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Reliability Analysis of Near Surface Disposal Facilities Using Collocation Based Stochastic Response Surface Method

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Abstract. The safety assessment and reliability analysis are the two critical factors that affect the design of a complex structured system. In this paper, safety assessment model is used and the radiation dose of radionuclides are evaluated. The model is developed based on single dumping mode of disposal. In the present paper, radioactive carbon (^{14}C) is considered as it delivered the maximum concentration in ground water. To consider variability in the model parameters, a probabilistic methodology is adopted. One of the efficient probabilistic methods, collocation based stochastic response surface methodology (CSRSM) is used. In this method, the complex analytical equations are approximated by a higher order polynomial (using polynomial chaos expansion (PCE)). Groundwater velocity, thickness of unsaturated zone, dispersivity and distribution coefficient are considered as random variables. Third order polynomial gave the best fit for the model and an R^2 of 0.99 is obtained for a third order polynomial. Reliability analysis was carried out and the probability of failure of annual radiation dose of ^{14}C (radioactive carbon) radionuclide exceeding the permissible limits was estimated for different scenarios. The results show that the probability of failure of the system is very low and the multi barrier system is safe.

Keywords. Safety assessment model, dumping modes, risk, collocation based stochastic response surface method CSRSM, reliability analysis

1. Introduction

A disposal facility should be designed to restrict the contaminant migration. In India, the radioactive waste management facilities have been operating for almost three decades. The waste generated during the operation of nuclear facilities, fuel fabrication, research reactor, fuel processing, isotope production and research laboratories from different locations are collected and disposed in the site (Kumar et al 2013). The Near Surface Disposal Facilities (NSDFs) are mainly earth trenches, Reinforced Cement Concrete (RCC) trenches or stone trenches. These disposal facilities are protected by a top cover and liner systems. The waste can be disposed and the process of dumping is of two types; single dump mode and multiple dump mode. In the present work, single dump mode of disposal is considered. The migration of contaminant is a complex process which involves advection, diffusion and dispersion. The ground water contamination through any of these mechanisms should be avoided (Rowe and Booker, 1985). For the post-closure phase of near surface repositories, the major safety issue is the possibility of radiation exposure and environmental impacts over time periods far into the future. Gradual leaching of radionuclides into groundwater and subsequent migration through environmental media and transfer to humans are some of the effects of contaminant migration. In addition, mathematical modelling implies many assumptions and estimations, which increase the uncertainty of the output of contaminant transport modelling. Therefore, the safety assessment of waste disposal using a deterministic approach could result in either an underestimation or overestimation of the repository performance. As a result stochastic methods have been established as viable tools for analysing contaminant transport in porous media. The safety assessment of waste disposal using a deterministic approach (Kim et al. 1993, Nair and Krishnamoorthy 1999) can result in either an underestimation or overestimation of the repository performance. As a result stochastic methods (Kim and Na 1997, Das and Zheng 2000, Huang et al. 2009 and Cadini et al. 2012) have been established as viable tools for analysing contaminant transport in porous media. The most common method to handle uncertainty problem is the classic Monte Carlo simulation method (MCS). MC simulations are perhaps the most intuitive, and for many site-specific studies, statistically the most accurate approach to uncertainty or risk analysis of contaminant transport in the subsurface (Harter 2000). As the number of simulations increases, the convergence to the actual solution is ensured. Recently many probabilistic methodologies/variance reduction techniques were adopted for reliability analysis which includes response surface methods (RSM). An improvement over the basic RSM methodologies were developed by vector projection of sampling points (Kim and Na 1997), constructed response surface in a cumulative manner and used in reliability analysis of plate structures (Das and Zheng 2000). In the present work, collocation based response surface method (CSRSM) method which is a probabilistic extension of response surface method is used to determine the statistical properties of ouput response (dose rate of radionuclide) and carried out reliability analysis radionuclide migrating through a multi barrier system (single dump). Reliability analysis is also carried out further, after quantifying the uncertainties through CSRSM for all the scenarios.

