Geotechnical Safety and Risk V
T. Schweckendiek et al. (Eds.)

doi:10.3233/978-1-61499-580-7-689

Safety Assessment of a Shallow Foundation Using the Random Finite Element Method

Łukasz ZASKÓRSKI, Wojciech PUŁA
Faculty of Civil Engineering, Wrocław University of Technology, Poland

Abstract. In this paper, the reliability assessment of the bearing capacity of a shallow strip footing was conducted using a reliability index β. In applicable standards (Eurocode 7) there is not presented any explicit method of an evaluation of characteristic values of soil parameters. Therefore some approaches of an estimation of characteristic values of soil properties were compared by evaluating values of reliability index β. Method of Orr and Breysse, Duncan's method, two Schneider's methods and suggestion included in Eurocode 7 were examined. Design values of the bearing capacity based on these approaches were referred to the stochastic bearing capacity estimated by the random finite element method (RFEM). A probability distribution of the bearing capacity was estimated basing on 3000 simulations. Assessed probability distribution was applied to compute design values of the bearing capacity and related reliability indices β. Conducted analysis were carried out for a cohesive soil. Hence a friction angle and a cohesion were defined as a random parameters and characterized by two dimensional random fields. Design values of the bearing capacity were evaluated for various widths and depths of a foundation in conjunction with design approaches DA defined in Eurocode 7.

Keywords. Eurocode 7, bearing capacity, random field, probability distribution, reliability index

1. Introduction

The explicit method of an evaluation of characteristic values of soil parameters cannot be found in the applicable standards - Eurocode 7. In general terms Eurocode 7 leaves wide scope for interpretation. Therefore the present study is focused on comparing commonly used approaches of estimation of characteristic values of soil parameters (Duncan's method, Schneider's method, Schneider's method with impact of a fluctuation scale, Orr and Breysse's method, method based on 5% quantile which is included in Eurocode 7) by assessing a shallow foundation bearing capacity in accordance with the guidelines contained in Eurocode 7. Reliability index β was adopted as a criterion for comparison of the above methods. Therefore it was tested what values of reliability index can be obtained using different methods for various foundation widths and depths. Values of reliability index β were obtained on the basis of the bearing capacity estimated by random finite element method (RFEM).

2. Characteristic Values of Parameters According to Eurocode 7

Any accurate algorithm of evaluation of characteristic values of soil parameters is not given in Eurocode 7. Only general guidelines can be found how these values should be estimated. Namely, Eurocode 7 mentions that the characteristic value of a soil property shall be selected as a cautious estimation of the value affecting the occurrence of the limit state. More precisely the characteristic value is a cautious estimation of a mean value over a certain volume of the ground governing the behavior of a geotechnical structure at a limit state. In fact this zone is more extensive than the zone in a soil tests. A crucial issue is estimation of soil parameters, which could represent whole area. Abovementioned guidelines leave much room for interpretation for designers, who determine values they adopt in calculations.

Eurocode 7 recommends that in the case of applying of statistical methods, the characteristic value should be estimated with the level of significance $\alpha = 0.05$. It means that the characteristic value of a parameter should be estimated as 5% quantile basing on a probability distribution of this parameter. Properties of materials, such as concrete or steel, are often described by a normal distribution, hence a characteristic value of a parameter X_k can be evaluated using the equation

$$X_k = \mu(X) - 1.645\sigma(X) \tag{1}$$

where $\mu(X)$ and $\sigma(X)$ are a mean value and a standard deviation of the parameter X, respectively. The factor 1.645 is directly connected with 5% quantile and comes from a standard normal distribution table. Although an assumption of a normal distribution to characterization of soil strength parameters is not adequate and can lead to unrealistic characteristic values of these parameters. It is connected with significant values of coefficient of variation of soil parameters which can result in negative characteristic values of parameters. The next reason for limited applicability of Eq. (1) is fact that the area of ground responsible for a collapse mechanism is greater than the zone in a soil test. It should be emphasized that a value of a parameter related with the occurrence of the limit state is a certain mean value corresponding with the slip surface and not a locally measured value. Further important factor is that geotechnical designing is generally based on small number of test results. In consequence a mean value and a standard deviation of a soil parameter obtained from in situ tests cannot be similar to a mean value and a standard deviation of this parameter corresponding with the zone responsible for the occurrence of the limit state. An adequate example was given by Orr and Breysse (2008).

