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Estimation of Probable Maximum Loss Index of Water Supply System

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Abstract. The numerous major earthquakes in Japan, such as the Great Hanshin-Awaji earthquake, Tohoku Regional Pacific Coast earthquake caused substantial casualties and property damages. Private companies/public utility enterprises should provide disaster reduction investment as part of risk management and prepare a strategic Business Continuity Plan. The significance of the Business Continuity Plan has been increasingly important in the connection of corporate social responsibility. This study focuses on the seismic risk analysis of the existing water supply system. The probable maximum loss index is employed to evaluate the seismic risk. An alternative method is proposed to evaluate Probable Maximum Loss index of the high order water supply network system.

Keywords. water supply system, seismic risk analysis, probable maximum loss

1. Introduction

The numerous major earthquakes in Japan, such as the Great Hanshin-Awaji earthquake, Tohoku Regional Pacific Coast earthquake caused substantial casualties and property damages. Private companies/public utility enterprises should provide disaster reduction investment as part of risk management and prepare a strategic Business Continuity Plan. The significance of the Business Continuity Plan has been increasingly important in the connection of corporate social responsibility (Hoshiya and Yamamoto, 2005).

This study focuses on the seismic risk analysis of the existing water supply system. Water supply system may be modeled as a network that consists of supply nodes (purification plants), demand nodes (pumping stations and service reservoirs) and links (buried pipes). It may be postulated that water can be supplied to a demand node as long as this node is connected at least to either one of supply nodes, and each demand node is connected to and is in charge of the corresponding supply area of a lower network. It is also postulated that links are subject to seismic risk and they have probability of two events, failure or success, whereas nodes are modeled as a system of components such as motors, water pipes, electric systems and so forth. Each component has probability of failure or success, and because of the combination of plural component probabilities, failure of a node system has different failure modes, and their probabilities must be evaluated, for example, by the event tree analysis.

In this research, probable maximum loss index is employed to evaluate the seismic risk of a lower network. An alternative method is proposed to avoid the complicated event tree analysis and evaluate probable Maximum Loss index of the high order water supply network system. Then a numerical analysis is carried out to demonstrate the efficiency of the proposed method.

2. Stochastic Model of Pipeline Failure

In this research, a geographical information system is used for the data analysis, all information installed as mesh data with (500m x 500m) mesh area. A water supply system of a blank City in the vicinity of Tokyo is employed as a numerical example.

The stochastic earthquakes and scenario earthquake that influence the water supply network and the data are installed from the data base of J-SHIS (National Research Institute for

Earth Science and Disaster Prevention Japan, 2013).

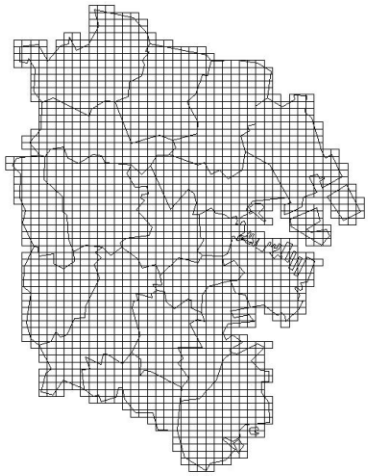

Figure 1. Water Supply Area

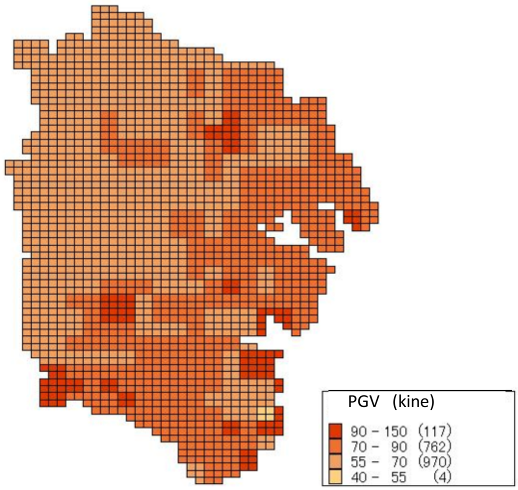

Figure 2. Peak Surface Ground Velocity

Figure 1 shows a water supply area with 500m square meshes. For example, figure 2 shows a peak ground velocity in kine (stochastic earthquake: probability of an earthquake excess in fifty years=0.05).

