown in Table 1 and Figure 1, respectively.

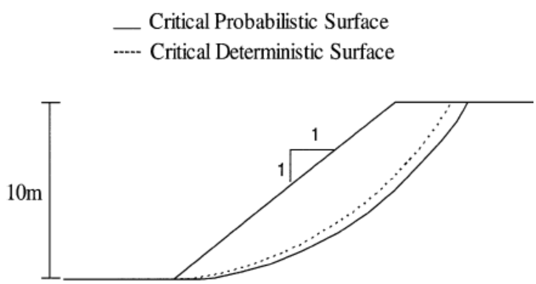

Figure 1. Geometry of the slope together with deterministic and probabilistic critical failure surfaces obtained by Bhattacharya et al. (2003)

Table 1. Material properties (Bhattacharya et al. 2003)

Parameter	Mean	COV (%)	Std. Dev.
c'	18 kN/m^2	20	3.6 kN/m^2
Φ'	30°	10	3°
r_u	0.2	10	0.02
γ	18 kN/m^3	5	0.9 kN/m^3

2.2. Finite Element Analysis

In order to carry out a Finite Element Analysis, Rocscience Phase2 v.8.0 is utilized. After defining the slope geometry in the software, both deterministic and probabilistic analyses are conducted separately. For the probabilistic analysis, Rosenblueth's Point Estimate Method, which is the only probabilistic method available in version 8.0 of the software, is used. The initial and boundary conditions are defined, and Mohr-Coulomb model is used for the constitutive model of the soil.

For the probabilistic analysis, effective cohesion and effective internal friction angle of the soil are used as random variables whose statistical parameters are given in Table 1. Other parameters are considered to be known with more certainty and used as deterministic input variables. The magnitude of Young's Modulus and Poisson's ratio do not affect the location of the critical failure surface and the value of the strength reduction factor (SRF) of the slope, since displacements and stresses are out of interest and location of the critical slip surface are determined by incremental shear stress values. Therefore, they are tried to be selected as reasonable values such as the Young's modulus of 50000 kPa and Poisson's ratio of 0.4. Dilation angle is taken as zero. Results from this section are reported in the comparison sub-section together with LEM results.

2.3. Limit Equilibrium Analysis

In order to carry out Limit Equilibrium Analyses, Rocsience Slide v.6.0 and Slope/W of GeoStudio 2012 are utilized. In Slope/W, both deterministic and probabilistic analysis with 1000 Monte-Carlo (MC) trials are carried out. For the sake of consistency, only the strength parameters, namely cohesion and friction angle, are used as random variables. Other variables are kept at their mean values.

For the probabilistic analyses in Rocscience Slide, one can use both "Global Minimum Method (GM)" and "Overall Slope Method (OS)" together with Monte-Carlo and Latin Hypercube (LH) sampling methods. GM is one of the probabilistic analyses type in which deterministic slope stability analysis is carried out using the mean values of all parameters and one critical slip surface is found. Then, using the generated samples of material properties as random variables, probabilistic analysis is carried out only for this slip surface. In the end, one slip surface with FS, PF, reliability index (RI) value is obtained. Unlike GM, however, entire search is repeated N times where N is the number of random samples generated for the OS type probabilistic analysis. It results in several slip surfaces and it can also generate the most critical failure surface. If a pre-defined slip surface is used for both methods, instead of searching for the most critical failure surface, both methods naturally give the same results.

For this particular case, OS type of analyses with both MC and LH are carried out since it is more likely to obtain different slip surfaces for different c-ϕ pairs. Among the many available limit equilibrium method of slices, such as Bishop, Swedish, Janbu and Morgenstern etc., Spencer's method is preferred since it satisfies all equilibrium conditions (e.g. overall moment, individual slice moment, horizontal and vertical force equilibriums) and it is suitable for slip surface of any shape (Pockoski and Duncan, 2000). For all limit equilibrium, as well as FEM analyses, Mohr-Coulomb strength type is used in the analyses. Normal distribution for random variables is chosen in the probabilistic analyses.

For this section, all analyses are carried by considering circular failure surfaces.

