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# Computing Subjective Expected Utility using Probabilistic Description Logics

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**Abstract.** We introduce a framework which is based on probabilistic Description logics (Prob-DL), to represent and solve multi-criteria discrete alternative problems by calculating expected utility. To our knowledge, this is the first ever approach for calculating expected utility using a Description logics based formalism.

**Keywords.** Description Logics, Probabilistic Description Logic, Prob-DL, Multicriteria Decision Making, Decision Theory, Utility Theory, Subjective Expected Utility, Probabilistic Ontology

## 1. Introduction

Since the first serious attention of *multi-attribute utility theory* (MAUT) in [7,4] to solve problems regarding *multi-criteria decision making* (MCDM), numerous approaches have been proposed, including probabilistic, possibilistic, fuzzy and graphical models [2,15,5] amongst others. In parallel, preference representation has become an ongoing research subject in artificial intelligence, gaining more popularity every day, which also lets the discipline to deal with the problems from Decision Theory. To represent preferences and encode decision-theoretic problems, a relatively new common approach stepping forward over the last decade is the use of logical languages [16,3,18,11,12,14,13].

Description Logics (DL) is a family of logic languages which is mainly based on decidable fragments of first order logic. It has been designed to be used as a formalism in the field of knowledge representation, and it has become one of the major approaches over the last decade. In the context of the Semantic Web, it embodies a theoretical foundation for the OWL Web Ontology Language, a standard defined by the World Wide Web Consortium.

In this paper, we introduce a formal framework which is based on probabilistic Description Logic Prob-DL ([10]), a family of DL languages designed to model subjective uncertainty. The aim of our framework is to encode and solve decision problems via computing expected utility using the inference services specific to

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the employed language (Prob-$\mathcal{ALC}$ in our case). To our knowledge, this is also the first DL-based framework aimed to calculate the expected utility. In our approach, we represent preferences of the decision maker (agent), from the utility theory perspective, where each *criteria* has an assigned utility value (weight). We consider *alternatives* in the form of ABoxes, and criteria as concepts. We represent decision maker's background knowledge via a Prob-DL knowledge base.

The framework can be applied to *multiple criteria discrete alternative problems* (see [17]). In general, it can be applied to every domain where background knowledge which is relevant for our decisions, can be shared, matched and related via knowledge bases in terms of ontologies. One motivation is that, within a DL-based decision making framework, one can express the dependency between attributes/criteria using the concept hierarchy and evaluate an *alternative* (a *choice*) in terms of its logical implications.

In the remainder of the paper, we first briefly present preliminaries in Prob-DL, in Section 2. Then, we introduce our framework and discuss an example in Section 3. In Section 4, we discuss the related works. We conclude the paper with a brief outline and ideas about future research in Section 5.

## 2. Basic Prob-DL

Probabilistic Description Logics family, Prob-DL is proposed in [10] as a fragment of First-Order Logic of *Type-2* probability (see [6]). Type-2 probability refers to subjective uncertainty, or *degree of belief* e.g., "Tweety the bird flies with probability greater than 0.9", whereas Type-1 probability refers to *statistical* probability. Therefore, a probabilistic logic which solely models Type-1 probabilities, fails to represent the above statement since it can be either true or false (i.e., $Flies(Tweety)$ holds with probability of either 0 or 1).

We assume that the reader has familiarity with the basic DL [1].To introduce the basic notions and notations, following [10], we give the definition of Prob-$\mathcal{ALC}$ as a probabilistic counterpart of $\mathcal{ALC}$. $N_C$, $N_R$, $N_I$ are denumerable sets of concept names, role names and individual names respectively. The syntax of the concepts in Prob-$\mathcal{ALC}$ extends $\mathcal{ALC}$ inductively as follows:

$$C ::= A \mid \neg C \mid C \sqcap D \mid \exists r.C \mid P_{\geq n}C \mid \exists P_{rel\ n}r.C \tag{1}$$

