

Parameter Identification of Consolidation Settlement Based on Multi-Objective Optimization

Ya Fei ZHENG^a, Lu Lu ZHANG^a and Jie ZHANG^b

^aState Key Laboratory of Ocean Engineering, Department of Civil Engineering, Shanghai Jiaotong University, Shanghai, China

^bTongji University, Shanghai, China

Abstract. Due to the complexity of natural ground condition, consolidation settlement is usually difficult to predict. In this study, a Pareto multi-objective optimization based back analysis method for consolidation settlement is presented in this study. The model is a coupled flow and deformation model for unsaturated soil foundation which is implemented in the interactive multiphysics software environment COMSOL. A multi-objective optimization algorithm AMALGAM is adopted to identify soil parameters based on multiple types of measurements. A case history of a highway trial embankment is used to demonstrate the proposed back analysis method. The observed displacement and pore-water pressures are utilized simultaneously to estimate the mechanical and hydraulic parameters of the soil. The results show that the bi-objective Pareto front exhibits a sharp rectangular pattern. When only displacement is used in back analysis, the numerical model with optimized soil parameters cannot simulate pore-water pressure very well, and vice versa. However, the back analyzed soil parameters of the compromise solution from the bi-objective back analysis can reasonably simulate both the displacement and pore-water pressure and predict the settlement well.

Keywords. Consolidation, settlement, multi-objective, optimization, back-analysis

1. Introduction

Due to the complexity of natural ground condition, consolidation settlement is difficult to predict. Numerical modeling provides a tool to examine field performance and predict future deformation. Recently, researchers investigated the consolidation settlement based on the theory of coupled flow and deformation in unsaturated soils (Wong et al., 1998; Wu et al., 2012; Qin et al. 2010; Shan et al. 2012; Khoshghalb and Khalili 2013).

The predicted consolidation behavior of in situ soft clay is quite different from field observations, mainly due to the approximate numerical modelling techniques used, as well as the uncertainties involved in soil properties and ground conditions. The laboratory or field measured soil parameters are subject to uncertainties of sample disturbance and measurement error. Even with accurately measured soil parameters, the predicted performance using numerical modeling may still deviate from the field observation due to inherent spatial variability of in situ soils and model error. Some researchers conducted back analyses to calibrate soil parameters for ground

settlement based on field response (Kim and Lee, 1997; Knabe et al., 2012; Park et al., 2009; Shibata et al. 2014). Nevertheless, previous studies were mostly based on a single objective function and only one type of field observation was used in the back-analysis. In a comprehensive field monitoring program of foundation engineering, usually different types of measurement, such as displacement and pore-water pressure, are measured. The use of a single objective may not be adequate to consider all the important characteristics of the field performance. To consider different types of measurements simultaneously, the back-analysis of consolidation settlement may be implemented within a multi-objective optimization framework. In this study, a back analysis method for consolidation settlement based on the Pareto multi-objective optimization is presented. A multi-objective optimization algorithm, the multi-algorithm genetically adaptive multi-objective method (AMALGAM), is implemented together with the finite element model of unsaturated consolidation in the interactive multiphysics software environment COMSOL, to identify soil parameters. The application of this method is illustrated by the

example of a highway pavement project. Two parameters of the model are estimated using the field data of settlement and pore water pressure. The trade-offs of the two objectives are analyzed based on the Pareto optimal solutions. The calibrated model is used to predict the settlement of the soft foundation after construction is completed.