2.4. Comparison of Results

In addition to Bhattacharya et al. (2003), this case was previously studied by Li and Lumb (1987) and Hassan and Wolff (1999) and their results are reported in Table 2 together with the results of aforementioned analyses. In Table 2, both deterministic and probabilistic analyses for aforementioned methods are tabulated and values corresponding to critical probabilistic slip surface (surface of minimum RI) are reported from previous studies for the sake of comparison.

Table 2. Comparison of different methods

Method	PF (%)	FS or SRF	RI
Li and Lumb (1987)	-	-	2.500
Hassan and Wolff (1999)	-	-	2.293
Bhattacharya et al. (2003)	-	1.337	2.239
Slide (Deterministic)	-	1.358	-
Slide (Monte-Carlo)	1.70	1.367	2.279
Slide (Latin Hypercube)	1.60	1.371	2.244
Slope/W (Monte-Carlo)	1.10	1.365	2.310
Phase2	3.37	1.350	-

Critical probabilistic and deterministic circular failure surfaces of Bhattacharya et al. (2003) are shown in Figure 1. Critical slip surfaces from other methods obtained in this study are shown in Figure 2, together with the background slip surface zone obtained from probabilistic FEM (Rocscience Phase2).

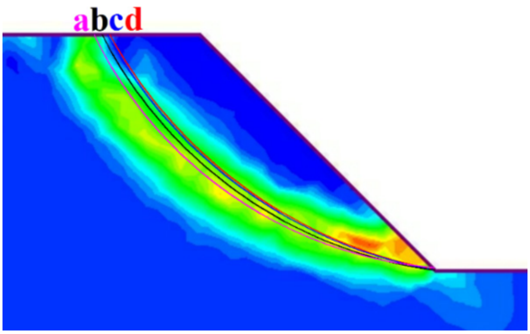

Figure 2. Comparison of the most critical slip surfaces for the case of r_u=0.2: (background) slip surface zone by Rocscience Phase2, (a) GeoStudio SlopeW, (b) Rocscience Slide-Deterministic, (c) Rocscience Slide Monte-Carlo (d) Rocscience Slide Latin-Hypercube.

As expected in homogeneous slopes, the most critical slip surfaces obtained from different probabilistic and deterministic limit equilibrium methods are practically the same. The values of the factor of safety obtained by different methods are very similar (Table 2). The values of the probability of failure by different methods are similar (they are in the range of 1.10 to 3.37%). However, as it can be seen from Table 2, FEM (e.g. Phase2) gives quite higher PF compared to other methods although their FS/SRF values are close to each other. The slip zone of the probabilistic finite element method is also slightly different from the rest of the slip surfaces (Figure 2).

3. Comparison between LEM and FEM (Dry Case)

The analyses in the previous section were for the case of pore pressure coefficient, r_u, value of 0.2. The similar analyses are reproduced here with the r_u value equal to zero (e.g. dry slope). Respective results are given in Table 3. Slip surface comparisons are shown in Figure 3. Similar result like the one obtained in the previous section is observed.

Table 3. Comparison of different methods (dry case)

Method	PF (%)	FS or SRF	RI
Slide (Deterministic)	-	1.617	-
Slide (Monte-Carlo)	0	1.659	3.447
Slide (Latin Hypercube)	0	1.656	3.411
Slope/W (Monte-Carlo)	0	1.630	3.536
Phase2	0.15	1.610	-

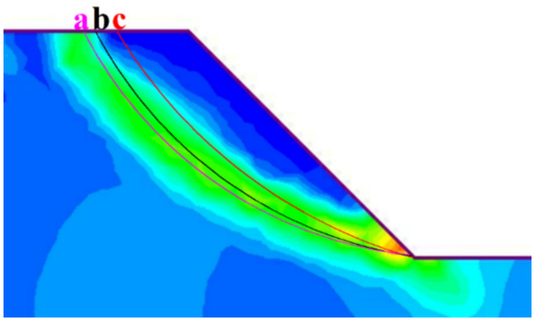

Figure 3. Comparison of the most critical slip surfaces for dry slope case: (background) slip surface zone by Rocscience Phase2, (a) GeoStudio SlopeW, (b) Rocscience Slide-Deterministic, (c) Rocscience Slide, Monte-Carlo and Latin-Hypercube

4. Relation between PF and FS

4.1. Analyses Procedure and Input Parameters

In this section, tendency of PF values for different safety levels is tried to be captured. For this purpose, slope and procedure given in the "section 2" is used. The procedure is repeated for dry case of the slope as well. For the analyses, GeoStudio Slope/W, Rocscience Phase2 and

Slide are utilized as explained in the "section 2.2. and 2.3."

