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Inconsistency resolution and global conflicts

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Abstract. Over the years, inconsistency management has caught the attention of researchers of different areas. Inconsistency is a problem that arises in many different scenarios, for instance, ontology development or knowledge integration. In such settings, it is important to have adequate automatic tools for handling conflicts that may appear in a knowledge base. We introduce an approach to consolidation of belief bases based on a refinement of kernel contraction that accounts for the relation among kernels using clusters instead. We define cluster contraction-based consolidation operators contraction by *falsum* on a belief base using cluster incision functions, a refinement of kernel incision functions.

1 Introduction

Inconsistency management is admittedly an important problem that needs to be faced in many modern computer systems. Several important approaches to handle inconsistency had been proposed in Artificial Intelligence (AI), specially in the areas of *belief revision* and *argumentation*. In particular, belief revision deals with the general problem of the dynamics of knowledge, *i.e.,* how belief states change and evolve through time, solving possible inconsistencies in the process. The work of Alchourrón, Gärdenfors and Makinson where the *AGM* model is presented [1], is currently considered the cornerstone from which belief revision theory has evolved (see [8]). *Contraction* is one of the basic change operators defined in the *AGM* model, the result of contracting a belief base K by a sentence α is a possibly smaller set of beliefs from which α can no longer be logically inferred. One approach to contraction, known as *kernel contraction* [3, 4], is based on removing elements from the α*-kernels* of K, which are (minimal) subsets of K that contribute to entailing α.

We focus on a different belief change operation called *consolidation*; this operation is inherently different from contraction as the ultimate goal of consolidation is to obtain a consistent belief base rather than removing a particular formula from it. Nevertheless, consolidation can be defined in terms of contraction; a natural way of achieving this is to take an inconsistent belief base and restore its consistency by attending every conflict in it, a process that is known in the belief revision literature as contraction by *falsum* [4]. In this work, we analyze consolidation operators that take an inconsistent belief base and apply special functions, called incisions functions, in order to restore consistency. Instead of treating conflicts locally as made in classic kernel contraction, we present incisions that account for global considerations to improve the efficiency of the consistency restoration process, avoiding unnecessary deletions that sometimes arise from the classic approach because of its generality.

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2 Kernel and Cluster Contraction-based Belief Base Consolidation

We first introduce some necessary notation that will be used throughout the paper. We assume a propositional language $\mathcal{L}$ built from a set of propositional symbols $\mathcal{P}$. This language is closed under classical propositional logic symbols. We denote propositional letters using lower-case Latin letters and propositional formulæ using lower-case Greek letters, possibly using subscripts; but, we reserve ρ and ϱ to represent incision functions.

An interpretation is a total function from $\mathcal{P}$ to $\{0, 1\}$, and the set of all interpretations is denoted with $\mathcal{W}$. An interpretation $\omega \in \mathcal{W}$ is a model of a formula α iff it makes α true in the classical way, denoted with $\omega \models \alpha$. The set of all models of a formula α is $\mathrm{mods}(\alpha) = \{\omega \in \mathcal{W} \mid \omega \models \alpha\}$. Furthermore, $\perp$ stands for an arbitrary contradiction. A belief base, denoted by K, is a finite set of propositional formulæ. The set of models of belief base K are $\mathrm{mods}(K) = \{\omega \in \mathcal{W} \mid \omega \models \alpha \text{ for all } \alpha \in K\}$. We denote with $K_{\mathcal{L}}$ the set of all belief bases that can be built from $\mathcal{L}$. Finally, a K is consistent iff $\mathrm{mods}(K) \neq \emptyset$; and K is inconsistent otherwise.

The work of Hansson in [3] describes how a contraction operator can be modeled by means of *incision functions*. These operators contract a belief base by a formula α by taking each minimal sets that entails α, and producing "incisions" on the set so α is no longer entailed from it. This approach is known as *kernel contraction*.

We define the consolidation process as the application of incision functions over the minimal inconsistent subsets of a belief base. We will call such sets kernels; we recall the formal definition from [3].

Definition 1 (Kernels) *Let K be a belief base. The set of kernels of K, denoted $K^{\perp\!\!\perp}\perp$, is the set of all $X \subseteq K$ such that $\mathrm{mods}(X) = \emptyset$ and for every $X' \subsetneq X$ it holds that $\mathrm{mods}(X') \neq \emptyset$.*

A *kernel incision function* takes a set of kernels and selects formulæ in them to be deleted from K [3].

