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Reliability Analysis of Infinite Slopes under Random Rainfall Events

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Abstract. Landslide events often occur after rainfall events during which the pore water pressure builds up within surficial soil layers, leading to shallow slope failure. The spatial variability of the permeability parameters of the soil causes high gradients in pore water pressure when the rainwater infiltrates into the slope. In this paper, we compute the reliability of infinite slopes under random rainfall events considering the spatial variability of the soil permeability. We model the infiltration process in HYDRUS-1D, which applies a numerical solution of Richard's equation, and combine this with a one-dimensional random field model of the hydraulic conductivity of the soil. The rainfall event is characterized in terms of its duration and average intensity and modeled through a self-similar random process. The reliability analysis of the infinite slope is based on the factor of safety concept for evaluating slope stability. To cope with the large number of random variables arising from the discretization of the random fields, we evaluate the slope reliability through subset simulation, which is an adaptive Monte Carlo method known to be especially efficient for such high dimensional problems. We study the influence of the duration and average intensity of the rainfall event on the slope reliability.

Keywords. Rainfall-induced landslide, random rainfall simulation, unsaturated flow, infinite slope, factor of safety, subset simulation.

1. Introduction

Rainwater infiltration leads to a redistribution of pore water pressure within the surficial soil layers. This redistribution is one of the governing factors of slope stability, because an increase of pore water pressure will reduce the resistant forces in potential slip planes and hence the factor of safety of the slope. Thus, landslides often occur during or after an intense rainfall event due to the pore water pressure building up (Rahardjo et al. 2005).

One-dimensional models of infiltration have been combined with the infinite slope failure model previously, and analytical as well as numerical solutions are available (Chen and Young 2006, Yeh et al. 2008, Muntohar and Liao 2010, Enrico and Antonello 2012). These studies concentrate on the time-dependent behavior of slope stability during a rainfall event in a deterministic way.

In most studies, the temporal distribution of rainfall is described by a uniform or triangle shape (Zhang et al. 2005, Huang et al. 2013). These rainfall models are usually parameterized by the rainfall intensity and duration. These approaches work well for simulating short time periods or extreme rainfall events, yet do not reflect the stochastic nature of the rainfall patterns. A more realistic representation of the rainfall event can be obtained through a random process modeling (Menabde and Murugesu 2000, Onof et al. 2000). The parameters of the random process can be estimated from rainfall data. Characteristic rainfall patterns from the region of interest can be obtained through simulating sample functions of the random rainfall process.

This paper assesses the reliability of an infinite slope subjected to random rainfall events. We model the rainfall event by a self-similar random process, as proposed in (Menabde et al. 1997, Menabde and Murugesu 2000). In addition, we apply a one-dimensional statistically homogeneous random field to describe the spatial variability of the saturated hydraulic conductivity (Phoon and Kulhawy 1999, Yuan et al. 2013). The random field is discretized by a multi-layer system with uniform values in each layer, which are described by correlated random variables. The failure of the slope is modeled by the factor of safety concept. The probability of failure is evaluated with subset simulation (Au and Beck 2001, Papaioannou et al. 2015), which is an efficient simulation method for estimation of small failure probabilities in problems with a large number of random variables. For each realization of the rainfall pattern and hydraulic conductivity, the pore water pressure is obtained by using HYDRUS-1D, which applies a numerical solution of Richard's equation. The stability analysis is evaluated by substituting the pore water pressure into the equation of the factor of safety for the infinite slope failure model.

2. Rainfall and Infiltration

2.1. Random Rainfall Events

Rainfall patterns are usually characterized by high intermittency combined with long-range correlation (Menabde et al. 1997). These phenomena can be captured accurately through application of self-similar random process theory. The basic idea of self-similarity is that the distribution of a quantity averaged over a given time period can be obtained by scaling down the distribution of the same quantity averaged over a longer time period. Self-similar random rainfall events can be simulated by application of the break down coefficients (BDC) concept (Menabde and Murugesu 2000). The time period of interest is divided into a number of basic cells. The simulation begins first with a uniformly distributed rainfall, whose pattern is described in terms of an average intensity and duration. At each simulation step, the rainfall event is scaled down into cells with shorter duration. The intensity of rainfall in each cell is then obtained as the average rainfall intensity multiplied by the corresponding break down coefficient (see Fig. 1).