2. Methodology

2.1. Theory of Coupled Flow and Deformation in Unsaturated Soils

Kim (2000) presented a formulation for coupled flow and deformation in unsaturated soils basically along the line of Biot’s (1941) poro-elastic theory of consolidation. It is assumed the air phase is continuous and the air pressure is atmospheric. The effective stress and pore water pressure (negative value for soil suction) are adopted as two independent stress state variables. The constitutive model for unsaturated soil is elastic. Under the plane strain condition, the governing equations can be simplified as:

$$\frac{\partial}{\partial x} \left(k_x \frac{\partial}{\partial x} (h + y) \right) + \frac{\partial}{\partial y} \left(k_y \frac{\partial}{\partial y} (h + y) \right) = \left(n \frac{dS_w}{dh} + nS_w\beta_w\gamma_w \right) \frac{\partial h}{\partial t} + \alpha_c S_w \frac{\partial}{\partial t} \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right] \tag{1}$$

$$\frac{\partial}{\partial x} \left[G \left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial x} \right) + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - (\alpha_c S_w \gamma_w h)^e \right] + \frac{\partial}{\partial y} \left[G \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] = 0 \tag{2}$$

$$\frac{\partial}{\partial y} \left[G \left(\frac{\partial v}{\partial y} + \frac{\partial v}{\partial y} \right) + \lambda \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) - (\alpha_c S_w \gamma_w h)^e \right] + \frac{\partial}{\partial x} \left[G \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \right] + [nS_w\rho_w + (1 - n)\rho_s]^e g = 0 \tag{3}$$

where u and v are displacement in the x and y direction respectively; h is the pore pressure

head; t is time; k_x and k_y are coefficient of permeability in the x and y direction respectively; n is the porosity; S_w is the degree of water saturation ($0 \leq S_w \leq 1$); β_w is the compressibility of water; γ_w is the unit weight of water; α_c is Biot’s hydromechanical coupling coefficient or the effective stress coefficient; G is the shear modulus (modulus of rigidity), $\lambda = E\nu / [(1 + \nu)(1 - 2\nu)]$, E is Young’s modulus, and ν is Poisson’s ratio; ρ_s is the solid density; ρ_w is the density of water. The superscripts e in Equations (2) and (3) denotes the incremental values of physical quantities.

In this study, the degree of saturation and the permeability of the unsaturated soil (van Genuchten, 1980) are defined as follows:

$$S_w = \begin{cases} S_{wr} + \frac{1 - S_{wr}}{\left\{ 1 + [\beta(-h)^\gamma] \right\}^\alpha} & h < 0 \\ 1 & h \geq 0 \end{cases} \tag{4}$$

$$k = \begin{cases} k_s \left\{ 1 + [a(-h)^b] \right\}^{-\alpha} & h < 0 \\ k_s & h \geq 0 \end{cases} \tag{5}$$

where S_{wr} is the residual saturation, α , β , γ , a and b are unsaturated hydraulic parameters.

2.2. Back Analysis and Parameter Estimation Based on Multi-objective Pareto Optimization

First, define the residual errors of a prediction model for consolidation settlement as:

$$\varepsilon_i = y'_i - y_i \tag{6}$$

which y'_i is the observed performance, y_i is the simulated output of a prediction model. In this study, a finite element model of consolidation settlement based on the coupled governing equations is developed in the commercial multiphysics modeling finite element program COMSOL.

Because of the uncertainties of the initial and boundary conditions, the structural inadequacies of the prediction model, and uncertainties of input parameters and measurement errors, the residual errors of a

prediction model are not expected to be equal to zero. The problem of back analysis to determine soil parameters is usually transferred into an optimization problem, where the objective function of optimization is defined as a function of residual errors. In this study, RMSE is considered as the objective function:

$$f(\mathbf{X}) = \sqrt{\frac{1}{d} \sum_{i=1}^d (y_i - y'_i)^2} \tag{7}$$

When multiple types of measurements are to be considered in the back-analysis simultaneously, the objective function can be written as a multi-objective optimization form (Deb 2001)

$$\begin{aligned} &\text{minimize } F(\mathbf{X}) = \\ &\text{minimize } (f_1(\mathbf{X}), f_2(\mathbf{X}), \dots, f_n(\mathbf{X})) \end{aligned} \tag{8}$$

where $F(\mathbf{X})$ is the vector of objective functions, where $f_m(\mathbf{X})$, $m = 1, 2, \dots, M$, is the m th objective function. For example, $f_1(\mathbf{X})$, $f_2(\mathbf{X})$ and $f_3(\mathbf{X})$ can be the RMSEs of wall deflection, ground surface settlement and the bottom heave, respectively. \mathbf{X} is the vector of model parameters, $\mathbf{X} = \{x_1, x_2, \dots, x_n\}$.

