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Compact Argumentation Frameworks

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Abstract. Abstract argumentation frameworks (AFs) are one of the most studied formalisms in AI. In this work, we introduce a certain subclass of AFs which we call compact. Given an extension-based semantics, the corresponding compact AFs are characterized by the feature that each argument of the AF occurs in at least one extension. This not only guarantees a certain notion of fairness; compact AFs are thus also minimal in the sense that no argument can be removed without changing the outcome. We address the following questions in the paper: (1) How are the classes of compact AFs related for different semantics? (2) Under which circumstances can AFs be transformed into equivalent compact ones? (3) Finally, we show that compact AFs are indeed a non-trivial subclass, since the verification problem remains coNP-hard for certain semantics.

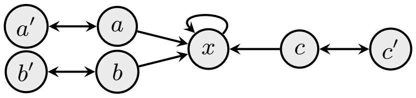
1 Introduction

In recent years, *argumentation* has become a major concept in AI research [5, 17]. In particular, Dung's well-studied *abstract argumentation frameworks* (AFs) [9] are a simple, yet powerful formalism for modeling and deciding argumentation problems. Over the years, various *semantics* have been proposed, which may yield different results (so called *extensions*) when evaluating an AF [9, 18, 6, 2]. Also, some subclasses of AFs such as acyclic, symmetric, odd-cycle-free or bipartite AFs, have been considered, where for some of these classes different semantics collapse [7, 10].

In this work we introduce a further class, which to the best of our knowledge has not received attention in the literature, albeit the idea is simple. We will call an AF *compact* (with respect to a semantics σ), if each of its arguments appears in at least one extension under σ. Thus, compact AFs yield a "semantic" subclass since its definition is based on the notion of extensions. Another example of such a semantic subclass are coherent AFs [11]; further examples are in [3, 14].

Importance of compact AFs mainly stems from the following two aspects. First, compact AFs possess a certain fairness behavior in the sense that each argument has the chance to be accepted, which might be a desired feature in some of the application areas AFs are currently employed in, such as decision support [1]. The second and more concrete aspect is the issue of normal-forms of AFs. Indeed, compact AFs are attractive for such a normal-form, since none of the arguments can be removed without changing the extensions.

Following this idea we are interested in the question whether an arbitrary AF can be transformed into a compact AF without changing the outcome under the considered semantics. It is rather easy to see that under the *naive* semantics, which is defined as maximal conflict-free sets, any AF can be transformed into an equivalent compact AF. However, as has already been observed in [12], this is not true for other semantics. As an example consider the following AF F_1, where nodes represent arguments and directed edges represent attacks.



The *stable* extensions (conflict-free sets attacking all other arguments) of F_1 are $\{a,b,c\}$, $\{a,b',c'\}$, $\{a',b,c'\}$, $\{a',b',c\}$, $\{a,b,c'\}$, $\{a',b,c\}$, and $\{a,b',c\}$. It was shown in [12] that there is no compact AF (in this case an F_1' not using argument x) which yields the same stable extensions as F_1. By the necessity of conflict-freeness any such compact AF would only allow conflicts between arguments a and a', b and b', and c and c', respectively. Moreover, there must be attacks in both directions for each of these conflicts in order to ensure stability. Hence any compact AF having the same stable extensions as F_1 necessarily yields $\{a',b',c'\}$ in addition. As we will see, all semantics under consideration share certain criteria which guarantee impossibility of a translation to a compact AF.

Like other subclasses, compact AFs decrease complexity of certain decision problems. This is obvious by the definition for credulous acceptance (does an argument occur in at least one extension). For skeptical acceptance (does an argument a occur in all extensions) in compact AFs this problem reduces to checking whether a is isolated. If yes, it is skeptically accepted; if no, a is connected to at least one further argument which has to be credulously accepted by the definition of compact AFs. But then, it is the case for any semantics which is based on conflict-free sets that a cannot be skeptically accepted, since it will not appear together with b in an extension. However, the problem of verification (does a given set of arguments form an extension) remains coNP-hard for certain semantics, hence enumerating all extensions of a compact AF remains non-trivial.