Definition 2 (Kernel Incision Function) *Let K be a belief base, and $K^{\perp\!\!\perp}\perp$ be the set of kernels for K. A* kernel incision function *is a function $\rho : 2^{K_{\mathcal{L}}} \mapsto K_{\mathcal{L}}$ such that (i) $\rho(K^{\perp\!\!\perp}\perp) \subseteq \bigcup(K^{\perp\!\!\perp}\perp)$, and (ii) for all $X \in K^{\perp\!\!\perp}\perp$, if $X \neq \emptyset$ then $(X \cap \rho(K^{\perp\!\!\perp}\perp)) \neq \emptyset$.*

Based on kernel incision functions we define kernel contraction-based belief consolidation operators as follows.

Definition 3 (Kernel Contraction-based Consolidation Operator) *Let K be a belief base, $K^{\perp\!\!\perp}\perp$ be the set of kernels for K and ρ be a kernel incision function. A kernel contraction-based consolidation operator Υ_ρ for K is defined as $\Upsilon_\rho(K) = K \setminus \rho(K^{\perp\!\!\perp}\perp)$.*

If we strive for minimal loss of information, then this operator has the important drawback of solving conflicts locally to every kernel;

even if the function only removes one formula from each kernel, the incisions may be too drastic from a global point of view:

Example 1 *Consider the following inconsistent knowledge base* $K = \{p, q, r, p \to \neg r, \neg(q \wedge r)\}$. *For belief base* K *we have that* $K^{\perp\!\!\perp}\perp = \{\kappa_1, \kappa_2\}$, *where* $\kappa_1 = \{p, r, p \to \neg r\}$ *and* $\kappa_2 = \{q, r, \neg(q \wedge r)\}$. *Now, suppose we have an incision function such that* $\rho(\kappa_1) = \{r\}$ *and* $\rho(\kappa_2) = \{q\}$*; we then have that* $\Upsilon_\rho(K) = K \setminus \rho(K^{\perp\!\!\perp}\perp) = K \setminus \{q, r\}$. *Then, for* κ_2 *we delete* q *from* K *in order to solve the conflict. However, this is unnecessary, as* r *will not be in the final belief base anyway, since it is deleted to solve the conflict in* κ_1*, and thus the conflict in kernel* κ_2 *is already solved, and there is no need to further remove propositions from* κ_2.

In the following we present an approach that avoids this by contemplating only incisions to the belief base that are globally optimal with respect to the amount of information lost. The proposal is based on the use of *clusters* [7, 5, 6]; which are built upon an overlapping relation θ: given two kernels $\kappa_1, \kappa_2 \in K^{\perp\!\!\perp}\perp$ we say they overlap, denoted $\kappa_1 \theta \kappa_2$, iff $\kappa_1 \bigcap \kappa_2 \neq \emptyset$. Furthermore, θ^* is the equivalence relation obtained through the reflexive and transitive closure of θ.

Definition 4 (Clusters [7]) *Let* K *be a belief base,* $K^{\perp\!\!\perp}\perp$ *be the set of kernels for* K*, and* θ *the overlapping relation. A cluster of* K *is a set* $\varsigma = \bigcup_{\kappa \in [\kappa]} \kappa$, *where* $[\kappa] \in K^{\perp\!\!\perp}\perp/\theta^*$. *We use* $K^{\perp\!\!\perp\!\!\perp}\perp$ *to denote the set of all clusters for* K.

Intuitively, a cluster groups together kernels that share formulæ, in a transitive way. The use of clusters instead of kernels can help to prevent situations like the one presented in Example 1 since they show a more global picture of how the formulæ are related in respect to conflicts. It is important to remember that by design the removal of any single formula within a kernel makes the set no longer inconsistent; however, this is not necessarily the case for clusters [5]. Therefore, in order to define incision functions over clusters, we cannot simply reuse Definition 2, instead, we introduce the notion of cluster incision functions, which are refinements of the ones introduced earlier.