The objective functions of multi-objective optimization constitute a multidimensional objective space. Different objective functions are often conflicting with each other and it is not possible to satisfy all the goals at a time. Hence, the multi-objective optimization yields a set of Pareto optimal solutions (Pareto 1897; Cohon 1978; Deb 2001). The Pareto optimal solutions are defined as follows:

$$\begin{aligned} \forall k \in [1, m] \quad &f_k(\mathbf{X}^{(i)}) \leq f_k(\mathbf{X}^{(j)}) \\ \exists k \in [1, m] \quad &f_k(\mathbf{X}^{(i)}) < f_k(\mathbf{X}^{(j)}) \end{aligned} \tag{9}$$

This formula means the solution $\mathbf{X}^{(i)}$ is no worse than $\mathbf{X}^{(j)}$ in all objectives and $\mathbf{X}^{(i)}$ is strictly better than $\mathbf{X}^{(j)}$ in at least one objective. Then the solution $\mathbf{X}^{(i)}$ is said to dominate $\mathbf{X}^{(j)}$. A solution is said to be Pareto optimal if it is not dominated by any other solution in the solution space.

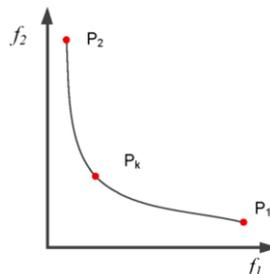


Figure 1. Pareto front in two dimensional space.

The set of Pareto optimal solutions forms a front in the objective space and the front is called the Pareto front and all the solutions in the Pareto front are considered as optimal solutions. Figure 1 illustrates the Pareto solution set for a simple problem in a two dimensional space which the aim is to simultaneously minimize two objectives (f_1, f_2). Here, three essential and most informative Pareto points are selected from the Pareto solutions. The first Pareto point is the best solution with respect to the first objective (i.e., minimize f_1). In Figure 1, we denote this point as P_1 . The second Pareto point is the best solution with respect to the second objective (i.e., minimize f_2). This point is denoted as P_2 . P_1 and P_2 represent the optimal solutions of single objective back analysis. The third point is the knee point in the Pareto front, where a small improvement in one objective would lead to a large deterioration in at least one other objective (Branke et al. 2004). In the graph, we denote the point as P_k and the solution is subsequently referred to as the compromise solution.

In this study, the multi-algorithm genetically adaptive multi-objective method (AMALGAM) developed by Vrugt and Robinson (2007) is adopted. The AMALGAM method uses the Nondominated Sorted Genetic Algorithm (NSGA-II, Deb et al. 2002), Particle Swarm Optimizer, Adaptive Metropolis Search (Haario et al. 2001), and Different Evolution (Storn and Price 1997) for population evolution and is more robust and efficient than the other multi-objective Pareto optimization methods for complex and higher dimensional problems. A detailed description of AMALGAM can be found in Vrugt and Robinson (2007) and hence is not repeated here. The multi-objective back-analysis is established by integration of AMALGAM with the finite element prediction model through MATLAB.

3. Case Study of a Highway Trial Embankment

3.1. Project Description and Monitoring Data

A trial embankment of a highway project in Zhejiang province of China is illustrated in this paper. The soil profile of at the test site consists of soft clay within the top of 3.0 m which is underlain by about 14.0 m of silty clay. Below this layer lies an approximately 3.0 m thick layer of soft silty clay. The distribution of the soil is shown in Figure 2. The embankment construction history is summarized in Table 1. For the ground improvement of the site, the vertical-drainage technique was employed and the drainage elements were installed in a square pattern with 1.5m×1.5 m spacing to the depth of 15m. Displacement and pore water pressure are measured at points *a* and *b* in Figure 2 respectively.