where $A \in N_C$ , $C$ and $D$ are concepts, $r \in N_R$, $rel \in \{\geq, >\}$ and $n \in [0, 1]$. $C \sqcup D$ is an abbreviation for $\neg(\neg C \sqcap \neg D)$, $\forall r.C$ for $\neg\exists.\neg C$, $\top$ for $C \sqcup \neg C$ and $\bot$ for $\neg\top$. Furthermore, $P_{<n}C$ is an abbreviation for $\neg P_{\geq n}C$, $P_{\leq n}C$ for $P_{\geq 1-n}\neg C$, and $P_{>n}C$ is for $P_{<1-n}\neg C$. A TBox is a finite set of axioms (concept inclusions) $C \sqsubseteq D$, which represents the ontology. A probabilistic ABox $\mathcal{A}$ is defined according to the following rule

$$\mathcal{A} ::= C(a) \mid r(a, b) \mid \neg\mathcal{A} \mid \mathcal{A} \wedge \mathcal{A}' \mid P_{\geq n}\mathcal{A} \tag{2}$$

where $C \in N_C$, $r \in N_R$, $a, b \in N_I$, $n \in [0, 1]$, $\mathcal{A}$ and $\mathcal{A}'$ ranges over probabilistic ABoxes. Abbreviations (i.e., $P_{rel\ n}A$) are defined similarly as for concepts. A *knowledge base* $\mathcal{K}$ is a pair $(\mathcal{T}, \mathcal{A})$ where $\mathcal{T}$ is a TBox and $\mathcal{A}$ is an ABox.

The semantics of Prob-DL is defined by generalizing the standard semantics of DL . In particular, a *probabilistic interpretation* has the form

$$\mathcal{I} = (\Delta^{\mathcal{I}}, W, (I_w)_{w \in W}, \mu), \tag{3}$$

where $\Delta^{\mathcal{I}}$ is the non-empty domain, $W$ is a non-empty set of *possible worlds*, $\mu$ is a discrete probability distribution on $W$, and for each $w \in W$, $\mathcal{I}_w$ is a classical DL interpretation with domain $\Delta^{\mathcal{I}}$. It is supposed that $a^{\mathcal{I}_w} = a^{\mathcal{I}_{w'}}$ for all $a \in N_I$ and $w, w' \in W$, therefore we write $a^{\mathcal{I}}$ in short. For $A \in N_C$, the probability that $a \in \Delta^{\mathcal{I}}$ is an $A$, is defined as

$$p_a^{\mathcal{I}}(A) = \mu(\{w \in W \mid a \in A^{\mathcal{I}_w}\}). \tag{4}$$

Similarly, for $r \in N_R$, the probability that $a, b \in \Delta^{\mathcal{I}}$ are related by $r$, is defined as

$$p_{a,b}^{\mathcal{I}}(r) = \mu(\{w \in W \mid (a, b) \in r^{\mathcal{I}_w}\}). \tag{5}$$

This is extended to complex concepts $C$, by defining the extension $C^{\mathcal{I}_w}$ of complex concepts by mutual recursion on $C$. The definition of $p_a^{\mathcal{I}}(C)$ is exactly as above (i.e., $A$ is replaced by $C$), and as the case for non-probabilistic concepts are defined in parallel to classical-DLs (e.g., $(C \sqcap D)^{\mathcal{I}_w} = C^{\mathcal{I}_w} \sqcap D^{\mathcal{I}_w}$), we give only the cases with probabilistic concepts:

$$\begin{aligned} (P_{rel\ n} C)^{\mathcal{I}_w} &= \{a \in \Delta^{\mathcal{I}} \mid p_a^{\mathcal{I}}(C) rel\ n\} \\ (\exists P_{rel\ n}.C)^{\mathcal{I}_w} &= \{\exists b \in C^{\mathcal{I}_w} : p_{a,b}^{\mathcal{I}}(r) rel\ n\} \end{aligned} \tag{6}$$