In order to see the effect of different slope safety levels (i.e. different FS values), on PF, keeping the slope height constant, slope angle of the geometry is changed systematically from 45° to 70°, therefore obtaining FS values in the range of 0.79 to 1.37, in the same soil. For each analysis, PF and FS values are obtained and FS vs. PF graphs are created (Figure 4). Slope angles used in the analyses are 45, 47.5, 50, 53 (4V:3H), 55, 57.5, 60, 63 (2V:1H), 65, 67.5 and 70.

4.2. Comparison of Results

The results of FS vs. PF graph for wet slope case ($r_u = 0.2$) is given in Figure 4 whereas that of dry case ($r_u = 0$) is given in Figure 5. From the figures, it can be seen that all approaches show the same pattern and tendency having two asymptotical values at each end. It can also be seen that GeoStudio Slope/W results cover narrower range than that of others. In wet slope case ($r_u = 0.2$), the PF corresponding to a factor of safety of 1.00 is about 44 to 62%, and the probability of failure for FS of 1.20 is 4 to 11%. As expected, as the safety level of the slope (FS) increases, the PF decreases (in both wet and dry slope cases, using all methods). In the case of wet slope ($r_u = 0.2$), the FS value of 1.20 which is greater than 1.00 and may be considered as "a safe slope" in engineering practice, still has a PF in the range of 4 to 11%. In dry slope case, all of the FS values are larger than 1.00; for FS values between 1.08 and 1.18 the PF is 12 to 30%.

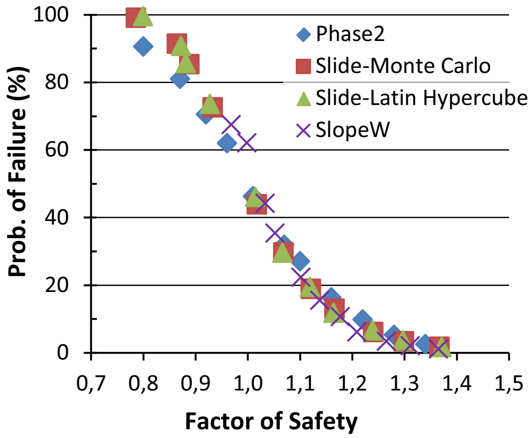

Figure 4. Relation between probability of failure and factor of safety for different approaches ($r_u = 0.2$)

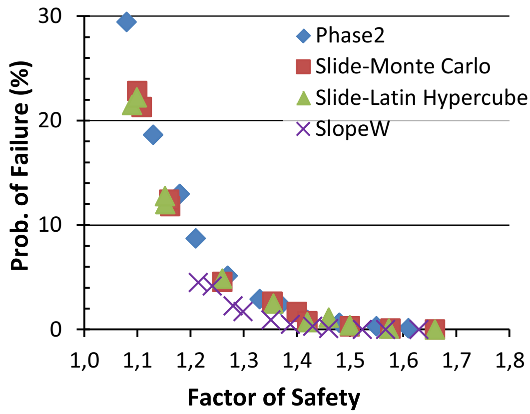

Figure 5. Relation between probability of failure and factor of safety for different approaches ($r_u = 0$)