Definition 5 (Cluster Incision Function) *Let* K *be a belief base and* $K^{\perp\!\!\perp}\perp$ *and* $K^{\perp\!\!\perp\!\!\perp}\perp$ *be the set of kernels and clusters for* K*, respectively. A* cluster incision function *is a function* $\varrho : 2^{K_\mathcal{L}} \mapsto K_\mathcal{L}$ *such that (i)* $\varrho(K^{\perp\!\!\perp\!\!\perp}\perp) \subseteq \bigcup(K^{\perp\!\!\perp\!\!\perp}\perp)$*, and (ii) for all* $X \in K^{\perp\!\!\perp\!\!\perp}\perp$ *and* $Y \in K^{\perp\!\!\perp}\perp$ *such that* $Y \subseteq X$ *it holds that for some* $\alpha \in Y$, $Y \cap \varrho(K^{\perp\!\!\perp\!\!\perp}\perp) = \{\alpha\}$.

Now, based on cluster incision functions we define a new operator, namely cluster contraction-based consolidation operator, as follows.

Definition 6 (Cluster Contraction-based Consolidation Operator) *Let* K *be a belief base,* $K^{\perp\!\!\perp}\perp$ *and* $K^{\perp\!\!\perp\!\!\perp}\perp$ *be the set of kernels and clusters for* K*, respectively, and* ϱ *be a cluster incision function. A cluster contraction-based operator* Ψ_ϱ *for* K *is defined as* $\Psi_\varrho(K) = K \setminus \varrho(K^{\perp\!\!\perp\!\!\perp}\perp)$.

Condition (ii) in Definition 5 ensures that all conflicts are solved once we delete the selected formulæ. Example 2 shows the behavior of a cluster contraction-based operator Ψ_ϱ with respect to the choice made in Example 1.

Example 2 *Consider belief base* K *from Example 1; we have the following set of clusters* $K^{\perp\!\!\perp\!\!\perp}\perp = \{\varsigma_1\}$, *with* $\varsigma_1 = \{p, q, r, p \to \neg r, \neg(q \wedge r)\}$, *because* r *belongs to both kernels in* $K^{\perp\!\!\perp}\perp$. *For a cluster contraction-based operator* Ψ_ϱ *based on a cluster incision function* ϱ*, possible incisions are* $\varrho(\varsigma_1) = \{p, q\}$, *and* $\varrho(\varsigma_1) = \{r\}$. *Consider option* $\varrho(\varsigma_1) = \{p, q\}$. *Thus,* $\Psi_\varrho(K) = K \setminus \varrho(K^{\perp\!\!\perp\!\!\perp}\perp) = K \setminus \{p, q\}$. *Note that if we were to choose* r *for deletion then to also choose* q *(i.e., the option considered in Example 1) is no longer a viable option for a cluster incision function, as if we choose both formulæ then the set* $\varrho(K^{\perp\!\!\perp\!\!\perp}\perp) \cap \kappa_2$ *will no longer be a singleton set, violating the second condition from Definition 5.*

Every cluster contraction-based consolidation operators is also a kernel contraction-based one.

Proposition 1 *Let* Ψ_ϱ *be a cluster contraction-based consolidation operator. Then,* Ψ_ϱ *is a kernel contraction-based consolidation operator.*

The converse does not hold, *i.e.,* a kernel contraction-based operator is not necessarily a cluster contraction-based operator, as kernel incision functions are not necessarily cluster incision functions.

As hinted by Examples 1 and 2, a clear benefit of using cluster-based consolidation operators is that some unnecessary deletions can be avoided by clustering conflicts. It can be shown that cluster contraction-based consolidation operators always remove at most the same number of formulæ than kernel contraction-based ones; the following proposition formalizes the result.

Proposition 2 *Let* K *be a belief base,* ϱ *and* ρ *be a cluster incision function and a kernel incision function, and* Ψ_ϱ *and* Υ_ρ *their respectively associated cluster and kernel contraction-based consolidation operators over* K. *Then, we have that* $|\Upsilon_\rho(K)| \leq |\Psi_\varrho(K)|$.

3 Conclusions and Future Work

In this paper we focus on an approach to consolidation of belief bases defined on terms of belief base contractions [1]. We have proposed and developed a new class of belief bases consolidation operators, called cluster contraction-based operators, and shown that they are more efficient in consistency restoration than the classic approach, from the point of view of minimal loss of information. Future work will involve a deeper analysis of the differences with both kernel contraction and the AGM approach, by exploiting the relation between incision and selection functions [2].

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