Orr and Breysse (2008) proposed method of an evaluation of a characteristic values which in better way cope with abovementioned issues. This method is based on a well-known formula for a confidence interval for a mean value. This equation takes the following form in case of an unknown standard deviation:

$$X_k = m(X) - [t \cdot s(X)]/\sqrt{N} \tag{2}$$

where

$$m(X) = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{3}$$

$$s(X) = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - m(X)\right)^2} \tag{4}$$

are estimators of a mean value and a standard deviation (unbiased estimators), respectively. A value t is carried out from the Students' t-distribution according to sample size N and a level of confidence.

The another approach used to estimate characteristic values was introduced by Schneider (1997) and is a simplified version of the Orr and Breysse's method. A characteristic value X_k is estimated as a mean value $m(X)$ reduced by a half of a standard deviation $s(X)$:

$$X_k = m(X) - 0{,}5s(X). \tag{5}$$

Schneider (2011) also proposed an algorithm whereby fluctuation scales δ of shear strength parameters and the zone of ground V responsible for the occurrence of the limit state can be considered. A characteristic value X_k in this case is expressed in the following form

$$X_k = \mu(X)\left(1 - kV_{in}(X)\sqrt{\delta/|V|}\right) \tag{6}$$

$V_{in}(X)$ denotes a coefficient of variation of a soil parameter associated with natural (inherent) uncertainty, and k is a factor defining 5% quantile from a probability distribution of this property. In this paper it is assumed that the maximum extent of the failure area was evaluated relying on Prandtl mechanism in order to evaluate the area of ground which is responsible for the occurrence of the limit state. It can be observed that the characteristic value of the shear strength parameter, given by Eq. (6), varies with change of a failure area which is associated with a foundation width.

The last method which is considered in this study, was presented by Duncan (2000), who based it on so called three-sigma rule. The three-sigma rule states that the occurrence of a value beyond the interval $[\mu(X) - 3\sigma(X), \mu(X) + 3\sigma(X)]$ is hardly possible. Therefore it can be assumed that for a bounded random variable, this value is included in an interval $[X_{min}, X_{max}]$. The standard deviation can be defined as

$$\sigma(X) = 1/6(X_{max} - X_{min}). \tag{7}$$

While the characteristic value can be evaluated from the Eq. (1) using the standard deviation given by the Eq. (7).

3. Random Fields and Random Finite Element Method

Random finite element method was presented by Griffiths and Fenton (1993), Fenton and Griffiths (2008). Within RFEM three main components can be distinguished – random field theory, finite element method and Monte Carlo simulations. If a soil property is characterized by a random field than this property at each point is a separate random variable. Moreover a certain correlation structure is given between abovementioned random variables. Such approach is more adequate than an assumption that a soil property is characterized by a single random variable in each layer. However considering ground area consisting of continuum amount of points with different parameters is not feasible to implement. Therefore in RFEM random fields are generated by local average subdivision – LAS method (Fenton and Vanmarcke, 1990) in order to allow effective computation.