Probability of failure PF_i^j of j th pipe category, which is characterized by such as pipe material and pipe diameter in i th mesh, conditioned on the stochastic and scenario earthquakes are evaluated based on the average damage rate n_i^j (total number of damage spots /km) which empirical formula is proposed by Japan water works association (1998).

$$n_i^j = C_p \times C_d \times C_g \times C_l \times R(V) \tag{1}$$

$$R(V) = 3.11 \times 10^{-3} \times (V - 15)^{1.30} \tag{2}$$

where coefficients C_p, C_d, C_g and C_l are associated respectively with pipe material, pipe diameter, surface ground condition and liquefaction in Tables.1(a) to 1(d). V is the maximum ground velocity. Figures 3 and 4 show estimated average failure spots induced by Miura Peninsula-Kinugasa fault scenario earthquake and stochastic earthquake with probability of an earthquake excess in fifty years=0.02.

Then, the failure events in a mesh area assumed to occur along each link being distributed in a Poisson distribution.

$$P_i^j(X_i^j = x_i^j) = \frac{(r_i^j)^x}{x_i^j!}\exp[-r_i^j] \tag{3}$$

in which $r_i^j = n_i^j \times L_i^j$ is an average failure spot, L_i^j is a pipe length of j th pipe category in i th mesh. X_i^j is a total number of damage spots with means $\overline{X}_i^j = r_i^j$ and variances $\sigma^2 X_i^j = r_i^j$.

Let $P^j(X_1^j = x_1^j, \cdots, X_m^j = x_m^j)$ be the multivariate Poisson probability function in j th pipe category, having the marginal Poisson probability functions $P_i^j(X_i^j = x_i^j)$ with parameters r_i^j given as follows.

$$P^j(X_1^j = x_1^j, X_2^j = x_i^j, \cdots, X_m^j = x_m^j) = \frac{(r_1^j)^{x_1^j}(r_2^j)^{x_2^j}\cdots(r_m^j)^{x_m^j}}{x_1^j! x_2^j! \cdots x_m^j} e^{-(r_1^j + r_2^j \cdots r_m^j)} \tag{4}$$

in which m = total number of existing mesh.

The average damage rate is evaluated by Eqs.(1) and (2), which include the spatially correlated parameters, such as maximum ground

Table 1 (a) Pipe material correction coefficients

Pipe material	Symbol	Correction coefficient C_p
Asbestos cement	ACP	3.0
Cast iron	CIP	1.1
Ductile cast iron	DCIP	0.3
Steel	SP	0.5

Table 1 (b) Pipe diameter correction coefficients

Pipe diameter (mm)	Correction coefficient C_d
– Φ75	1.6
Φ100 – 150	1.0
Φ200 – 450	0.8
Φ600 –	0.5

Table 1 (c) Ground correction coefficients

Surface ground	Correction coefficient C_g
Modified mountainous region	1.1
Modified hilly areas	1.5
Valley, old water routes	3.2
Alluvial plain	1.0
Good ground	0.4

Table 1(d) Liquefaction effect correction coefficients

Hazard level	Correction coefficient C_l
No liquefaction	1.0
Medium liquefaction	2.0
Significant liquefaction	2.4