5. Effect of the Spatial Correlation Distance in RFEM

5.1. Procedure and Input Parameters

As explained in the introduction section, RFEM is an extension to FEM. In this method, random fields of material properties are generated and mapped onto the finite element mesh. The spatial variation of material properties can be correlated to each other by using "spatial correlation length", which is sometimes referred to in the literature, as, the "scale of fluctuation". This parameter describes the distance over which spatially random variables will tend to be significantly correlated (Griffiths and Fenton, 2004). Therefore, large values of spatial correlations length means smoothly varying (more uniform) field. Theoretically, the value of infinity would mean a homogeneous field. This value roughly means that soil samples taken close to each other will be more likely to have similar material properties than that of faraway samples. There is also “anisotropic spatial correlation” in which soil is likely to have longer spatial correlation lengths in the horizontal direction compared to vertical direction since most soils are deposited vertically (Griffiths and Fenton, 2004). However, the literature about spatial correlation length is quite insufficient and not well documented. In the case of an insufficient data, the spatial correlation length can be used as 0.1 to 0.25 of the size of the problem geometry in each direction (Griffiths and Fenton, 2004). One of the distinct features of this method is that it can seek-out the weakest, most critical path and it does not have to be a certain shape such as a circular surface (Griffiths and Fenton, 2004).

In the literature, there are some comments about the results of RFEM compared with simple approaches without spatial correlation. Some researchers (e.g. El-Ramly et al. 2002; Ji et al. 2013) concluded that ignoring the spatial variability results in overestimation of PF. In other words, not accounting for the spatial variability will result in higher PF and will overestimate the risk of failure compared to classical methods without spatial correlation. Some researchers claim the opposite (e.g. deWolfe et al. 2010). Griffiths and Fenton (2004) stated that ignoring the spatial variability will overestimate PF when COV is relatively small, whereas it will underestimate the PF when COV is relatively high.

In order to carry out the RFEM analyses, Mrslope2D software created by G.A. Fenton and D.V. Griffiths in 1992 is used. It is an open-source coded, publically available, and free of charge software. For this section, slope geometry and parameters given in Figure 1 and Table 2 are used with the exception of r_u which is used as 0 (dry case). All three parameters in Table 2 are used as a random variables. To be on the verge of failure, slope angle is used as 63° (2V:1H) like in section 4 and several correlation distances are used for analyses. In other words, only the effect of correlation distance is tried to be observed. COV values are kept as given in Table 2. By having different ratios between correlation distance in x-direction and y-direction, anisotropic behavior is also tried to be seen. Correlation distances and ratios used for the analyses are given in Table 4. Young’s Modulus and Poisson’s ratio is taken as 50000 kPa and 0.4, respectively, like in section 2.2. Dilation angle is taken as zero.

Table 4. Spatial Correlation lengths and ratios used in the analyses

Corr.Length_x/Corr.Length_y	Corr.Length_x (meter)
1	6, 8, 10, 12, 14, 16, 18, 20
2	6, 8, 10, 12, 14, 16, 18, 20
4	6, 8, 10, 12, 14, 16, 18, 20
8	6, 8, 10, 12, 14, 16, 18, 20
10	6, 8, 10, 12, 14, 16, 18, 20

5.2. Comparison of RFEM Results

After the analyses, spatial correlation length in x-direction vs. PF graph for each ratio is plotted in Figure 6. From the figure, it can be said that, given that the other parameters are constant, as the correlation length in the x-direction increases, probability of failure also increases. It can be also said that as the ratio of correlation lengths in x and y direction decreases (e.g. correlation length in y-direction increases), probability of failure also increases.

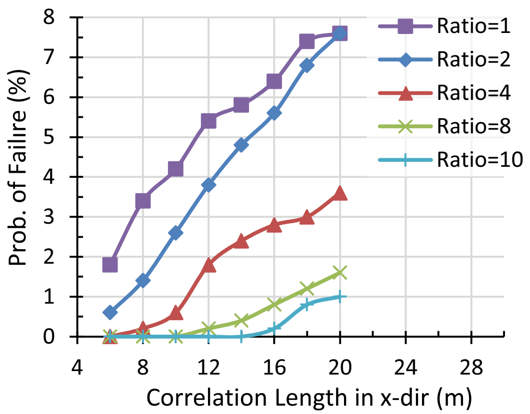

Figure 6. Effect of spatial correlation length and its anisotropy in probability of failure

6. Conclusion

In this study, different probabilistic approaches are utilized for a simple slope geometry and their results are compared in terms of slip surface, probability of failure, reliability index and factor of safety. In addition to that, concept of Random Finite Element Method is used to demonstrate the effect of spatial correlation distance on the probability of failure. The effect of anisotropy in spatial correlation length is also investigated.

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