Dry periods during the rainfall event are neglected in this study, and therefore the random rainfall event is constructed by a series of discretized rainfall cells without interruption. The duration of each basic cell is set to 0.1 hour ($s_0 = 0.1\text{hr}$). The break down coefficient is defined as:

$$U(s,t) = \frac{R_s(t_1)}{R_t(t_2)} \quad s < t \tag{1}$$

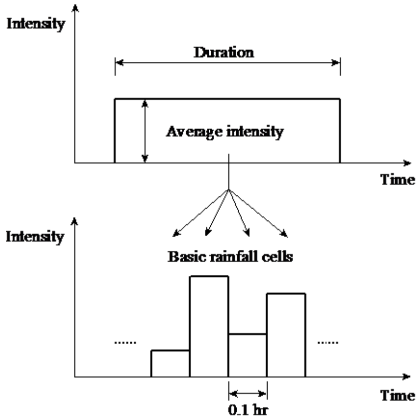


Figure 1. Random rainfall generation process

in which $R_s(t_1)$ and $R_t(t_2)$ are the accumulated precipitations over the rainfall durations s and t and centered at t_1 and t_2, respectively. The coefficient $U(s,t)$ is a random variable, whose distribution depends on the ratio s/t. Here, U is modeled by a Beta distribution with a unique parameter a, denoted by $U \backsim B(a,a)$. The parameter a changes with the timescale parameter s following (Menabde and Murugesu 2000):

$$a(s) = a_0 s^{-H} \tag{2}$$

in which a_0 is a dimensionless constant estimated from regional data; H is the constant Hurst parameter, which relies on the data regression analysis for a certain region.

At each step of the simulation procedure, the rainfall event is divided into two cells of equal duration. The break down coefficient of each cell with duration s is obtained by simulating two random variables Y_1 and Y_2, which follow the Beta distribution with parameter $a(s)$ and satisfy $Y_1 + Y_2 = 1$. The latter condition ensures that the accumulative precipitation of the random rainfall cells is unchanged. The process is repeated N times, until the duration of each cell reaches the duration of the basic cell s_0. The rainfall intensity in each basic cell is obtained as

$$R_i = RU_i \quad i = 1,2,\dots,2^N \tag{3}$$

in which R is the average precipitation intensity and i represents the number of the cell. The break down coefficient is:

$$U_i = \prod_{j=1}^{N} W_{j,i} \tag{4}$$

in which $W_{j,i}$ is either Y_1 or Y_2 for the jth scaling down step corresponding to the ith rainfall cell. Notice there are $2+4+\cdots+2^N = 2^{N+1}-2$ random variables $W_{j,i}$ during the rainfall cell generation. Figure 2 shows a random realization of a rainfall event with $N=5$ and duration $2^N s_0 = 3.2$hr.

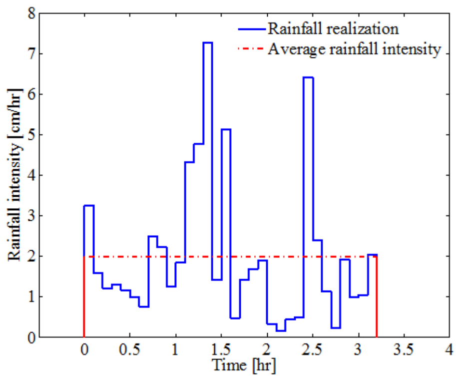


Figure 2. One realization of the random rainfall event.