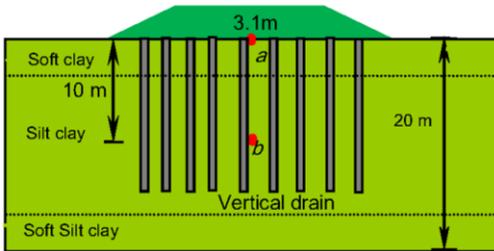


Figure 2. Geological profile of a highway trial embankment.

Table 1. Construction history of the embankment.

Stage	Total fill thickness(m)	Fill period (d)	Rest period (d)
1	2.2	0-18	18-29
2	3.1	29-44	44-63
3	3.3	63-80	80-present

3.2. Numerical Model and Soil Parameters

The finite element mesh of the numerical model is shown in Figure 3. The model is 160 m long

and 20 m deep, 375 elements totally. EF is assumed to be drained as vertical drainage installed below the embankment in the ground. AE and DF are assumed to be impervious boundary. A constant ground water level which is 2 m below ground is applied on the boundaries of AB and CD. Because of the relatively low permeability of underlying soft clay, BC is also assumed to be impervious boundary. Construction of the embankment is simulated by a surcharge load. Previous numerical studies have shown that the Young’s modulus E and the saturated permeability k_s are the most important factors in the consolidation of the unsaturated soil. In this study, the observed data of ground settlement and pore water pressure are used to estimate these two parameters. The logarithm of E and k_s are adopted in the optimization to reduce the numerical errors. The prior distribution and ranges of $\log(E)$ and $\log(k_s)$ are listed in Table 2.

Table 2. The initial ranges and distribution of estimated variables

Parameter	Unit	Distribution	Upper bound	Lower bound
$\log(E)$	kPa	Uniform	2.7	3.3
$\log(k_s)$	m/min	Uniform	-4.5	-7.5

4. Results and Discussion

The field data of settlement and pore water pressure of consolidation are used to estimate model parameters. Observed data during construction (80 days) are used in this back analysis, then the calibrated parameters are used to predict the settlement and pore water pressure after construction finished which is called validation period. The population size of AMALGAM is 200 for each generation and the maximum number of generation is set as 30. Therefore, the total number of samples is 6000.

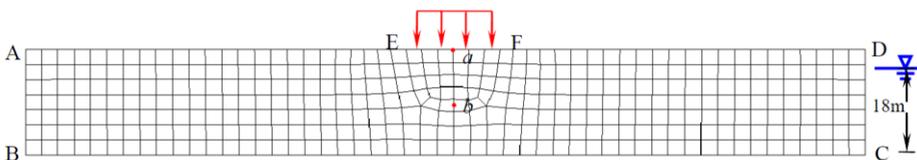


Figure 3. Finite element mesh and boundary conditions of the numerical model

4.1. Characteristics of Pareto Front

The two objective functions converge to a Pareto front after 20th generation. The Pareto front of the last generation has a sharp rectangular shape which indicates that the model can simultaneously meet both two objectives adequately. Figure 4 presents the trade-off of the $f_1 - f_2$ bi-objective Pareto front for the model. All the numerical simulated solutions are marked as black triangle points. The Pareto solutions of the last generation are marked as blue circles.