An exact characterization of the collection of all sets of extensions which can be achieved by a compact AF under a given semantics σ seems rather challenging. We illustrate this on the example of stable semantics. Interestingly, we can provide an exact characterization under the condition that a certain conjecture holds: Given an AF F and two arguments which do not appear jointly in an extension of F, one can always add an attack between these two arguments (and potentially adapt other attacks in the AF) without changing the stable extensions. This conjecture is important for our work, but also an interesting question in and of itself.

To summarize, the main contributions of our work are:

- We define the classes of compact AFs for some of the most prominent semantics (namely naive, stable, stage, semi-stable and preferred) and provide a full picture of the relations between these classes. Then we show that the verification problem is still intractable for stage, semi-stable and preferred semantics.
- Moreover we use and extend recent results on maximal numbers of extensions [4] to give some impossibility results for *compact*

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realizability. That is, we provide conditions under which for an AF with a certain number of extensions no translation to an equivalent (in terms of extensions) compact AF exists.

- Finally, we study *signatures* [13] for compact AFs exemplified on the stable semantics. An exact characterization relies on the open explicit-conflict conjecture mentioned above. However, we give some sufficient conditions for an extension-set to be expressed as a stable-compact AF. For example, it holds that any AF with at most three stable extensions possesses an equivalent compact AF.

2 Preliminaries

In what follows, we briefly recall the necessary background on abstract argumentation. For an excellent overview, we refer to [2].

Throughout the paper we assume a countably infinite domain $\mathfrak{A}$ of arguments. An *argumentation framework* (AF) is a pair $F = (A, R)$ where $A \subseteq \mathfrak{A}$ is a non-empty, finite set of arguments and $R \subseteq A \times A$ is the attack relation. The collection of all AFs is given as $AF_{\mathfrak{A}}$. For an AF $F = (B, S)$ we use A_F and R_F to refer to B and S, respectively. We write $a \mapsto_F b$ for $(a, b) \in R_F$ and $S \mapsto_F a$ (resp. $a \mapsto_F S$) if $\exists s \in S$ such that $s \mapsto_F a$ (resp. $a \mapsto_F s$). For $S \subseteq A$, the *range* of S (wrt. F), denoted S^+_F, is the set $S \cup \{b \mid S \mapsto_F b\}$.

Given $F = (A, R)$, an argument $a \in A$ is *defended* (in F) by $S \subseteq A$ if for each $b \in A$, such that $b \mapsto_F a$, also $S \mapsto_F b$. A set T of arguments is defended (in F) by S if each $a \in T$ is defended by S (in F). A set $S \subseteq A$ is *conflict-free* (in F), if there are no arguments $a, b \in S$, such that $(a, b) \in R$. We denote the set of all conflict-free sets in F as $cf(F)$. $S \in cf(F)$ is called *admissible* (in F) if S defends itself. We denote the set of admissible sets in F as $adm(F)$.

The semantics we study in this work are the naive, stable, preferred, stage, and semi-stable extensions. Given $F = (A, R)$ they are defined as subsets of $cf(F)$ as follows:

- $S \in naive(F)$, if there is no $T \in cf(F)$ with $T \supset S$
- $S \in stb(F)$, if $S \mapsto_F a$ for all $a \in (A \setminus S)$
- $S \in pref(F)$, if $S \in adm(F)$ and $\nexists T \in adm(F)$ s.t. $T \supset S$
- $S \in stage(F)$, if $\nexists T \in cf(F)$ with $T^+_F \supset S^+_F$
- $S \in sem(F)$, if $S \in adm(F)$ and $\nexists T \in adm(F)$ s.t. $T^+_F \supset S^+_F$

We will make frequent use of the following concepts.

Definition 1. Given $\mathbb{S} \subseteq 2^{\mathfrak{A}}$, $Arg_{\mathbb{S}}$ denotes $\bigcup_{S \in \mathbb{S}} S$ and $Pairs_{\mathbb{S}}$ denotes $\{(a, b) \mid \exists S \in \mathbb{S} : \{a, b\} \subseteq S\}$. $\mathbb{S}$ is called an *extension-set* (over $\mathfrak{A}$) if $Arg_{\mathbb{S}}$ is finite.

3 Compact Argumentation Frameworks

Definition 2. Given a semantics σ, the set of *compact argumentation frameworks* under σ is defined as $CAF_\sigma = \{F \in AF_{\mathfrak{A}} \mid Arg_{\sigma(F)} = A_F\}$. We call an AF $F \in CAF_\sigma$ just σ-compact.