2.2. Infiltration Analysis

The pore water pressure redistribution within the soil slope is mainly influenced by vertical infiltration rather than by horizontal flow (Santoso et al. 2011). In this study, we model the infiltration process in HYDRUS-1D, which applies a one-dimensional flow model. The saturated hydraulic conductivity of the soil column $K_s(z)$ is modeled by a one-dimensional homogeneous lognormal random field. The autocorrelation coefficient function of the underlying Gaussian field is defined as:

$$\rho(\tau) = e^{-\frac{2\tau}{r}} \tag{5}$$

where r is the scale of fluctuation; τ is the distance between two locations in the z direction (see Fig. 3). The soil column is divided into equal thickness soil layers and the random field $K_s(z)$ is discretized by the midpoint method (Der Kiureghian and Ke 1988). The unsaturated hydraulic conductivity $K(h,z)$ is a time-dependent function, expressed as:

$$K(h,z) = K_r(h)K_s(z) \tag{6}$$

in which K_r is the relative hydraulic conductivity and h is the pressure head, which changes throughout the infiltration procedure. It is assumed that no hysteresis occurs during wet and dry process of the water flow. The relative hydraulic conductivity can be expressed in terms of the pressure head as (Van Genuchten 1980):

$$K_r(h) = \begin{cases} \frac{\{1-(\alpha|h|)^{n-1}[1+(\alpha|h|)^n]^{-m}\}^2}{[1+(\alpha|h|)^n]^{m/2}}, h \le 0 \\ 1, h > 0 \end{cases} \tag{7}$$

in which α, n, m are emprical parameters that depend on the soil type and $m = 1 - 1/n$.

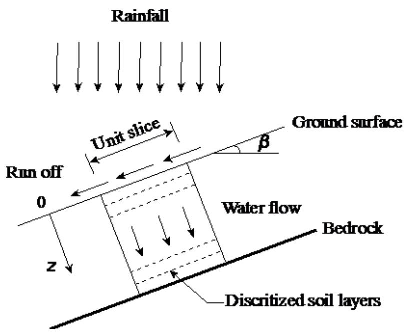


Figure 3. Infiltration model

The governing equation of one-dimension water flow in the inclination plane in HYDRUS-1D is described as a modified Richards' Equation (Šimůnek et al. 2009):

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial z}[K(\frac{\partial h}{\partial t} + cos\beta)] \tag{8}$$

in which $\theta \in [\theta_r, \theta_s]$ is the water content of the soil layer; h is the pressure head in the inclination plane; z is the depth of the soil column, which is perpendicular to the ground surface; β is the inclination angle the slope (see Fig. 3); θ_r and θ_s are the water content of the soil layer in the very dry and saturated condition, respectively. The random precipitation is set as boundary condition in HYDRUS-1D, which accounts for the runoff with respect to the water capacity within the soil layers. The solution of HYDRUS-1D provides the pressure head h at each time step and computational layer. The pore water pressure p_w is expressed in terms of pressure head as $p_w = \gamma_w h$, in which γ_w represents the unit weight of water.

3. Stability and Reliability Analysis

3.1. Stability Analysis

The rainwater infiltration has a significant influence on the surficial pore water pressure redistribution. Therefore, the critical slope failure plane is located at shallow depths and can be assumed parallel to the ground surface. Hence, the shallow slope failure mechanism is often sufficiently described by the infinite slope model (Enrico and Antonello, 2012). The factor of safety of the infinite slope along the potential slip plane at a certain depth z (see Fig. 3) can be written as:

$$FS = \frac{c' + (\gamma z cos\beta - p_w) tan\varphi'}{\gamma z sin\beta} \tag{9}$$

in which c' is effective cohesion of the soil; φ' is the effective friction angle of the soil; γ is the unit weight of soil mass.

Through the numerical solution of Eq. (8), one obtains the pressure distribution at the soil layers at a number of discrete time steps. At each time step, the point-in-time factor of safety is evaluated as the minimum factor of safety from all potential slip surfaces, located at the bottom of each discrete layer. The factor of safety at the end of the entire time period is approximated as the minimum of the point-in-time factors of safety of each discrete time step.