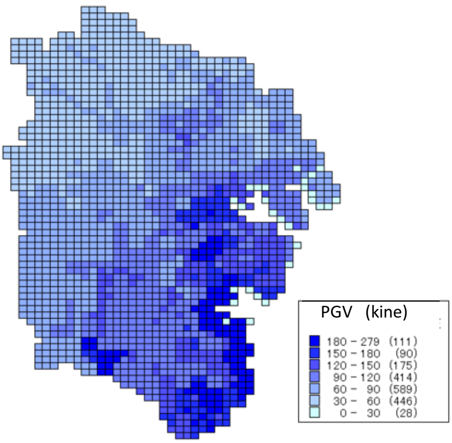

Figure 3. Estimated Average Damage Spots
Miura (Peninsula-Kinugasa fault scenario earthquake)

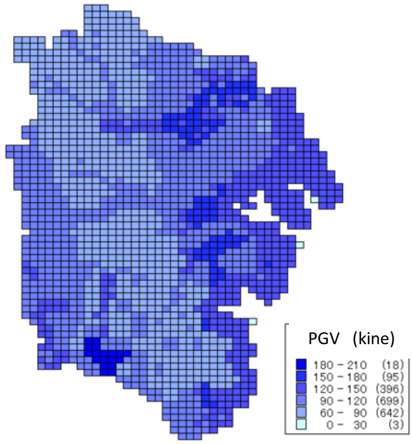

Figure 4. Estimated Average Damage Spots
(Stochastic earthquake with probability of an earthquake excess in fifty years=0.02)

velocity, liquefaction effect correction coefficients. This means that we have to consider the spatial correlation effect of failure spots in each mesh. The probability density function required enable the spatial correlation.

Lakshminarayana et al. (1999) proposed a bivariate Poisson distribution as a product of Poisson marginal with a multiplicative factor. We employ the multivariate correlated Poisson distribution given as.

$$P^{*j}(X_1^j = x_1^j, X_2^j = x_i^j, \cdots, X_m^j = x_m^j)$$
$$= \frac{(r_1^j)^{x_1^j}(r_2^j)^{x_2^j}\cdots(r_m^j)^{x_m^j}}{x_1^j!x_2^j!\cdots x_m^j} e^{-(r_1^j + r_2^j \cdots r_m^j)}$$
$$\times[1 + \sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\alpha_{i,k}^j(e^{-x_i^j} - e^{-r_i^j \cdot c})(e^{-x_k^j} - e^{-r_k^j \cdot c})](i \neq k) \tag{5}$$

Where $c = 1 - e^{-1}$, $\alpha_{i,k}^j$ is a range factor .

The mean vector is given as

$$\overline{X}^j = E\begin{bmatrix} X_1^j \\ \vdots \\ X_m^j \end{bmatrix} = \begin{bmatrix} r_1^j \\ \vdots \\ r_m^j \end{bmatrix} \quad (6)$$

The elements of covariance matrix $\overline{X}^j$ is given as follows.

$$\begin{aligned} &\text{cov}(i,k) = \alpha_{i,k}^j r_i^j r_k^j c^2 e^{-(r_i^j + r_k^j)c} \quad (i \neq k) \\ &\text{cov}(i,i) = r_i^j \end{aligned} \quad (7)$$

Hence the correlation coefficient turns out to be

$$\rho_{i,k}^j = \alpha_{i,k}^j \sqrt{r_i^j r_k^j}\, c^2 e^{-(r_i^j + r_k^j)c} \quad (8)$$

3. Estimation of Physical Loss of Pipeline Failure

The failure events in a mesh area (500m x 500m), stochastic nature of failure spots X_i^j are given by correlated Poisson distribution. The total number of failure spots of whole area Z^j is given by as follows.

$$Z^j = \sum_{i=1}^{m} X_i^j \quad (9)$$

Then, physical loss of in whole area is evaluated as.

$$\begin{aligned} W &= a_1 \cdot Z^1 + a_2 \cdot Z^2 + \cdots + a_\ell \cdot Z^\ell \\ &= \sum_{j=1}^{\ell} \sum_{i=1}^{m} a_j \cdot X_i^j \end{aligned} \quad (10)$$

in which a_j is a re-construction cost of one damage spot.