A probabilistic interpretation $\mathcal{I}$ satisfies a concept inclusion $C \sqsubseteq D$ if $C^{\mathcal{I}_w} \subseteq D^{\mathcal{I}_w}$ for all $w \in W$. The interpretation $\mathcal{I}$ is a *model* of a TBox $\mathcal{T}$ if it satisfies all inclusions in $\mathcal{T}$. Similarly, $\mathcal{I}_w$ satisfies assertions parallel to classical DLs (i.e., $\mathcal{I}_w \models C(a)$ iff $a^{\mathcal{I}} \in C^{\mathcal{I}_w}$), and this is defined inductively for ABoxes. Again we provide the probabilistic case;

$$\mathcal{I}_w \models P_{rel\ n}(\mathcal{A}) \text{ iff } p^{\mathcal{I}}(\mathcal{A}) rel\ n \tag{7}$$

where $p^{\mathcal{I}}(\mathcal{A})$ is the probability that an ABox $A$ holds and it is defined as

$$p^{\mathcal{I}}(\mathcal{A}) = \mu(\{w \in W \mid \mathcal{I}_w \models \mathcal{A}\}). \tag{8}$$

Note that, in this semantics $P_{rel\ n}(C(a))$ and $(P_{rel\ n}C)(a)$ are equivalent. It is said that $\mathcal{I}$ is a model of $\mathcal{A}$ if $\mathcal{I}_w \models \mathcal{A}$ for some $w$, and is a model of $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ if it is a model of both $\mathcal{T}$ and $\mathcal{A}$. A knowledge base $\mathcal{K}$ is *consistent* if it has a model. For convenience, we restrict ourselves to the language Prob-$\mathcal{ALC}_c$ which does not allow probabilistic roles, that is $\exists P_{rel\ n}$.C. This is because Prob-$\mathcal{ALC}_c$ is expressive enough for our purpose and also expectedly it provides much better complexity results (the consistency in full Prob-$\mathcal{ALC}$ is 2-EXPTIME-hard, whereas for Prob-$\mathcal{ALC}_c$ it is EXPTIME-complete) [10]. For the procedure for consistency check and details we refer the reader to [10].

As mentioned in [10], standard semantics of Prob-$\mathcal{ALC}$ does not support deducing the probability of *independent events* (i.e., $p(A \wedge B) = p(A) \cdot p(B)$). For that reason, Prob-$\mathcal{ALC}^{\text{indep}}$ is introduced (see [10]) as an extension of Prob-$\mathcal{ALC}$, which allows *independence constraints* in the form of indep$(C, D)$ in the TBox; $C$, $D$ being concepts, $p_d^{\mathcal{I}}(C) \cdot p_d^{\mathcal{I}}(D) = p_d^{\mathcal{I}}(C \sqcap D)$.

## 3. Decision Bases and Expected Utility

We model a discrete multi-criteria decision problem from the agent's perspective in the sense that *in the light of background knowledge and ranked outcomes which choice (alternative) should the agent take?* Here, ranking of outcomes are projected by agent's subjective utility values, and background knowledge is represented by a probabilistic knowledge base. We note that, although we assume the basic Prob-$\mathcal{ALC}_c$ as our base for clarity, the main working principle of our framework is not based on a specific Prob-DL language. We also note that, in this paper we do not concern ourselves with problems regarding elicitation. We assume that agent's preferences are sufficiently elicited.

### *3.1. Representing Discrete Multicriteria Decision Problems*

We represent the background knowledge of the agent by the Prob-DL knowledge base $\mathcal{K}$, which includes the hierarchy $\mathcal{T}$ of probabilistic concepts and *assertions* about individuals which are represented in $\mathcal{A}$. The choice set $\mathcal{C}$ represents *a priori* alternatives which utilities yet are unknown to the agent. $\mathcal{U}$ is the criteria set where each of its element consists of non-probabilistic concept denoted by $Y_i$ and a corresponding value denoted by $u_i$.