In this study a cohesive soil is examined. Therefore a friction angle and cohesion are described by random fields in order to evaluate the random bearing capacity. The cohesion is characterized by a lognormal distribution which can be obtained by the transformation $X = \exp\{Z\}$. Z is a normally distributed random field. The probability density function of X is given by the following equation

$$f(x) = \frac{1}{x\sigma_{lnX}\sqrt{2\pi}} \cdot \exp\left\{-\frac{1}{2}\left(\frac{\ln x - \mu_{lnX}}{\sigma_{lnX}}\right)^2\right\} \quad (8)$$

where μ_{lnX} is a mean value and σ_{lnX} denotes a standard deviation of an underlying Gaussian distribution of Z. The friction angle is a parameter which adopts values within bounded range. Thus this property is described by a bounded distribution which the probability density function takes form

$$f_x(x) = \frac{\sqrt{\pi}(b-a)}{\sqrt{2}s(x-a)(b-x)} \cdot \exp\left\{-\frac{1}{2s^2}\left[\pi \ln\left(\frac{x-a}{b-x}\right) - m\right]^2\right\} \quad (9)$$

where a and b are min. and max. values of a parameter, s is a scale factor correlated with a standard deviation of the property, m is a location parameter and $x \in (a, b)$. Details can be found in Fenton and Griffiths (2008). Generally any random field is characterized by its correlation structure. Within this study the ellipsoidal correlation function was considered

$$\rho(\tau) = \exp\left(-\sqrt{\left(\frac{2|\tau_2|}{\theta_x}\right)^2 + \left(\frac{2|\tau_1|}{\theta_y}\right)^2}\right), \quad (10)$$

where τ_1 and τ_2 denote the vertical and horizontal distances respectively, between two points in two-dimensional space. Furthermore θ_x and θ_y are fluctuation scales along directions x and y. In this study the anisotropic case was considered and fluctuation scales were assumed relying on geotechnical literature and took following values – $\theta_x = 10.0$m and $\theta_y = 1.0$m.

4. Computations

4.1. General Assumptions

This study considers a shallow strip foundation with infinite length and the plane strain situation. Analysis were conducted for various foundation widths *B*=1.0m; 1.2m; 1.4m; 1.6m; 1.8m; 2.0m and for two values of foundation depth *H*=0.5m and *H*=1.5m in order to test the influence of geometric parameters on the reliability index β. In this paper a kaolin clay which occurs in the vicinity of Wrocław was considered. Statistical data concerning effective shear strength parameters were taken from PhD thesis (Thao, 1984), where 67 in situ tests were gathered. A mean value and a standard deviation of soil parameters – a friction angle and a cohesion, were the main information received from abovementioned paper. It was necessary to estimate the variability interval of a friction angle, as its distribution was assumed to be bounded (Eq. 9). It was done using the three-sigma rule as in the case of Duncan's method described in section 2. The symmetry of a variability interval with respect a mean value implies that the location parameter m in Eq. (9) is equal to zero. Other properties such as a soil unit weight, Young's modulus, Poisson's ratio were assumed as deterministic values: $E = 40\text{MPa}$, $\upsilon = 0.3$ and $\gamma = 19.9\text{kN/m}^3$. Their variability has insignificant influence on the random bearing capacity. Such conclusions were achieved by Puła and Zaskórski (2014) who made sensitivity analysis to examine which random fields affect the random fluctuation of the bearing capacity. Parameters of the friction angle and the cohesion used in the analysis are provided in Table 1.

Table 1. Parameters of the friction angle and cohesion applied in analysis

Effective soil parameter	Friction angle ϕ'	Cohesion c'
Mean value	12.41°	29kPa
Stand. deviation	1.15°	7kPa
Max. value	16.80°	-
Min. value	8.00°	-
Number of in situ tests	67	67

4.2. Deterministic Computations (Eurocode 7)

Firstly effective soil parameters from in situ tests were used to compute the characteristic values of a friction angle and a cohesion for considered methods of estimation of characteristic values described in section 2. The results were gathered in Table 2.