Of course the contents of CAF_σ differ with respect to the semantics σ. Concerning relations between the classes of compact AFs note that if for two semantics σ and θ it holds that $\sigma(F) \subseteq \theta(F)$ for each AF F, then also $CAF_\sigma \subseteq CAF_\theta$. Our first important result provides a full picture of the relations between classes of compact AFs under the semantics we consider.

Proposition 1. *1. $CAF_{sem} \subset CAF_{pref}$;*
2. $CAF_{stb} \subset CAF_\sigma \subset CAF_{naive}$ for $\sigma \in \{pref, sem, stage\}$;
3. $CAF_\theta \not\subseteq CAF_{stage}$ and $CAF_{stage} \not\subseteq CAF_\theta$ for $\theta \in \{pref, sem\}$.

Proof. (1) $CAF_{sem} \subseteq CAF_{pref}$ is by the fact that, in any AF F, $sem(F) \subseteq pref(F)$. Properness follows from the AF F' in Figure 1 (including the dotted part)[4]. Here $pref(F') = \{\{z\}, \{x_1, a_1\}, \{x_2, a_2\}, \{x_3, a_3\}, \{y_1, b_1\}, \{y_2, b_2\}, \{y_3, b_3\}\}$, but $sem(F') = (pref(F') \setminus \{\{z\}\})$, hence $F' \in CAF_{pref}$, but $F' \notin CAF_{sem}$.
(2) Let $\sigma \in \{pref, sem, stage\}$. The $\subseteq$-relations follow from the fact that, in any AF F, $stb(F) \subseteq \sigma(F)$ and each σ-extension is, by being conflict-free, part of some naive extension. The AF $(\{a, b\}, \{(a, b)\})$, which is compact under naive but not under σ, and AF F from Figure 1 (now without the dotted part), which is compact under σ but not under stable, show that the relations are proper.
(3) The fact that F' from Figure 1 (again including the dotted part) is also not *stage*-compact shows $CAF_{pref} \not\subseteq CAF_{stage}$. Likewise, there is an AF (to be found in the long version) which is *sem*-compact, but not *stage*-compact. Finally, the AF $(\{a, b, c\}, \{(a, b), (b, c), (c, a)\})$ shows $CAF_{stage} \not\subseteq CAF_\theta$ for $\theta \in \{pref, sem\}$. ☐

[4] The construct in the lower part of the figure represents symmetric attacks between each pair of arguments.

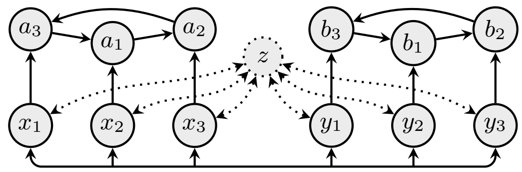


Figure 1. AFs illustrating the relations between various semantics.

Considering compact AFs obviously has effects on the computational complexity of reasoning. While credulous and skeptical acceptance are now easy (as discussed in the introduction) the next theorem shows that verifying extensions is still as hard as in general AFs.

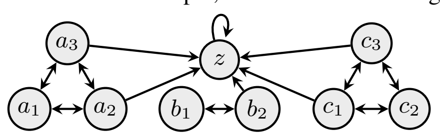
Theorem 2. *For $\sigma \in \{pref, sem, stage\}$, AF $F = (A, R) \in CAF_\sigma$ and $E \subseteq A$, it is* coNP*-complete to decide whether $E \in \sigma(F)$.*