**Definition 1** (Decision Base)**.** *A decision base, $\mathcal{D} = (\mathcal{K}, \mathcal{C}, \mathcal{U})$ is a triple with;*

- $\mathcal{K} = (\mathcal{T}, \mathcal{A})$ *is Prob-DL* knowledge base *(background knowledge) in which $\mathcal{T}$ is a probabilistic general acyclic TBox and $\mathcal{A}$ is a probabilistic ABox,*
- $\mathcal{C} = \{Ch_1, \dots, Ch_n\}$ *is a* choice box*, a non-empty finite set of* choices*, each being a probabilistic ABox,*
- $\mathcal{U} = \{\langle Y_1, u_1\rangle, \dots, \langle Y_m, u_m\rangle\}$ *is a* utility box *(UBox), a finite set of non-probabilistic concepts $Y_i$ (criterion) and a corresponding* basic utility $u_i \in \mathbb{R}^+$ *with* $Y_i \equiv_{\mathcal{T}} Y_j \implies u_i = u_j$,

*with the restrictions that no nested probabilistic constructor occurs in $\mathcal{D}$ (e.g., $P_{\geq n}(P_{>n}(C))$), and for each probabilistic concept $P_{rel\ n}C$, $rel \in \{\geq, >\}$ and $n \in (0, 1]$.*

In a description logic decision base, we refer to a choice or alternative, as a list of specifications about individual(s) (DL), e.g., for a car buyer, choices can be technical specifications about cars whereas for a medical doctor, they can be treatment/medication alternatives. Notice that, one could also give an alternative definition that allows probabilistic concepts to occur in UBox. However, this in turn yields so much expressivity which is not very intuitive and without immediate obvious benefits. Also, we did not prefer to allow concept or role assertions in UBox, for a simpler and intuitive exposition of the framework (including the sequel). However, to make distinction between individuals and provide more expressivity from the utility perspective, one can extend the definition. Recall that basic utility values are solely subjective, serving our purpose. Also, we have restricted them to be non-negative reals for the sake of a simpler exposition in the sequel. This restriction can optionally be removed to model a particular decision problem conveniently.

### *3.2. Subjective Expected Utility*

Given that Prob-DL is developed to represent subjective uncertainty, and basic utility values for each criterion is specified, now we can define the subjective expected utility of a choice. For that purpose, let us introduce few useful notions:

- $clash_{\mathcal{K}}(C) = \{C'(a) \mid \{C(a), C'(a)\}$ is inconsistent $: \mathcal{K} \models C'(a) \wedge a \in Ind(\mathcal{A})\}$ where $Ind(\mathcal{A})$ is the set of individuals occuring in $\mathcal{A}$,
- The function $int : \mathcal{A} \rightarrow 2^{[0,1]}$ for $int(C(a)) = [n, 1]$ if $C$ is in the form of $P_{\geq\ n}D(a)$; $int(C(a)) = [1, 1]$ if $C$ is a non-probabilistic concept. Similarly, for the concepts with other probabilistic concept constructors (e.g., $int(C(a)) = (n, 1]$ if $C$ is in the form of $P_{>\ n}D(a)$),
- $\wp^{rel}(\mathcal{A}) = \inf\{\bigcap_{C(a)\in\mathcal{A}}\{int(C(a))\}\}$ where $rel \in \{\geq, >\}$,

Informally, for any given concept $C$, $clash_{\mathcal{K}}(C)$ denotes the set of all entailed assertions (from knowledge base $\mathcal{K}$) which yields a clash. A clash is considered as the form of $[C(a), \neg C(a))]$, or in particular $[P_{<\alpha}C(a), P_{\geq\beta}C(a)]$ where $\beta \geq \alpha$, due to abbreviations of probability constructors. The function $int$ outputs the probability interval for a given assertion. The function $\wp^{rel}$ gives the infimum of the intersection interval of all concept assertions in a given ABox. Contrary to *Analysis*, we leave infimum of an empty set as undefined.