3.2. Reliability Analysis

In reliability analysis, failure F is defined through limit state functions g, so that a negative value of g indicates failure, and a positive value of g the safe state. The limit state function defining slope failure is:

$$g(\mathbf{X}) = \min[FS(\mathbf{X})] - 1 \tag{10}$$

in which **X** is the vector of all random variables, which includes the break down coefficients for generating the random rainfall events and the saturated hydraulic conductivities at the discrete soil layers; $\min[FS(\mathbf{X})]$ represents the minimum value of the factor of safety among all potential slip planes during the observation time period $t \in [0, S]$, where S is the duration of the rainfall event. The probability of failure of the infinite slope during S reads:

$$P_f = \Pr[g(\mathbf{X}) < 0] \tag{11}$$

The probability in Eq. (11) is estimated by application of subset simulation (Au and Beck 2001), which is an efficient simulation method for estimating probabilities of rare failure events in problems with large number of random variables. Subset simulation expresses the small failure probability as a product of larger conditional probabilities, which are estimated by Markov chain Monte Carlo methods (Papaioannou et al. 2015).

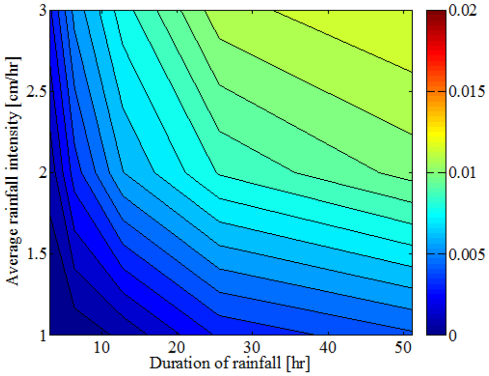
4. Case Study

We consider an infinite slope in sandy soil subjected to a random rainfall event. The sandy layer has 5m depth to the bedrock. Assume the infinite sandy slope is distributed with the uniform water content θ_0 before the rainwater infiltration. The slope angle and effective friction angle of the sandy soil are $\beta = 18°$ and $\varphi' = 30°$, respectively, while the effective cohesion is assumed zero, i.e. $c' = 0$. The unit weight of sandy soil is 20kN/m^3. The median of the saturated hydraulic conductivity K_s is 3.6cm/hr. The standard deviation in log-scale, i.e. the standard deviation of $\log_{10} K_s$, is 0.3. The random field is discretized with 100 equal thickness layers. The permeability parameters of the sandy soil are summarized in Table 1. We perform the analysis for two different scales of fluctuation, namely $r = 1\text{m}$ and $r = 0.05\text{m}$. For the generation of the random rainfall event, the parameters of the self-similar random process, Eq. (2), are chosen as $a_0 = 12.27$and $H = 0.47$ following Menabde and Murugesu (2000).

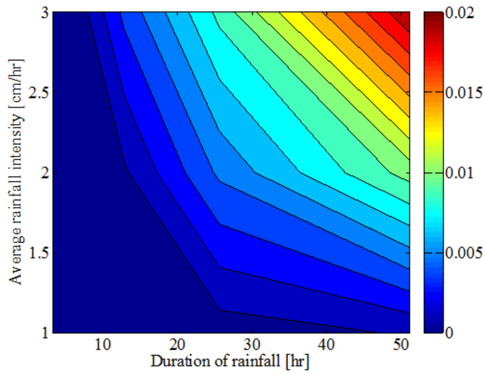
Table 1. Permeability parameters

Parameters	α	n	θ_s	θ_0	θ_r
Values	0.145	2.68	0.437	0.2	0.02

The duration of each basic rainfall cell is set to 0.1hr and the time step for evaluation of the point-in-time factor of safety is chosen as 0.2hr. We vary the number of steps N in the simulation of the rainfall event as $N = 5, 6, 7, 8$ and 9 and evaluate the probability of failure for the corresponding time periods 3.2hr, 6.4hr, 12.8hr, 25.6hr and 51.2hr. The average rainfall intensity is varied as 1cm/hr, 2cm/hr and 3cm/hr. The number of samples per level for Subset Simulation is set to 10^3, which was found to give acceptable coefficient of variations of the probability estimates for the cases considered. Figure 4 depicts the probability of failure with respect to the average rainfall intensity and rainfall duration in a contour plot for the two considered scales of fluctuation.



