ECAI 2014
T. Schaub et al. (Eds.)

doi:10.3233/978-1-61499-419-0-957

DL-Lite and Interval Temporal Logics: a Marriage Proposal

A. Artale[1] and **D. Bresolin**[2] and **A. Montanari**[3] and **G. Sciavicco**[4] and **V. Ryzhikov**[1]

Abstract. Description logics of the *DL-Lite* family are widely used in knowledge representation because of their low computational complexity and rather good expressivity sufficient to capture important conceptual modelling constructs and the OWL2 QL profile of the Ontology Web Language (OWL). Recently, various point-based temporal extensions of *DL-Lite* have been investigated. Here, we propose to extend *DL-Lite* with fragments of Halpern and Shoham's interval logic of Allen's relations ($\mathcal{HS}$). We formally define such extensions and show how they can be successfully used in knowledge representation. In the quest for a decidable logic, we discuss the challanges in combining decidable fragments of $\mathcal{HS}$ with *DL-Lite*.

1 INTRODUCTION

The name *DL-Lite* identifies a family of description logics (DLs) characterized by a nice computational behaviour combined with a relatively high expressive power [1, 7]. *DL-Lite* is the basis of the OWL 2 QL language (a W3C standard) and is able to capture conceptual modeling formalisms like UML class diagrams and ER diagrams. We focus our attention to the *supremum* (w.r.t. the expressive power) formalism, namely, *DL-Lite*$^{\mathcal{HN}}_{bool}$, where the sub-script *bool* stands for full Boolean operators and $\mathcal{HN}$ indicates that number restrictions and role inclusions are fully available. A comprehensive survey of the *DL-Lite* family of languages and their computational properties and applications can be found in [1].

Temporal extensions of *DLs* have been extensively studied in the literature [2,4,8,10]. In particular, logics of the *DL-Lite* family have been combined with a variety of *point-based* temporal logics ranging from Future and Past LTL to full LTL with Since and Until [3]. Different logics, generically denoted by T_{TL}*DL-Lite*, have been investigated, which are characterized by the following parameters: *(i)* the fragment of LTL to be used, *(ii)* the fragment of *DL-Lite* taken as the basis for the extension, and *(iii)* whether roles can be temporalized or not. Complexities of the different logics range from NLOGSPACE to undecidable (in particular, when roles can be temporalized or role inclusions and number restrictions can interact without constraints).

In this paper, we propose *interval-based* extensions of *DL-Lite* based on fragments of Halpern and Shoham's logic $\mathcal{HS}$ [9]. Propositional $\mathcal{HS}$ encompasses a modality $\langle X\rangle$ for each Allen relation depicted in Fig. 1 plus all their transposes (denoted as $\langle\overline{X}\rangle$), giving

$\langle A\rangle$	$[i,j]r_A[i',j'] \Leftrightarrow j = i'$
$\langle L\rangle$	$[i,j]r_L[i',j'] \Leftrightarrow j < i'$
$\langle B\rangle$	$[i,j]r_B[i',j'] \Leftrightarrow i = i', j < j'$
$\langle E\rangle$	$[i,j]r_E[i',j'] \Leftrightarrow j = j', i < i'$
$\langle D\rangle$	$[i,j]r_D[i',j'] \Leftrightarrow i < i', j' < j$
$\langle O\rangle$	$[i,j]r_O[i',j'] \Leftrightarrow i < i' < j < j'$

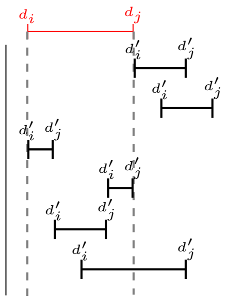


Figure 1. Allen's interval relations and $\mathcal{HS}$ modalities.