Proof. For all three semantics the problem is known to be in coNP [6, 8, 15]. For hardness we only give a (prototypical) proof for *pref*. We use a standard reduction from CNF formulas $\varphi(X) = \bigwedge_{c \in C} c$ with each clause $c \in C$ a disjunction of literals from X to an AF F_φ with arguments $A_\varphi = \{\varphi, \bar{\varphi}_1, \bar{\varphi}_2, \bar{\varphi}_3\} \cup C \cup X \cup \bar{X}$ and attacks (i) $\{(c, \varphi) \mid c \in C\}$, (ii) $\{(x, \bar{x}), (\bar{x}, x) \mid x \in X\}$, (iii) $\{(x, c) \mid x \text{ occurs in } c\} \cup \{(\bar{x}, c) \mid \neg x \text{ occurs in } c\}$, (iv) $\{(\varphi, \bar{\varphi}_1), (\bar{\varphi}_1, \bar{\varphi}_2), (\bar{\varphi}_2, \bar{\varphi}_3), (\bar{\varphi}_3, \bar{\varphi}_1)\}$, and (v) $\{(\bar{\varphi}_1, x), (\bar{\varphi}_1, \bar{x}) \mid x \in X\}$. It holds that φ is satisfiable iff there is an $S \neq \emptyset$ in $pref(F_\varphi)$ [8]. We extend F_φ with four new arguments $\{t_1, t_2, t_3, t_4\}$ and the following attacks: (a) $\{(t_i, t_j), (t_j, t_i) \mid 1 \leq i < j \leq 4\}$, (b) $\{(t_1, c) \mid c \in C\}$, (c) $\{(t_2, c), (t_2, \bar{\varphi}_2) \mid c \in C\}$ and (d) $\{(t_3, \bar{\varphi}_3)\}$. This extended AF is in CAF_{pref} and moreover $\{t_4\}$ is a preferred extension thereof iff $pref(F_\varphi) = \{\emptyset\}$ iff φ is unsatisfiable. ☐

4 Limits of Compact AFs

Extension-sets obtained from compact AFs satisfy certain structural properties. Knowing these properties can help us decide whether – given an extension-set $\mathbb{S}$ – there is a compact AF F such that $\mathbb{S}$ is exactly the set of extensions of F for a semantics σ. This is also known as *realizability*: A set $\mathbb{S} \subseteq 2^{\mathfrak{A}}$ is called *compactly realizable* under semantics σ iff there is a compact AF F with $\sigma(F) = \mathbb{S}$.

Among the most basic properties that are necessary for compact realizability, we find numerical aspects like possible cardinalities of σ-extension-sets. As an example, consider the following AF F_2:



Let us determine the stable extensions of F_2. Clearly, taking one a_i, one b_i and one c_i yields a conflict-free set that is also stable as long as it attacks z. Thus from the $3 \cdot 2 \cdot 3 = 18$ combinations, only one (the set $\{a_1, b_1, c_2\}$) is not stable, whence F_2 has $18 - 1 = 17$ stable extensions. We note that this AF is not compact since z occurs in none of the extensions. Is there an equivalent stable-compact AF? The results of this section will provide us with a negative answer.

In [4] it was shown that there is a correspondence between the maximal number of stable extensions in argumentation frameworks and the maximal number of maximal independent sets in undirected graphs [16]. Recently, the result was generalized to further semantics [13]. To set the scene for the subsequent results building upon it, we recall the result below (Theorem 3). For any natural number n we define:[5]

$$\sigma_{\max}(n) = \max\{|\sigma(F)| \mid F \in \text{AF}_n\}$$

$\sigma_{\max}(n)$ returns the maximal number of σ-extensions among all AFs with n arguments. Surprisingly, there is a closed expression for $\sigma_{\max}$.

Theorem 3. *The function $\sigma_{max}(n) : \mathbb{N} \to \mathbb{N}$ is given by*

$$\sigma_{max}(n) = \begin{cases} 1, & \text{if } n = 0 \text{ or } n = 1, \\ 3^s, & \text{if } n \geq 2 \text{ and } n = 3s, \\ 4 \cdot 3^{s-1}, & \text{if } n \geq 2 \text{ and } n = 3s+1, \\ 2 \cdot 3^s, & \text{if } n \geq 2 \text{ and } n = 3s+2. \end{cases}$$

What about the maximal number of σ-extensions on connected graphs? Does this number coincide with $\sigma_{\max}(n)$? The next theorem provides a negative answer to this question and thus gives space for impossibility results as we will see. For a natural number n define

$$\sigma^{\text{con}}_{\max}(n) = \max\{|\sigma(F)| \mid F \in \text{AF}_n, F \text{ connected}\}$$

$\sigma^{\text{con}}_{\max}(n)$ returns the maximal number of σ-extensions among all *connected* AFs with n arguments. Again, a closed expression exists.