Table 2. Characteristic values of soil parameters for various methods of their evaluation

Approach			ϕ' [°]	c' [kPa]
Duncan's method			10.00	21.32
Orr & Breysse's method			12.13	27.29
5% quantile method			10.40	19.06
Simplified Schneider's method			11.84	22.00
Extended Schneider's method	B [m]	1.00	10.87	21.38
		1.20	11.00	22.04
		1.40	11.10	22.56
		1.60	11.19	22.97
		1.80	11.26	23.32
		2.00	11.32	23.61

The characteristic value of the friction angle differ in a relatively small degree from the mean value based on in situ tests due to a small coefficient of variation (~10%). In the case of the cohesion the characteristic value is considerable smaller than the mean value as a consequence of relatively large coefficient of variation (~25%). Values carried out by Orr and Breysse's algorithm are the most similar to mean values of the friction angle and the cohesion. It is due to a relatively large sample size.

In the next step the deterministic values of the bearing capacity were calculated basing on characteristic shear strength parameters given in Table 2. Deterministic computations were carried out by applying design situations DA1.C1, DA1.C2, DA2* and DA3 provided in Eurocode 7. Case DA2* is a modification of DA2, which was introduced in the national annex in Poland.

In this paper, the simplification is made to allow comparison of results obtained by means of each design situation with results from numerical analysis. Namely, authors assumed that a footing is loaded solely by permanent load, for which partial safety factors are equal to 1.35 or 1.00 depending on a design situation. In case of design situations where a partial safety factor is equal to 1.35, the bearing capacity was divided by 1.35. Such procedure causes that deterministic values of the bearing capacity become comparable regardless of the design approach.

4.3. Stochastic Computations (RFEM)

The stochastic analysis based on RFEM were carried out in RBEAR2D software (freely available on www.engmath.dal.ca/rfem/). In this study 2D case was considered hence the strain plane situation was assumed. At the beginning the width of the mesh used in RFEM was assumed basing on the range of a failure mechanism in analytical approach. The depth was adopted as two times greater than the width of the foundation. However precise mesh size was conducted by calibration analysis. Furthermore the mesh size was specified so that the influence of boundary conditions on the random bearing capacity could be neglected. The mesh consists of 4800 rectangular elements of size 0.1x0.1m. The boundary conditions used in finite element method are as follows: right and left sides of the mesh are constrained against horizontal displacement and bottom boundary is fixed. An application of RBEAR2D code required calibration of other program parameters such as the displacement increment, maximal displacement step and maximal number of iterations. These features are related with computation of the bearing capacity of the soil by displacing the footing into the soil.

Parameters used in RFEM simulations are provided in Table 1 and Table 3.

Table 3. Program parameters applied in RFEM analysis

Parameter		Unit	Value
Number of elements	x	[m]	120
	y	[m]	40
Element size		[m]	0.1x0.1
Displacement increment		[m]	0.007
Maximal displacement step		[-]	50
Maximal iterations		[-]	240

Numerical analysis in conjunction with RFEM were carried out to assess reliability of a shallow footing and specify which method of evaluation of characteristic soil parameters gives most appropriate results in terms of the Eurocodes. The reliability index β is used as it is one of measures of structure reliability described in Eurocode 0 and is related to the failure probability p_f by equation

$$p_f = \Phi_0(-\beta) \tag{11}$$

where Φ_0 is the standard normal cumulative distribution function. The probability of failure can be defined as a probability that the random bearing capacity q_f will take a lower value than the deterministic design value of the bearing capacity Q_d (Q_d corresponds to exact value of reliability index). Therefore p_f is given by following form

$$p_f = \mathrm{P}\left|q_f < Q_d\right| \tag{12}$$

An estimation of the reliability index β and the related deterministic design value of the bearing capacity Q_d is based on a probability distribution of the random bearing capacity q_f. The bearing capacity probability distribution was estimated according Monte Carlo simulation with 3000 runs.