rise to a very expressive multi-modal logic. We make the following assumptions: *(i)* we take $\mathbb{Z}$ as the temporal domain; *(ii)* we consider only $\mathcal{HS}$ fragments that can express the so-called *length constraints*; *(iii)* we distinguish between *rigid* roles, that is, roles that are time-invariant, and *flexible* roles, but we do not allow the computationally-expensive *temporalised* roles. All logics studied here can be considered as fragments of the combination of *DL-Lite*$^{\mathcal{HN}}_{bool}$ and $\mathcal{HS}$, that is, $T_{\mathcal{HS}}$*DL-Lite*$^{\mathcal{HN}}_{bool}$. Since $\mathcal{HS}$ is undecidable over $\mathbb{Z}$, it easily follows that $T_{\mathcal{HS}}$*DL-Lite*$^{\mathcal{HN}}_{bool}$ is undecidable as well. However, a number of recent results show that various fragments of $\mathcal{HS}$ offer a good balance between expressiveness and decidability/complexity [5, 6], suggesting that we can weaken the temporal part by considering decidable fragments of $\mathcal{HS}$ such as $\mathsf{AB\overline{B}\overline{L}}$ and *metric* $\mathsf{A\overline{A}}$ (denoted by MPNL), without sacrificing expressiveness too much.

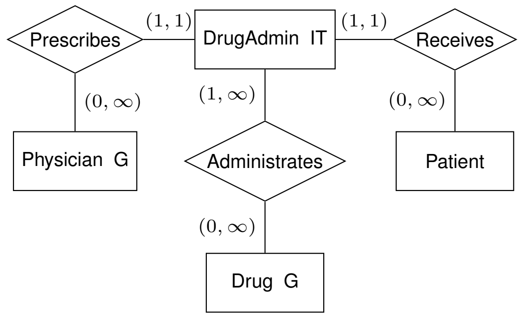
2 THE LANGUAGE $T_{\mathcal{HS}}$*DL-Lite*$^{\mathcal{HN}}_{bool}$

The logic $T_{\mathcal{HS}}$*DL-Lite*$^{\mathcal{HN}}_{bool}$ can describe concepts denoting set of individuals possibly changing over time intervals. The syntax of the language can be naturally obtained from the two components *DL-Lite*$^{\mathcal{HN}}_{bool}$ and $\mathcal{HS}$, and it contains *object names* $a_0, a_1, \ldots$, *concept names* $A_0, A_1, \ldots$, *flexible role names* $P_0, P_1, \ldots$, and *rigid role names* $G_0, G_1, \ldots$ *Role names* S, *roles* R, *basic concepts* B, and *concepts* C are formed by the following grammar:

$$S ::= P_k \mid G_k \qquad R ::= S \mid S^-$$
$$B ::= \bot \mid A_k \mid \geq q\, R \qquad C ::= B \mid \neg C \mid C_1 \sqcap C_2 \mid \langle X_k\rangle C$$

where $\langle X_k\rangle$ is one of the $\mathcal{HS}$-modalities. A TBox $\mathcal{T}$ is a finite set of concept and role *inclusions* of the form: $C_1 \sqsubseteq C_2$ and $R_1 \sqsubseteq R_2$. An ABox is a finite set of statements of the following form: $A_k(a_m, [i,j])$ (i.e., '$A_k(a_m)$ *holds on the interval* $[i,j]$'), $\neg A_k(a_m,[i,j])$, $S_k(a_m,a_n,[i,j])$, and $\neg S_k(a_m,a_n,[i,j])$. A $T_{\mathcal{HS}}$*DL-Lite*$^{\mathcal{HN}}_{bool}$ KB is a pair $\mathcal{K} = (\mathcal{T},\mathcal{A})$. A *temporal interpretation* is a pair $\mathcal{I} = (\Delta^{\mathcal{I}}, \cdot^{\mathcal{I}([i,j])})$, where $\Delta^{\mathcal{I}}$ is a non empty

[1] KRDB Research Center, Free University of Bozen-Bolzano (Italy) {*artale*|*ryzhikov*}*@inf.unibz.it*
[2] Department of Computer Science and Engineering, University of Bologna (Italy) *davide.bresolin@unibo.it*
[3] Department of Mathematics and Computer Science, University of Udine (Italy) *angelo.montanari@uniud.it*
[4] Department of Information Engineering and Communications, University of Murcia (Spain) *guido@um.es*



$$\mathit{DrugAdmin} \sqsubseteq \exists \mathit{Prescribes}^-, \quad \geq 2\ \mathit{Prescribes}^- \sqsubseteq \bot \qquad (1)$$
$$\mathit{Physician} \sqsubseteq [G]\mathit{Physician}, \quad \mathit{Drug} \sqsubseteq [G]\mathit{Drug} \qquad (2)$$
$$[\mathrm{G}]\mathit{DrugAdmin} \sqsubseteq \bot \qquad (3)$$
$$\mathit{Therapy} \sqsubseteq \langle B \rangle \mathit{DrugAdmin} \sqcap \langle E \rangle \mathit{DrugAdmin} \qquad (4)$$
$$\mathit{DrugAdmin} \sqsubseteq [O]\neg \mathit{DrugAdmin} \qquad (5)$$