The mean value $\overline{W}$ and variance $E[(W-\overline{W})^2]$ is given by,

$$\overline{W} = \sum_{j=1}^{\ell} \sum_{i=1}^{m} a_j \cdot r_i^j \quad (11)$$

$$\sigma^2{}_W = \sum_{j=1}^{\ell} a^2{}_j \cdot \sigma^2{}_{zj} + \sum_{j}^{\ell} \sum_{\neq k}^{\ell} a_j a_k \cdot \text{cov}(z^j, z^k) \quad (12)$$

We consider the physical loss of the network system. There are numerous ways to evaluate the physical loss, among that most common approaches is event tree analysis with considering the connectivity of network system. We must evaluate failure or safety in each link. This means, if total number of existing mesh size is m, we have to evaluate 2^m combinations. If m becomes a large number, this is an unreasonable requirement.

The case of interest to us is when the failure spot in each mesh is mutually independent Poisson distribution, the total number of failure spots of whole area Z^j is also becomes Poisson distribution by reproductive property. IN each mesh, an average failure spot is a very small number, however the total amount of failure spot in whole area becomes a large number such as over 1000 spots. Then, stochastic nature of the failure spots Z^j is converge to the Gaussian distribution by Central limit theorem.

If stochastic variable Z^j is a Gaussian in nature, an analytical solution of the Probable Maximum Loss index may be obtained by the following way.

Normalizing W into $Y = (W - \overline{W})/\sigma_W$, the analytical solution of probability of excess as follows.

$$P = 1 - \int_{-\infty}^{\beta} \frac{1}{\sqrt{2\pi}} \exp(-\frac{Y^2}{2}) dY \quad (13)$$

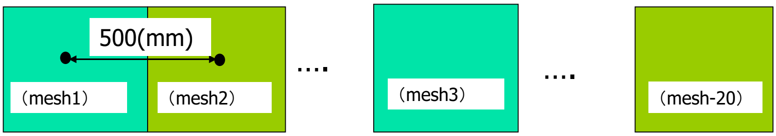

Figure 5. One dimensional Spatial Field

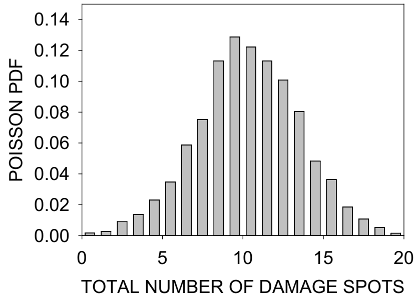

Figure 6. Probability Density Function (Generated by Poisson Distribution)

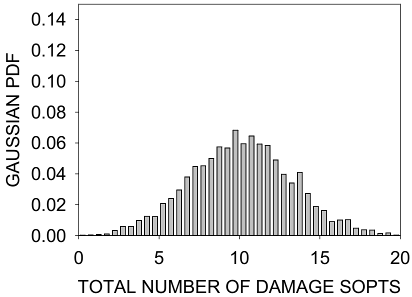

Figure 7. Probability Density Function (Generated by Gaussian Distribution)

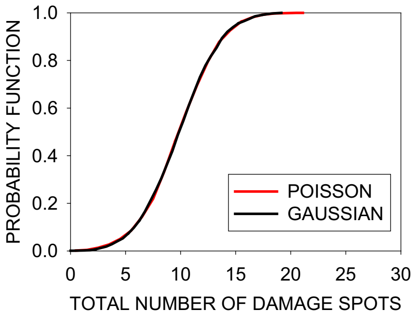

Figure 8. Probability Distribution Function (independent)

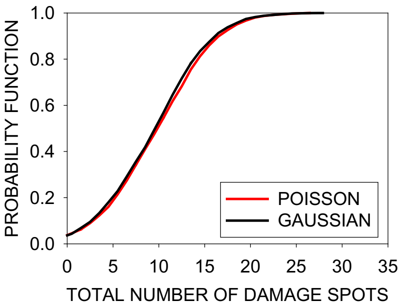

Figure 9. Probability Distribution Function (Correlated)