Empirical distributions for considered widths and depths of the foundation were obtained from conducted simulations. Subsequently several commonly used theoretical probability distributions were tested to find the most accurate one which fits the best empirical distribution of the bearing capacity. Lognormal distribution turned out to fit empirical data obtained from stochastic analysis regardless of the width and depth of a foundation. Similar results were presented in earlier papers (Cherubini et al. 2009) which focused on cohesive soil. It is worth mentioning that assumption of lognormal distribution of the bearing capacity of a cohesionless soil is inadvisable. In previous study (Puła and Zaskórski, 2014) authors suggested that if a soil strength is described only by the friction angle random field the Weibull distribution is the best-fitted distribution to the empirical distribution of the random bearing capacity.

4.4. Comparison of Deterministic and Stochastic Computations

Guidelines concerning minimum values of reliability index (ultimate limit states) for various types of structures and reference periods are indicated in Eurocode 0. Reliability class RC2 corresponds to the β value greater than 3.8 for a 50 year reference period. Figure 1 a-d presents comparison of design values of the bearing capacity obtained by using various approaches in reference to reliability indices β evaluated by RFEM in terms of the Eurocodes for the foundation depth *H*=0.5m.

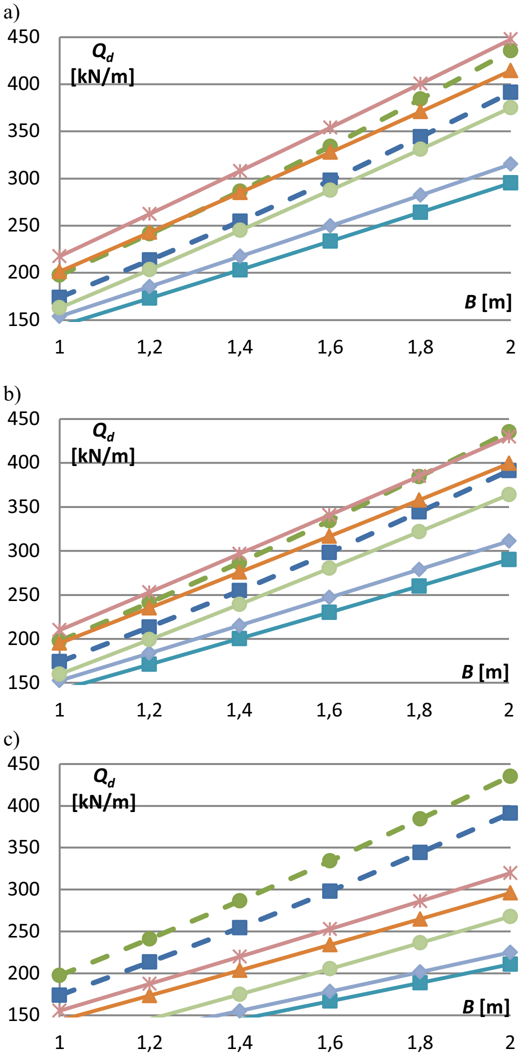
In case of the depth *H*=1.5m the character of curves representing each method in relation with reliability indices $\beta = 3.8$ and 3.0 is similar.

5. Conclusions

The lowest values of the bearing capacity were achieved for methods, which gave the safest (the most conservative) estimation of characteristic values of shear strength parameters regardless the design situation – method of 5% quantile and Duncan's method.

Application of design situations DA2* and DA3 leads to the lowest values of the bearing capacity. The reason is directly connected with types and amount of unfavourable partial safety factors used in these design situations. Unfavourable safety factors concern actions and a resistance in case of DA2*, actions and soil parameters in case of DA3. Design situations DA1.C1 and DA1.C2 give similar results basing on stochastic analysis. In both approaches extended method of Schneider (method concerning an influence of fluctuation scales) gives the most effective evaluation of the bearing capacity with respect to the index $\beta = 3.8$.