Figure 2. The conceptual data model of the medical example.

domain and $\mathcal{I}([i,j])$ is a standard *DL* interpretation for each interval $[i,j] \in \mathbb{I}(\mathbb{Z})$. We assume that rigid role names and object names have a time-invariant interpretation, i.e., $G^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$ and $a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$. The interpretation of flexible role names $P^{\mathcal{I}([i,j])}$ and concept names $A^{\mathcal{I}([i,j])} \subseteq \Delta^{\mathcal{I}}$ depends on the interval $[i,j]$ of evaluation. We interpret temporal concepts as: $(\langle X_k \rangle C)^{\mathcal{I}([i,j])} = \bigcup_{[i,j] r_{X_k} [i',j']} C^{\mathcal{I}([i',j'])}$, where r_{X_k} is the Allen's relation that corresponds to the modality $\langle X_k \rangle$. Members of the TBox are interpreted *globally*: $\mathcal{I} \Vdash C_1 \sqsubseteq C_2$ iff $C_1^{\mathcal{I}([i,j])} \subseteq C_2^{\mathcal{I}([i,j])}$, for all $[i,j] \in \mathbb{I}(\mathbb{Z})$ (similarly for role axioms), while ABox is interpreted locally.

3 A MOTIVATING EXAMPLE

The example of Fig. 2 represents a part of a medical information system. The ER diagram represents an entity *DrugAdmin* that is prescribed by some *Physician* to some *Patient*. A *DrugAdmin* consists of the administration of some *Drug*. We can model the diagram in $T_{\mathcal{HS}}$*DL-Lite*$^{\mathcal{HN}}_{bool}$ by considering *DrugAdmin*, *Physician*, *Patient*, and *Drug* as concept names, and *Prescribes*, *Receives*, and *Administrates* as role names. All atemporal constraints can be encoded in *DL-Lite*, e.g., cardinality constraints such as *'a drug administration is prescribed by exactly one physician'* can be expressed with axioms (1). The fact that *Physician* and *Drug* are *time-invariant* entities—i.e., global entities holding at every interval—is represented with the *timestamp* G (standing for *global*) and is enforced by using the global temporal operator $[G]$ (which can be expressed by various $\mathcal{HS}$ fragments) and the corresponding axioms (2). *Time-varying* entities are represented by marking them with the *timestamp* IT with the meaning that they have a limited life-span. The fact that *DrugAdmin* is time-varying can be captured using axiom (3). Moreover, the interval modalities of $\mathcal{HS}$ allow us to express complex relations between the lifespan of time-varying concepts. For instance, we can introduce the concept *Therapy* as an *event* made of drug admnistration *subevents* using axiom (4), and we can force *'drug administrations not to overlap inside the same therapy'* with axiom (5). The above example makes use of $\mathcal{HS}$ modalities that belong to fragments which are not always decidable. Under reasonable assumptions, such as, for example, *'a therapy is always shorter than k time units'*, we can rewrite the above formulas using only the interval modalities of the decidable fragments MPNL or $\mathsf{AB\overline{B}\overline{L}}$.