2. Model for Multi Barrier System

The multi barrier system includes top cover, waste container, waste form, backfill material, bottom cover and the near field geosphere in a sequence. After proper conditioning, the radioactive waste in the solidified form (waste form) is packed in steel drums (waste container) and buried in the facility. Though the barrier system is designed for safe environment, a failure scenario may be encountered due to infiltration. The radionuclide release to the groundwater is estimated by considering the sequential failure of the barrier system by means of infiltration from rainfall water. The failure of the top cover (Barrier a) begins with infiltration due to rainfall. This would result in contact of the waste container (Barrier b) with water and corroding the mild steel. As the corrosion proceeds, water will interact with the solidified waste (Barrier c) resulting in leaching of radionuclides from the waste form. The leached radioactivity will begin to migrate through the backfill (Barrier d) and after the failure of the bottom cover (Barrier e) reaches the geosphere (Barrier f). If the system is assumed to operate without any repair and the failure is random, the probability density function *f(t)* is expressed as (Kim et al.,1993).

$$f(t) = \lambda \exp(-\lambda t) \tag{1}$$

where, λ is the conditional failure rate and t is the operation time of the barrier (y).
The distribution of the time of release of the radionuclides in to the groundwater can be determined analytically as (Nair et al.,1999) ,

$$f_s(t) = (\prod_{i=a}^{f} \lambda_i)(\sum_{i=a}^{f} \frac{e^{-\lambda_i t}}{\prod_{j \neq i}(\lambda_j - \lambda_i)}) \tag{2}$$

where $f_s(t)$ is the exponential failure probability density of the barrier system (y^{-1}).

The release rate of the radionuclide into groundwater for single dump mode is calculated as (Nair et al.,1999),

$$R_s(t) = S_s(t) f_s(t) \tag{3}$$

where $S_s(t) = M \exp(-\lambda_p t)$ is the source term in Becquerel (Bq), M is the inventory (Bq) of the radionuclide corresponding to 50 GWe.y energy production, λ_p is the decay constant of the radionuclide (y^{-1}) and t is the time elapsed after disposal (y).

The time dependent concentration of the radionuclide in the groundwater for single dump mode can be evaluate as convolution integral (Nair et al.,1999),

$$C_{gs}(x,t) = \int_0^t R_s(t-\tau) C_g(x,\tau) d\tau \tag{4}$$

The one dimensional solution of concentration of radionuclide in groundwater for the instantaneous release of unit activity is considered in the model as,

$$C_g(x,t) = \frac{\exp(-\lambda_p t)\exp(-(x-U_x t)^2/4D_x t}{2\pi A R_g \theta_g \sqrt{D_x t}} \tag{5}$$

where D_x is the retarded longitudinal dispersion coefficient (cm^2/y), D_y is the retarded lateral dispersion coefficient (cm^2/y), $U_x$is the retarded groundwater velocity (cm/y), A is the cross sectional area of aquifer (cm^2), $R_g$is the retardation factor which is 1+($K_d\rho_b/\theta_g$) where K_d is the distribution coefficient (ml/g), $\rho_b$is the bulk density (g/cc), $H_g$is the aquifer thickness (cm) and θ_g is the effective porosity. The radionuclide dependent parameters of carbon (half life-5730 years) used in the model are given in Table 1. The Mean time to failure (MTTF) for all the barriers is presented in Table 2. The independent parameters of the radionuclides are given in Table 3.

The radiation dose due to the radionuclide through the drinking water pathway is calculated as the product of concentration of radionuclide in the ground water (obtained from equation 4), drinking water intake and the ingestion dose coefficient. The ingestion dose coefficients applicable to general population along with the water intake of 2.2 l/day is used in the evaluation.