Further conclusion from conducted deterministic analysis, is fact that the choice of a method of an estimation characteristic values of soil parameters is of great importance, as evaluated characteristic values of properties can vary drastically. Consequently it can lead to significantly different values of the bearing capacity. Difference between bearing capacities obtained in one design situation reaches 35%. It should be considered as a high discrepancy. Particularly bearing in a mind small variation between effective values of soil parameters.



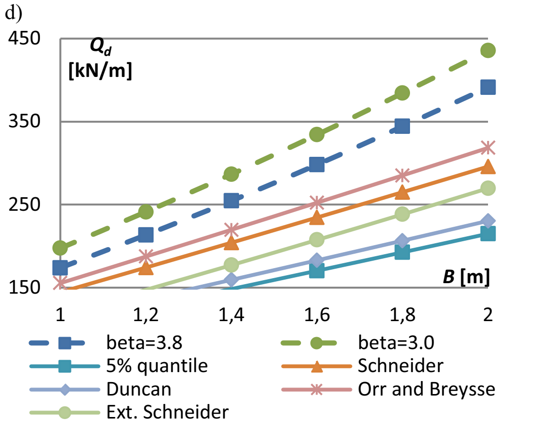


Figure 1. Comparison of design values of the bearing capacity Q_d – the depth of foundation H=0.50m; the design situation a) DA1.C1; b) DA1.C2; c) DA2* d) DA3.

To conclude, an application of RFEM allowed to consider a random spatial variability of soil properties and confront design values of the bearing capacity obtained by deterministic methods with reliability indices. It can be used as a supporting tool of a process of a foundation design that can help to select the most adequate characteristic values of soil strength parameters. Of course the foundation safety also relies on partial safety factors that lead to design values. However, partial safety factors are fixed within specific design approach and therefore they can only in a bounded range reflect spatial variability of soil properties.

References

Cherubini, C., Vessia, G., Pieczyńska, J., Puła, W. (2009). Application of random finite element method to bearing capacity design of strip footing, *Journal of GeoEngineering*, 4(3), 103-112.

Duncan, J. M. (2000). Factors of safety and reliability in geotechnical engineering. *Journal of Geotechnical and Geoenvironmental Engineering*, 126(4), 307-316.

EN 1997-1:2004. Eurocode 7: Geotechnical Design. Part 1: General rules. CEN, European Committee for Standardization, Brussels.

EN 1990:2002. Eurocode: Basis of structural design. CEN, European Committee for Standardization, Brussels.

Fenton, G. A., Vanmarcke, E. H. (1990). Simulation of random fields via local average subdivision. *Journal of Engineering Mechanics*, 116(8), 1733-1749.

Fenton, G. A., Griffiths, D. V. (2008). Risk assessment in geotechnical engineering. New York: John Wiley & Sons, ISBN: 978-0-470-17820-1.

Griffiths, D. V., & Fenton, G. A. (1993). Seepage beneath water retaining structures founded on spatially random soil. *Géotechnique*, 43(6), 577-587.

Orr, T. L. L. & Breysse, D. (2008). *Eurocode 7 and reliability-based design*. Reliability Based Design in Geotechnical Engineering. London and New York : Taylor and Francis.

Puła, W., & Zaskórski Ł. (2014). Estimation of the probability distribution of the bearing capacity of cohesionless soil using the random finite element method. *Structure and Infrastructure Engineering*, (online).

Schneider, H. R. (1997). Definition and determination of characteristic soil properties. *Proc. XII Intern. Conf. on Soil Mech. and Geotech. En*., Hamburg. Rotterdam, 2271-2274.

Schneider, H. R. (2011). Dealing with uncertainties in EC7 with emphasis on characteristic values. *Proc. of Workshop on Safety Concepts and Calibr. of Partial Factors in Eur. and N. Amer. Codes of Prac.*, Delft.

Thao, N. T. P. (1984). Geotechnical analysis of a chosen region of the Wrocław city by statistical method (unpublished doctoral dissertation). Wrocław University of Technology, Wrocław.