**Definition 2** (Expected Utility). *The* expected utility $U_{rel}$ *of a choice* $Ch$ *w.r.t* $\mathcal{D} = (\mathcal{K}, \mathcal{C}, \mathcal{U})$ *is,*

$$U_{rel}(Ch) = \Sigma_{\{\langle Y_i, u\rangle\in\mathcal{U}\}}\wp^{rel}(clash_{\mathcal{K}\cup Ch}(P_{\leq 0}Y)) \cdot u$$

*where* $Ch \in \mathcal{C}$, $\mathcal{K} \cup Ch$ *is consistent and* $K \cup Ch \cup P_{\leq 0}Y(a)$ *is inconsistent for an* $a \in Ind(Ch \cup \mathcal{A})$, $rel \in \{\geq, >\}$.

Informally, (subjective) expected utility of a choice is the sum of the products between the infimum of the intersection of intervals of probabilities that criteria are satisfied, and basic utilities of criteria in $\mathcal{U}$, that are satisfied by that choice w.r.t. the knowledge base. Intuitively, a knowledge base and a choice ($\mathcal{K} \cup Ch$) will entail a criterion $Y$ with a degree of probability more than zero, if $P_{\leq 0}Y$ yields inconsistency (w.r.t. $\mathcal{K} \cup Ch$) for any $a \in Ind(Ch \cup \mathcal{A})$. Semantically, $U_{rel}(Ch)$ is interpreted as $rel$ utility of a choice, e.g., $U_{\geq}(Ch) = 40$ means that $Ch$ has utility of at least or equal to 40. Note that $U_{rel}$ yields a complete and transitive *preference relation* $\succeq$ over choices;

$$Ch_1 \succeq Ch_2 \iff U_{rel}(Ch_1) \geq U_{rel}(Ch_2). \tag{9}$$

In a similar fashion, an upper bound for the expected utility w.r.t. $<, \leq$ can be defined via the help of using sup in $\wp^{rel}$ instead. We conjecture that the use of *sup* and *inf* can induce a characterisation of *risk-seeking* and *risk averse* agents respectively, once the *maximum expected utility* is defined. We leave this to future work. In the sequel, we will drop $rel$, and write just $U$ instead for the sake of simplicity.

From the definition above, it follows that the utility of an inconsistent alternative/choice (with respect to the knowledge base) is undefined. Thus we restrict ourselves to assess only the consistent decisions. This naturally provides us a service to eliminate alternatives which can cause inconsistencies.

Notice that calculating the utility of a choice, can be thought of as answering a series of consistency checking problems. We speculate that it is at least of the complexity of the consistency checking problem (of the employed Prob-DL language) in the size of the given UBox. We leave a detailed formal investigation on complexity issues to future work.

Now, given the decision base and the expected utility of a choice $Ch$, one can easily define the type of the problem such as calculating the **maximum expected utility**:

$$Ch_{max} = \arg\max_{Ch}\{U(Ch) \mid Ch \in \mathcal{C}\} \tag{10}$$

This can be generalised in terms of picking up the best $n$ choices together.

$$Ch^n_{max} = \arg \max_{(Ch_1,\ldots,Ch_n)}\{U(\bigcup_{i=1}^{n} Ch_i) \mid Ch_1, \ldots, Ch_n \in \mathcal{C} \text{ and } n \leq |\mathcal{C}|\} \tag{11}$$

Or it can be logically restricted to a situation that agent can pick up at most one choice (mutually exclusive), with the following definition.