Table 1. Radionuclide dependent parameters used in the model (Nair et al.,1999)

Waste inventory (Bq/GWe.y)	Distribution coefficient K_d(ml/g)	Ingestion dose coefficient (Sv/Bq)
4.81x10^{12}	20	6.20x10^{-12}

Table 2. MTTF of barriers (Nair et al.,1999)

Notation	Barrier	MTTF (years)
A	Top cover	25
B	Waste container	12.5
C	Waste form	300
D	Backfill	30
E	Bottom cover	15
F	Near field geosphere	$R_d T_r$

Table 3. Radionuclide independent parameters (Nair et al.,1999).

Parameter	Unit	Value
Bulk density (ρ_b)	g/cc	1.7
Porosity (θ_g)	-	0.3
Longitudinal distance parallel to the flow (x)	cm	160000
Groundwater velocity	cm/s	1.157x10^{-4}
Dispersivity (α)	cm	100
Thickness of unsaturated zone (z)	cm	200
Water intake	l/day	2.2
Risk factor	mSv^{-1}	7.3x10^{-5}
Aquifer thickness (H)	cm	600
Aquifer cross sectional area (A)	cm^2	1.0 x 10^6
Seepage velocity in unsaturated zone (U_z)	cm/s	1.157 x 10^{-8}

R_d represents the retardation factor and T_r the travel time (=z/U_z) in years.

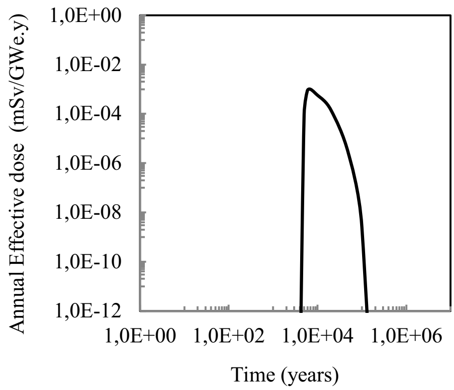

Figure 1. Annual effective dose verses time

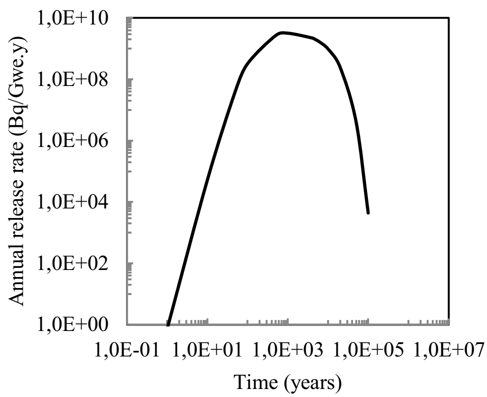

Figure 2. Annual release rate verses time

Figure 1 and Figure 2 show the annual dose and release rate of carbon with time. The maximum risk over time for single dump mode is obtained as 8.32×10^{-7} y-1 and this is lower than the risk observed from industrial accidents and natural catastrophes (1×10^{-3}-1×10^{-4} y^{-1}) (Nair et al.,1999).

3. Stochastic Response Surface Method

The stochastic response surface method (SRSM), an extension of the traditional response surface method (Li et. al, 2011) has been successively applied in many areas of research. Isukapalli et. al, (1998) introduced SRSM to uncertainty propagation analysis for environmental and biological systems in which all uncertain inputs are represented by random variables. SRSM based on polynomial expansion of model with an independent identically distributed (iid) sequence of input random variables, can replace the complex contaminant transport model with an approximated less-expensive surrogate model. In CSRSM the unknown output vector is represented as a polynomial chaos basis with unknown coefficients (Tatang 1995, Huang et al. 2009). The iid sequence of standard random variables are represented as $\{\delta\}_{i=1}^{n}$, where n is the number of independent inputs, and each δ_i has zero mean and unit variance. The output vector represented as a multidimensional Hermite polynomials is given by the equation

$$F(\xi) = a_0 + \sum_{i_1=1}^{n} a_{i_1}\Gamma_1(\delta_{i_1}) + \sum_{i_1=1}^{n}\sum_{i_2=1}^{i_1} a_{i_1 i_2}\Gamma_2(\delta_{i_1},\delta_{i_2})\cdots \qquad (6)$$