Theorem 4. *The function $\sigma^{con}_{max}(n) : \mathbb{N} \to \mathbb{N}$ is given by*

$$\sigma^{con}_{max}(n) = \begin{cases} n, & \text{if } n \leq 5, \\ 2 \cdot 3^{s-1} + 2^{s-1}, & \text{if } n \geq 6 \text{ and } n = 3s, \\ 3^s + 2^{s-1}, & \text{if } n \geq 6 \text{ and } n = 3s+1, \\ 4 \cdot 3^{s-1} + 3 \cdot 2^{s-2}, & \text{if } n \geq 6 \text{ and } n = 3s+2. \end{cases}$$

A further interesting question concerning arbitrary AFs is whether all natural numbers less than $\sigma_{\max}(n)$ are compactly realizable.[6] The following theorem shows that there is a serious gap between the maximal and second largest number. For any positive natural n define

$$\sigma^2_{\max}(n) = \max(\{|\sigma(F)| \mid F \in \text{AF}_n\} \setminus \{\sigma_{\max}(n)\})$$

$\sigma^2_{\max}(n)$ returns the second largest number of σ-extensions among all AFs with n arguments. Graph theory provides us with an expression.

Theorem 5. *Function $\sigma^2_{max}(n) : \mathbb{N} \setminus \{0\} \to \mathbb{N}$ is given by*

$$\sigma^2_{max}(n) = \begin{cases} \sigma_{max}(n) - 1, & \text{if } 1 \leq n \leq 7, \\ \sigma_{max}(n) \cdot \frac{11}{12}, & \text{if } n \geq 8 \text{ and } n = 3s+1, \\ \sigma_{max}(n) \cdot \frac{8}{9}, & \text{otherwise.} \end{cases}$$

Example 1. Recall that the (non-compact) AF F_2 we discussed previously had the extension-set $\mathbb{S}$ with $|\mathbb{S}| = 17$ and $|Arg_{\mathbb{S}}| = 8$. Is there a stable-compact AF with the same extensions? Firstly, nothing definitive can be said by Theorem 3 since $17 \leq 18 = \sigma_{\max}(8)$. Furthermore, in accordance with Theorem 4 the set $\mathbb{S}$ cannot be compactly σ-realized by a connected AF since $17 > 15 = \sigma^{\text{con}}_{\max}(8)$. Finally, using Theorem 5 we infer that the set $\mathbb{S}$ is not compactly σ-realizable because $\sigma^2_{\max}(8) = 16 < 17 < 18 = \sigma_{\max}(8)$.

[5] In this section, unless stated otherwise we use σ as a placeholder for stable, semi-stable, preferred, stage and naive semantics.

[6] We sometimes speak about realizing a natural number n and mean realizing an extension-set with n extensions.

The compactness property is instrumental here, since Theorem 5 has no counterpart in non-compact AFs. More generally, allowing additional arguments as long as they do not occur in extensions enables us to realize any number of stable extensions up to the maximal one.

Proposition 6. *Let n be a natural number. For each $k \leq \sigma_{max}(n)$, there is an AF F with $|Arg_{stb(F)}| = n$ and $|stb(F)| = k$.*

Now we are prepared to provide possible short cuts when deciding realizability of a given extension-set by initially simply counting the extensions. First some formal definitions.

Definition 3. Given an AF $F = (A, R)$, the component-structure $\mathcal{K}(F) = \{K_1, \ldots, K_n\}$ of F is the set of sets of arguments, where each K_i coincides with the arguments of a weakly connected component of the underlying graph; $\mathcal{K}_{\geq 2}(F) = \{K \in \mathcal{K}(F) \mid |K| \geq 2\}$.

The component-structure $\mathcal{K}(F)$ gives information about the number n of components of F as well as the size $|K_i|$ of each component. Knowing the components of an AF, computing the σ-extensions can be reduced to computing the σ-extensions of each component and building the cross-product. The AF resulting from restricting F to component K_i is given by $F{\downarrow_{K_i}} = (K_i, R_F \cap K_i \times K_i)$.