**Definition 3** (Mutual Exclusion)**.** *A decision base $\mathcal{D} = (\mathcal{K}, \mathcal{C}, \mathcal{U})$ is* mutually exclusive *if for every $Ch_i, Ch_j \in \mathcal{C}$ with $i \neq j$, $Ch_i \cup Ch_j \cup \mathcal{K}$ is inconsistent.*

In general, in order to model the concerned type of a decision problem, one can bring some restrictions on $\mathcal{C}$ and $\mathcal{U}$. Also, in $\mathcal{U}$ one can express *complement attributes* that is the utility of having both attribute is greater than sum of each, e.g, $\langle TechnicallySkilled, 20\rangle$ and $\langle FluentInEnglish, 30\rangle$ whereas $\langle TechnicallySkilled \sqsubseteq FluentInEnglish, 70\rangle$. Similarly one can express *substitute attributes* i.e., having both attribute has a lower basic utility than each.

### *3.3. Example: Hiring an employee*

We give an example on an agent (employer) giving a decision on hiring an employee based on criteria such as friendliness, punctuality, being technically skilled

$$
\begin{aligned}
\mathcal{T} = \quad & \{\exists hasPassed.Toefl \sqsubseteq P_{\geq 0.7} FluentInEnglish, \\
& Australian \sqsubseteq FluentInEnglish, \\
& \exists hasStudied.Math \sqsubseteq P_{\geq 0.9} TechnicallySkilled, \\
& \exists hasStudied.Business \sqsubseteq P_{\geq 0.4} TechnicallySkilled, \\
& \exists StudiedIn.EliteUni \sqsubseteq P_{\geq 0.8} Smart, \\
& \exists StudiedIn.AverageUni \sqsubseteq P_{\geq 0.5} Smart, \\
& P_{\geq 0.8} Friendly \sqcap Punctual \sqsubseteq Reliable\} \\
\mathcal{A} = \quad & \{EliteUni(UniB), \\
& AverageUni(UniA)
\end{aligned}
$$

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$$
\begin{aligned}
Ch_1 = \{ \quad & StudiedIn(Alice, UniB), & \qquad Ch_2 = \{ \quad & StudiedIn(Bob, UniA), \\
& Australian(Alice), & & hasStudied.Math(Bob), \\
& hasStudied.Business(Alice), & & hasPassedToefl(Bob)\} \\
& Punctual(Alice), & & \\
& P_{\geq 0.8} Friendly(Alice)\} & &
\end{aligned}
$$

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$$
\begin{aligned}
\mathcal{U} = \{ \quad & \langle TechnicallySkilled, 90\rangle, \\
& \langle FluentInEnglish, 70\rangle \\
& \langle Smart, 55\rangle, \\
& \langle Friendly, 20\rangle, \\
& \langle Reliable, 60\rangle\}
\end{aligned}
$$

**Figure 1.** Employer's background knowledge $\mathcal{K} = (\mathcal{T}, \mathcal{A})$, choices set CBox $\mathcal{C} = \{Ch_1, Ch_2\}$ representing two candidates for the *job position*, and UBox $\mathcal{U}$ on criteria and respective weights representing the preferences for the *position*.

etc. It can be thought of as impressions of the employer about two candidates after interviews.

Some of the information (subjective) given in Figure 1, is if someone studied in a elite university, she is smart at least with the probability 0.8. If an *individual* is known to be an Australian than she is certainly fluent in English. Moreover it is defined that $P_{\geq 0.8} Friendly(Alice)$ and a $Punctual$ $individual$ is $Reliable$. It can be interpreted that the agent also has the impression that Alice is punctual (which might follow from a possible scenario that she appeared on time to the interview) etc.

The interested reader can check that the expected utility of both choices and see that $Ch_1 \succeq Ch_2$ since $U(Ch_1) \geq 70 + 90 \times 0.4 + 55 \times 0.8 + 0.8 \times 20 + 60 = 226$ whereas $U(Ch_2) \geq 0.9 \times 90 + 0.5 \times 55 + 0.7 \times 70 = 157$, which is not a surprise as Bob fails to satisfy reliability and friendliness.