where F refers to an output vector,
$a_{i_1,\ldots i_n}$ are coefficients to be evaluated,
Γ individual polynomials of the basis
$\delta = (\delta_1, \delta_2 \ldots \delta_n)$ is the vector of independent standard normal random variables, and
$\Gamma_p(\delta_1, \delta_2 \ldots \delta_n)$ is the multi-dimensional Hermite polynomial of degree p given by

$$\Gamma_p(\delta_1, \delta_2 \ldots \delta_n) = (-1)^n e^{\frac{1}{2}\delta^T\delta} \frac{\partial^p}{\partial\delta_{i_1}\partial\delta_{i_2}\ldots\partial\delta_{i_p}} e^{-\frac{1}{2}\delta^T\delta} \qquad (7)$$

The number of unknown coefficients for the polynomial of order p is given by

$$P = \frac{(n+p)!}{n!\,p!} \qquad (8)$$

Here, four random variables are considered and for a third order four dimensional polynomial, there are 35 terms (from equation 8).

In general, as the order of polynomial increases the convergence to the actual solution increases. In this method, the deterministic response evaluation and stochastic analysis are de coupled (Huang et al., 2009). In the present work, CSRSM described above is employed to develop polynomial chaos equations (meta-models) for the annual dose rate of radionuclide. The coefficients of the PCE are determined by solving the linear system of equations using MATLAB and verified with excel add-in developed by Huang et al, 2009.

3.1. Parametric Uncertainty

In the paper, four critical parameters i.e., groundwater velocity, distribution coefficient, longitudinal dispersivity and thickness of unsaturated zone and their variability are considered. The type of distribution and ranges of values that are considered are given in Table 4.

Table 4. Statistical properties of uncertain parameters for Carbon radionuclide (Nair et al, 1999)

Parameter	Mean	Standard deviation	Distribution
Distribution coefficient (ml/g)	30	10	Uniform
Ground water velocity(cm/sec)	0.0005	0.00025	Lognormal
Longitudinal Dispersivity (cm)	255	122.5	Lognormal
Thickness of unsaturated zone (cm)	250	75	Normal

3.2. Results and Discussion

The accuracy of CSRSM is demonstrated by considering annual dose rate problem. To get the output distribution of annual dose rate, it is expressed as a function of four variables. CSRSM is used and, second and third order expansions are done. To check the accuracy of the polynomial functions, direct simulation is carried out by using Monte Carlo simulation (10000 simulations). In CSRSM, only 81 and 625 collocation points (Huang et al, 2009) were sufficient to generate the second and third order polynomials respectively. The resulting curves matched quite well with direct simulation. The cumulative distribution function (CDF) of dose rate of radionuclide (^{14}C) is shown in the figures 3 and 4. From these graphs, it is observed that CSRSM simulated the output distribution (expressed as a function of four variables) from less number of simulations when compared to direct simulation through Monte Carlo method. Similar trends were observed for single dump 2D. R^2 value for 3rd order polynomial is 0.99 which implies that the best fit was obtained in that case. The R^2 values for different polynomials are tabulated in Table 5.

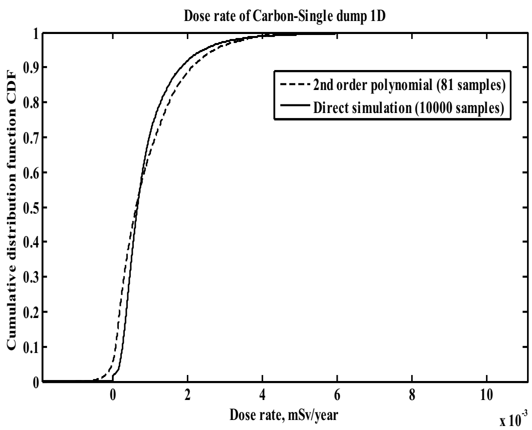

Figure 3. Comparison of Direct simulation with CSRSM (2nd order polynomial)