(a) $r = 0.05$m



(b) $r = 1$m

Figure 4. Contour plot of the probability of failure with different scale of fluctuations

As expected, the probability of failure increases with increasing rainfall duration and with increasing average rainfall intensity. The density of the contour lines represents the changing rate of the probability of failure. In the case with smaller scale of fluctuation ($r = 0.05$m), rapid changes of the probability occur when the average rainfall intensity increases from 1cm/hr to 2cm/hr. On the other hand, in the case with a larger scale of fluctuation ($r = 1$m), the probability of failure increases faster at higher rainfall intensities. This indicates that when the spatial variability of the soil is high, fast changes in p_w at shallow depths are likely to occur at shorter rainfall duration.

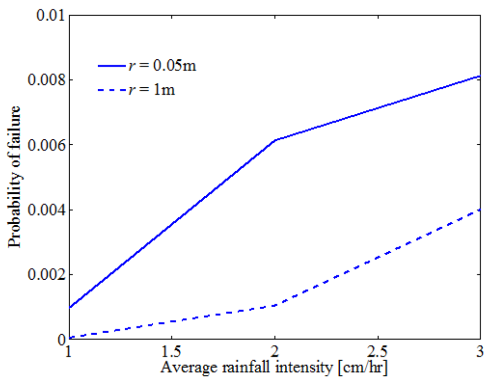


Figure 5. Comparison of probability of failure in cases with different scales of fluctuation

Figure 5 illustrates the influence of the scale of fluctuation on the probability of failure for a given time period of 12.8hr. For lower rainfall intensity, the probability of failure is larger in the case with smaller scale of fluctuation ($r = 0.05$m). This is because soils with high spatial variability will favor the occurrence of low permeability layers, and hence p_w is easier to build up at shallow depths. Therefore, in such cases, the critical slip plane is likely to be located near the ground surface. This implies a simultaneous decrease of z and increase of p_w in Eq. (9), which will result in a reduction of the critical factor of safety.

When the average rainfall intensity approaches the median of the saturated hydraulic conductivity (3.6cm/hr), the probability of failure rises slowly in the soil with smaller scale of fluctuation. This occurs because for high average rainfall intensity, part of the rainwater will become runoff and the remaining part will form a saturated flow within the sandy soil. This effect is more evident in soils with high spatial variability because of the more often occurrence of low values of the hydraulic conductivity, which lead to saturated flow. For low average rainfall intensities, the sandy slope will absorb the entire rainfall supply since it is far below the water capacity. In the case of $r = 1\mathrm{m}$, which implies a slowly varying hydraulic conductivity, a significant increase of the probability of failure occurs as the average rainfall intensity reaches $3\mathrm{cm/hr}$. This is due to the fact that a low spatial variability will result in most of the rainwater infiltrating, even though the rainfall supply is close to the infiltration capacity.

5. Conclusion

We presented the evaluation of the reliability of an infinite slope subjected to random rainfall events. The infiltration analysis is carried out in HYDRUS-1D and the spatial variability of the saturated hydraulic conductivity is considered through a random field modeling. In this paper, the probability of slope failure was evaluated for different average intensities and durations of the rainfall event. It can be concluded that the probability of failure increases as the scale of fluctuation of the hydraulic conductivity decreases. The influence of the water capacity cannot be neglected for larger average rainfall intensities, since part of rainwater forms runoff and hence does not affect the slope reliability. This effect is more pronounced at small scales of fluctuation, because they favor the occurrence of layers with low hydraulic conductivity that may result in saturated flow.

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