4 DISCUSSION AND FUTURE WORK

From the decidability point of view, we already know that propositional $\mathcal{HS}$ is undecidable. Therefore, we restrict our attention to combinations *DL-Lite* with decidable fragments of $\mathcal{HS}$, such as $\mathsf{AB\overline{B}\overline{L}}$ or MPNL. In analogy to [3], where the combination of *DL-Lite* with Linear Temporal Logic (LTL) over $\mathbb{Z}$ was studied, one can show that a $T_{\mathcal{HS}}$*DL-Lite*$^{\mathcal{HN}}_{bool}$ KB $\mathcal{K}$ can be encoded as an equi-satisfiable formula $\mathcal{K}^{\dagger}$ of the first-order $\mathcal{HS}$ language that uses *unary predicates only*. The key point of such an encoding is to use predicates $E_qS(x)$ and $E_qS^-(x)$ encoding, respectively, the concepts $\geq q\ S$ and $\geq q\ S^-$ occurring in $\mathcal{K}$, and an axiom $[G](\exists x\, E_1S(x) \leftrightarrow \exists x\, E_1S^-(x))$. In LTL extensions, it is possible to further encode $\mathcal{K}^{\dagger}$ into an equi-satisfiable *propositional* (LTL-)formula $\mathcal{K}^{\ddagger}$. In fact, the only axiom of $\mathcal{K}^{\dagger}$ not purely universal is the above one, and it can be "skolemized" by introducing "witnesses" d_{S^-} and d_S such that d_{S^-} makes the predicate ES^- true at time 0 whenever ES is true on some object at some time (similarly for d_S). A similar reduction to a decidable fragment $\mathcal{F}$ of $\mathcal{HS}$, which would allow us to prove the decidability of the fragment $T_{\mathcal{F}}$*DL-Lite*$^{\mathcal{HN}}_{bool}$, does not work. Indeed, we cannot "skolemize" $\mathcal{K}^{\dagger}$ by using finitely many witnesses because ES may hold on intervals of various lengths, and it is not possible to use a witness of a fixed length, say n, as a witness of length $n+1$ or $n-1$. This problem is intrinsic to interval-based temporal logics and did not occur in the point-based case.

As for future work, we will investigate the decidability problem for those $T_{\mathcal{HS}}$*DL-Lite*$^{\mathcal{HN}}_{bool}$ fragments based on decidable fragments of $\mathcal{HS}$. We will also study so-called sub-Boolean fragments of these languages, in the quest for *tractable* combinations of *DL-Lite* with fragments of $\mathcal{HS}$. Such low complexity sub-Boolean languages have been recently identified for LTL extensions of *DL-Lite* [3].

ACKNOWLEDGMENTS We acknowledge the support from the Spanish fellowship program *'Ramon y Cajal' RYC-2011-07821* (G. Sciavicco) and the Italian GNCS project *'Automata, games and temporal logics for verification and synthesis of safety-critical systems'* (D. Bresolin and A. Montanari).

References

[1] A. Artale, D. Calvanese, R. Kontchakov, and M. Zakharyaschev, 'The DL-Lite family and relations', *JAIR*, **36**, 1–69, (2009).

[2] A. Artale and E. Franconi, 'Temporal description logics', in *Handbook of Temporal Reasoning in Artificial Intelligence*, Foundations of Artificial Intelligence, 375–388, Elsevier, (2005).

[3] A. Artale, V. Ryzhikov, R. Kontchakov, and M. Zakharyaschev, 'A cookbook for temporal conceptual data modelling with description logics', *ACM Transactions on Computational Logic*, **15**(3), (2014).

[4] F. Baader, S. Ghilardi, and C. Lutz, 'LTL over description logic axioms', *ACM Transactions on Computational Logic*, **13**(3), (2012).

[5] D. Bresolin, D. Della Monica, A. Montanari, P. Sala, and G. Sciavicco, 'Interval temporal logics over strongly discrete linear orders: Expressiveness and complexity', *Theoretical Computer Science*. To appear.

[6] D. Bresolin, A. Montanari, P. Sala, and G. Sciavicco, 'Optimal decision procedures for MPNL over finite structures, the natural numbers, and the integers', *Theoretical Computer Science*, **493**, 98–115, (2013).

[7] D. Calvanese, G. De Giacomo, D. Lembo, M. Lenzerini, and R. Rosati, 'Tractable reasoning and efficient query answering in description logics: The DL-Lite family', *JAR*, **39**(3), 385–429, (2007).

[8] D. Gabbay, A. Kurucz, F. Wolter, and M. Zakharyaschev, *Many-dimensional modal logics: theory and applications*, Studies in Logic. Elsevier, 2003.

[9] J. Halpern and Y. Shoham, 'A propositional modal logic of time intervals', *J. of the ACM*, **38**(4), 935–962, (1991).

[10] C. Lutz, F. Wolter, and M. Zakharyaschev, 'Temporal description logics: A survey', in *Proc. of TIME'08*. IEEE Computer Society, (2008).