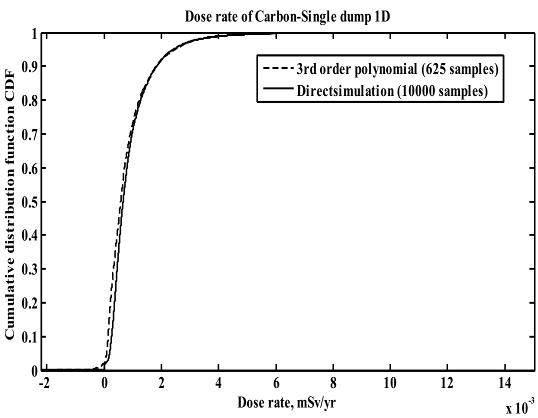

Figure 4. Comparison of Direct simulation with CSRSM (2nd order polynomial)

Table 5. R^2 for different polynomials

SNo	Order of polynomial	R^2 Single dump 1D	R^2 Single dump 2D
1	2nd order polynomial	0.92	0.902
2	3rd order polynomial	0.99	0.99

4. Reliability Evaluation Using MCS

In a complex structural system like a near surface disposal facility, the amount of radionuclide released into the drinking water pathway post closure is a major concern for the design. To estimate the effect of migration, it is worthwhile to know the probability of the radiation dose in the drinking water pathway of a particular radionuclide reaching an expected value as the parameters for the estimation of dose are random variables. Thus reliability analysis is carried out. The limit state/performance function, g(X) is defined in terms of the basic random variables X_i, and the functional relationship among them. The failure condition is defined as

$$g(X) = [D_r - D(X)] < 0 \tag{9}$$

where D_r is maximum permissible radiation dose in the drinking water pathway and $D(X)$ is the function of the four uncertain input parameters. The “probability of failure”, is regarded as the probability that the performance function will yield unacceptable values for the analytical and statistical models adopted. In this study, reliability evaluation has been done using Monte-Carlo simulations. The permissible value of dose rate is considered as the maximum annual dose rate obtained from the model. To get a good estimate of probability of failure through this method, 10,000 simulations were done. These values are used for the analysis to assure the reliability of system for the dose rates of that magnitude. The probability of failure(P_f) for all the cases is of the order of 10^{-3}. The details of the simulations are tabulated below.

Table 6. Comparison of probability of failure

Dumping mode	Permissible value of radiation dose (mSv/GWe.y)	Time for Computation mathematical equation (seconds)	Time for computation CSRSM (seconds)
Single dump 1D	0.0044	2182.4	1.061
Single dump 2D	9.2215×10^{-4}	2357.5	1.072

Note: Permissible value- maximum dose obtained for the radionuclide

From the table, it is observed that the simulation time taken for determining the reliability of the system when the analytical solution was used is very high when compared to that when CSRSM (i.e., 3rd order PCE) is used. The computational effort and time required is very less in the latter case and hence CSRSM can be used as an efficient methodology than direct simulation. Since the P_f is low, the barrier system is designed efficiently and the effect of leaching of radionuclide (^{14}C) through drinking water pathway is negligible and the system is safe.

5. Concluding Remarks

A safety assessment model was used to determine the annual dose rate and release rate of radiocarbon. Variability of the input parameters from deterministic analysis (aquifer thickness, dispersivity, groundwater velocity and thickness of unsaturated zone) was considered in the model. CSRSM (collocation based stochastic response surface method) was used. It is observed that with lesser number of simulations and lesser duration of time, this method achieved desirable results and an illustration of its accuracy is also presented. Third order polynomial could give a very good fit with the direct simulation curve. Finally, the reliability of the system as a function of radiation dose through drinking water pathway is also determined. A probability of failure of 10^{-3} is obtained. As it is low, it can be assured that the barrier system is designed efficiently and the effect of leaching of radionuclide (^{14}C) through drinking water pathway is negligible and the system is safe.

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