Lemma 7. *Given an AF F with component-structure $\mathcal{K}(F) = \{K_1, \ldots, K_n\}$ it holds that the extensions in $\sigma(F)$ and the tuples in $\sigma(F{\downarrow_{K_1}}) \times \cdots \times \sigma(F{\downarrow_{K_n}})$ are in one-to-one correspondence.*

Given an extension-set $\mathbb{S}$ we want to decide whether $\mathbb{S}$ is realizable by a compact AF under semantics σ. For an AF $F = (A, R)$ with $\sigma(F) = \mathbb{S}$ we know that there cannot be a conflict between any pair of arguments in $Pairs_{\mathbb{S}}$, hence $R \subseteq \overline{Pairs_{\mathbb{S}}} = (A \times A) \setminus Pairs_{\mathbb{S}}$. In the next section, we will show that it is highly non-trivial to decide which of the attacks in $\overline{Pairs_{\mathbb{S}}}$ can be and should be used to realize $\mathbb{S}$. For now, the next proposition implicitly shows that for argument-pairs $(a, b) \notin Pairs_{\mathbb{S}}$, although there is not necessarily a direct conflict between a and b, they are definitely in the same component.

Proposition 8. *Let $\mathbb{S}$ be an extension-set. (1) The transitive closure of $\overline{Pairs_{\mathbb{S}}}$, the set $\left(\overline{Pairs_{\mathbb{S}}}\right)^*$, is an equivalence relation, that is, it is reflexive, symmetric, and transitive. (2) For each AF $F \in CAF_\sigma$ that compactly realizes $\mathbb{S}$ under semantics σ (that is, $\sigma(F) = \mathbb{S}$), the component structure $\mathcal{K}(F)$ of F is given by the equivalence classes of $\left(\overline{Pairs_{\mathbb{S}}}\right)^*$, that is, $\mathcal{K}(F)$ is the quotient set of $Arg_{\mathbb{S}}$ by $\left(\overline{Pairs_{\mathbb{S}}}\right)^*$.*

We will denote the component-structure induced by an extension-set $\mathbb{S}$ as $\mathcal{K}(\mathbb{S})$. Note that, by Proposition 8, $\mathcal{K}(\mathbb{S})$ is equivalent to $\mathcal{K}(F)$ for every $F \in CAF_\sigma$ with $\sigma(F) = \mathbb{S}$. Given $\mathbb{S}$, the computation of $\mathcal{K}(\mathbb{S})$ can be done in polynomial time. With this we can use results from graph theory together with number-theoretical considerations in order to get impossibility results for compact realizability.

Proposition 9. *Given an extension-set $\mathbb{S}$ where $|\mathbb{S}|$ is odd, it holds that if $\exists K \in \mathcal{K}(\mathbb{S}) : |K| = 2$ then $\mathbb{S}$ is not compactly realizable under semantics σ.*

Example 2. Consider the extension-set $\mathbb{S} = \{\{a, b, c\}, \{a, b', c'\}, \{a', b, c'\}, \{a', b', c\}, \{a, b, c'\}, \{a', b, c\}, \{a, b', c\}\} = stb(F_1)$ where F_1 is the non-compact AF from the introduction. There, it took us some effort to argue that $\mathbb{S}$ is not compactly stb-realizable. Proposition 9 now gives an easier justification: $Pairs_{\mathbb{S}}$ yields $\mathcal{K}(\mathbb{S}) = \{\{a, a'\}, \{b, b'\}, \{c, c'\}\}$. Thus $\mathbb{S}$ with $|\mathbb{S}| = 7$ cannot be realized.

We denote the set of possible numbers of σ-extensions of a compact and *connected* AF with n arguments as $\mathcal{P}^c(n)$. Although we know that $p \in \mathcal{P}^c(n)$ implies $p \leq \sigma^{\text{con}}_{\max}(n)$, there might be $q \leq \sigma^{\text{con}}_{\max}(n)$ with $q \notin \mathcal{P}^c(n)$. Having to leave the exact contents of $\mathcal{P}^c(n)$ open, we can still state the following result:

Proposition 10. *Let $\mathbb{S}$ be an extension-set that is compactly realizable under semantics σ where $\mathcal{K}_{\geq 2}(\mathbb{S}) = \{K_1, \ldots, K_n\}$. Then for each $1 \leq i \leq n$ there is a $p_i \in \mathcal{P}^c(|K_i|)$ such that $|\mathbb{S}| = \prod_{i=1}^{n} p_i$.*