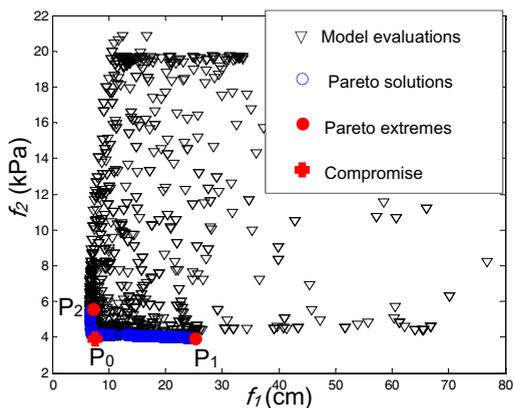


Figure 4. Pareto optimal solutions, Pareto extremes and compromise solution

4.2. Results of Back-Analysis and Prediction

The three Pareto extreme points (P_1 , P_2 and P_0) are selected from the Pareto solutions. P_1 , P_2 represent the optimal solutions of single objective back-analysis. The knee point P_3 is the compromise solution of multi-objective back analysis. For extremes P_1 , P_2 , and compromise solution P_0 , the corresponding parameters are listed in Table 3. As is shown in Table 3, the back estimated parameters from single objective back analysis are different from the compromise solution. Both E and k_s are smaller from multi-objective back analysis.

Table 3. Optimal parameters obtained from back-analysis

Parameters	unit	P_1	P_2	P_0
E	kPa	1501	1987	1186
k_s	m/min	10^{-5}	1.45×10^{-6}	2.7×10^{-6}

The simulated settlement is compared with the filed observation as shown in Figure 5. It can

be seen that the estimated settlement by P_1 simulates much better than the one by P_2 , but the predictions of validation are both worse than P_0 . RMSEs of the validation period with respect to P_1 , P_2 and P_0 are 15.64 cm, 25.73 cm and 6.63 cm respectively. The result illustrates when only one observed information is used, the predictions of validation are not very well. However the parameters of compromise solution can simulate the settlement reasonably during validation period.

Figure 6 shows the comparing of pore water pressure using three kind of solution. The model bias is relatively large during both construction period and validation period when Pareto extreme P_1 is used. Simulations of construction period and prediction of the validation are both acceptable. RMSEs of the validation period with respect to P_1 , P_2 and P_0 are 15.42 kPa, 2.23 kPa and 2.23 kPa respectively.

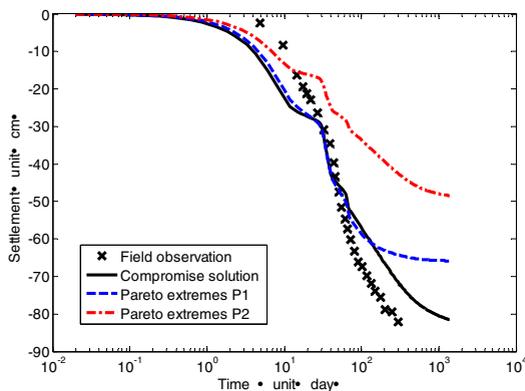


Figure 5. Comparison of measured and predicted settlement using Pareto extremes

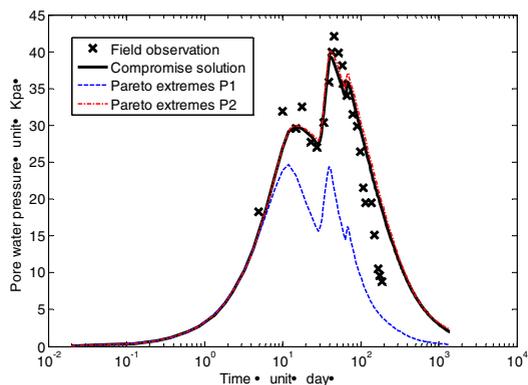


Figure 6. Comparison of measured and predicted pore water pressures using Pareto Conclusion

5. Conclusion

This paper presented a Pareto multi-objective optimization-based back-analysis method for foundation engineering. Major conclusions are listed below.

1. The trade-off of Pareto front base on the Pareto optimal theory exhibits a rectangular shape, illustrating that compromise solution can be obtained.

2. The soil parameters obtained by single objective back-analysis method cannot simulate well on the other objectives. The prediction of compromise solution by multi-objective back-analysis is acceptable with the RMSEs of the validation period are relatively small.

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