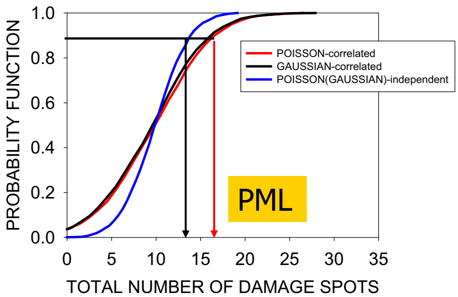

Figure 10. Estimated Probable Maximum Loss

In this paper, the value of loss correspond to probability of excess P=0.1 is defined as Probable Maximum Loss index (PML).

The β value in Eq.(13) is estimated approximately 1.3, then we can evaluate physical loss of the Probable Maximum Loss index as follows.

$$W_{PML} = \beta\sigma_W + \overline{W} \tag{14}$$

Note that we consider the correlated Poisson stochastic variables in Eq.(5), it must be examined Central limit theorem for correlated variables. Next, we will discuss in the numerical example.

4. Numerical Example

Numerical Examples are demonstrated by employing one dimensional spatial field (Figure 5). The area is divided into twenty contiguous mesh area, each mesh size is 500(m) x 500(m).

First, the accuracy of the Central Limit theorem of uncorrelated field is investigated by Monte Carlo simulation. Here, we assume that failure spots in each mesh is statistically independent Poisson distribution with common mean value 0.5 and variance 0.5, respectively. In the numerical analysis, each sample realizations are obtained by Eqs.(4) and (10). Figure 6 illustrates probability density function of W in terms of total number of sample realizations of 5000. In this case $a = 1.0$ in money terms in Eq.(10) is used for the numerical examples, so the physical loss is identical to total number of damage spots. Also, Figure 7 illustrates probability density function, each sample realizations is obtained Gaussian distribution N(0.5,0.5), where the first argument 0.5 is the mean, and the second argument 0.5 is the variance. Figure 8 shows a cumulated probability function of physical loss. They are directly calculated form sample distributions as shown in Figs.6 and 7. It is confirmed that estimated probability functions are almost identical by the Central Limit Theorem.

Next, the applicability of Central Limit theorem in correlated case is investigated. It is assumed that failure spots in each mesh is statistically correlated Poisson distribution (Eq.(5)) with common mean value 0.5 and variance 0.5. As for correlation coefficient is first assigned by following equation.

$$\rho = \exp(-\frac{|\Delta|}{d}) \tag{15}$$

In which $\Delta =$ relative distance of two mesh center, $d =$ correlation distance.

Then, the correlation coefficients of Eq.(8) is obtained by adjusted by parameter $\alpha_{i,k}^{j}$. Each set of sample realizations based on Eq.(5) is obtained by Markov Chain Monte Carlo method (W.R. Gilks et. Al. 1996). Figure 9 illustrates probability density function of in terms of total number of sample realizations of 5000. This result examined by another set of sample realizations obtained from multivariate Gaussian distribution with same mean, variance and correlation. It is observed that even though in correlated Poisson distribution case, estimated probability functions are converged to the Gaussian distribution, so far as this numerical example, the Central Limit Theorem may applicable.

Figure 10 shows an estimated probable maximum loss, estimated by independent and correlated case. It is observed that normal expected loss is almost identical, however probable maximum loss is varied.

5. Concluding Remarks

This study focuses on the seismic risk analysis of the existing water supply system. The Probable maximum Loss index is employed to evaluate the seismic risk. The proposed method may have a potential to evaluate the Probable Maximum Loss Index. In the presentaion, more detail numerical examples of a blank city will be shown to demonstrate the efficiency of the proposed method.

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