Notice that using Prob-$\mathcal{ALC}_c^{\text{indep}}$, one can calculate the expected utility w.r.t. an extended UBox including a criterion such as $\langle TechnicallySkilled \sqcap FluentInEnglish, 100\rangle$. In this case, the probability of $TechnicallySkilled \sqcap FluentInEnglish$ is implicitly inferred to be $\geq 0.63$ for Bob, in turn, getting an additional score of 63.

## 4. Related Work

Preference representation using logical languages has become popular over the last decade. Many of these approaches are based on propositional logic [3,8,18]. DL languages are used for preference representation in [9,11,12,14,13,16], yet most of them are not including uncertainty. Regarding the first use of DL in the context of MAUT, [14,13], Ragone et al focus on multi-issue bilateral negotiation. In [11, 12], they mainly discuss how to compute utilities (without uncertainty), where preferences are represented by weighted DL-formulas (*preference set*), just as UBox in our approach.

According to their terminology, our approach can be understood as an *implication-based* approach. They define logical implication in terms of membership, i.e., $m \models C$ iff $m \in C^{\mathcal{I}}$. The *minimal model* that they introduced in order to define the *minimal utility value* is more restrictive than ordinary models in DL. They change this definition to ordinary models in their next paper [12], while keeping the formal machinery the same (except the way they compute utilities). Hence, in addition to not dealing with uncertainty, the main difference of our approach is the formal extension to multiple alternatives and the use of ABoxes, which provides considerable expressivity.

To our knowledge, the only work which attempts to ground MCDM problems to (fuzzy) DL formally is [16]. The main difference is the choice of fuzzy DL as formalism to deal with uncertainty. Although the terms *utility* and *preference* are not explicitly used, it consists of preferences implicitly. They base their work on a standard MCDM feature, a *decision matrix* wherein the performance score of each alternative over each criteria is explicitly stated. Criteria are expressed as fuzzy concepts. Among alternatives, the optimal alternative (w.r.t the fuzzy knowledge base) is the one with the highest *maximum satisfiability degree*. In the explained framework, authors do not explicitly make a distinction between the knowledge base and the set of criteria. In general, the focus of the work is to show the potential and flexibility of fuzzy DL in encompassing the usual numerical methods used in MCDM, rather than leveraging the practicality of description logics in MCDM for expressing relations and handling inconsistencies between criteria, alternatives, and the knowledge base.

## 5. Conclusion and Further Plan

We have introduced a description logic based framework, to effectively express and solve decision problems of multi-attribute discrete alternatives.

As the major part of the utility theory and decision making literature is concerned with uncertainty, we based our approach on probabilistic description logics Prob-DL ([10]). In particular, the probabilistic extension allowed us to compute the subjective expected utility of choices in terms of their logical implications. This will allow us in our further work, to access the essential utility theory literature from the DL perspective, along with lots of new application possibilities.

One major direction is to investigate the decision theoretic properties of the utility function $U$, and the expressivity of $\mathcal{D}$, defining some restrictions on the UBox. Another major research direction is to extend the framework to sequential decisions (e.g. $\mathcal{D}_i \rightarrow \mathcal{D}_{i+1}$, sequence of decision bases). Once sequential decisions are defined, we will be able to represent policies and define a planner. Furthermore, it can be extended to represent collaborative decision making scenarios as well as game theoretical set-ups by considering more than one agent and specifying restrictions between their choice sets and knowledge bases. For instance in an arbitrary set-up, rules of the game could be a subset of intersection of both agent's knowledge bases, then the knowledge bases would get extended according to each players choices if each player can see what others choose. It can be checked whether a game-theoretical condition is satisfied, in terms of some corresponding conditions on ontologies.

Currently, we are working on the implementation of the basic framework as a Protégé[2] plug-in. Our plugin is planned to consist of an editor for the definition of UBoxes and choices, while the background knowledge is loaded via the standard interfaces of Protégé. Our extension will then be able to compute the utility of the given choices w.r.t background knowledge and display a ranking of choices. The development of our Protégé plugin is motivated by the idea to demonstrate the benefits of our approach to a set of different application scenarios where decision making is involved.

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