Example 3. Consider the extension-set $\mathbb{S}' = \{\{a,b,c\}, \{a,b',c'\}, \{a',b,c'\}, \{a',b',c\}\}$. (In fact there exists a (non-compact) AF F with $stb(F) = \mathbb{S}'$). We have the same component-structure $\mathcal{K}(\mathbb{S}') = \mathcal{K}(\mathbb{S})$ as in Example 2, but since now $|\mathbb{S}'| = 4$ we cannot use Proposition 9 to show impossibility of realization in terms of a compact AF. But with Proposition 10 at hand we can argue in the following way: $\mathcal{P}^c(2) = \{2\}$ and since $\forall K \in \mathcal{K}(\mathbb{S}') : |K| = 2$ it must hold that $|\mathbb{S}| = 2 \cdot 2 \cdot 2 = 8$, which is obviously not the case.

In particular, we have a straightforward non-realizability criterion whenever $|\mathbb{S}|$ is a prime number: the AF (if any) must have at most one weakly connected component of size greater than two. Theorem 4 gives us the maximal number of σ-extensions in a single weakly connected component. Thus whenever the number of desired extensions is larger than that number and prime, it cannot be realized.

Corollary 11. *Let extension-set $\mathbb{S}$ with $|Arg_{\mathbb{S}}| = n$ be compactly realizable under σ. If $|\mathbb{S}|$ is a prime number, then $|\mathbb{S}| \leq \sigma^{con}_{max}(n)$.*

We can also make use of the derived component structure of an extension-set $\mathbb{S}$. Since the total number of extensions of an AF is the product of these numbers for its weakly connected components (Lemma 7), each non-trivial component contributes a non-trivial amount to the total. Hence if there are more components than the factorization of $|\mathbb{S}|$ has primes in it, then $\mathbb{S}$ cannot be realized.

Corollary 12. *Let extension-set $\mathbb{S}$ be compactly realizable under σ and $f_1^{z_1} \cdot \ldots \cdot f_m^{z_m}$ be the integer factorization of $|\mathbb{S}|$, where $f_1, \ldots, f_m$ are prime numbers. Then $z_1 + \ldots + z_m \geq |\mathcal{K}_{\geq 2}(\mathbb{S})|$.*

5 Capabilities of Compact AFs

The results in the previous section made clear that the restriction to compact AFs entails certain limits in terms of compact realizability. Here we provide some results approaching an exact characterization of the capabilities of compact AFs with a focus on stable semantics.

5.1 C-Signatures

The *signature* of a semantics σ is defined as $\Sigma_\sigma = \{\sigma(F) \mid F \in AF_{\mathfrak{A}}\}$ and contains all possible sets of extensions an AF can possess under σ (see [13] for characterizations of such signatures). We first provide some alternative, yet equivalent characterizations of these signatures in Proposition 13. Then we strengthen the concept of signatures to "compact" signatures (c-signatures), which contain all extension-sets realizable with compact AFs.

The most central concept when structurally analyzing extension-sets is captured by the *Pairs*-relation from Definition 1. Whenever two arguments a and b occur jointly in some element S of extension-set $\mathbb{S}$ (i.e. $(a,b) \in Pairs_{\mathbb{S}}$) there cannot be a conflict between those arguments in an AF having $\mathbb{S}$ as solution under any standard semantics. $(a,b) \in Pairs_{\mathbb{S}}$ can be read as "evidence of no conflict" between a and b in $\mathbb{S}$. Hence, the *Pairs*-relation gives rise to sets of arguments that are conflict-free in any AF realizing $\mathbb{S}$.

Definition 4. Given an extension-set $\mathbb{S}$, we define $\mathbb{S}^{cf} = \{S \subseteq Arg_{\mathbb{S}} \mid \forall a, b \in S : (a,b) \in Pairs_{\mathbb{S}}\}$, and $\mathbb{S}^+ = \max_{\subseteq} \mathbb{S}^{cf}$.

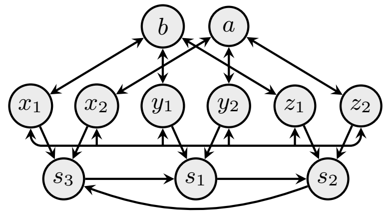


Figure 2. AF compactly realizing an extension-set $\mathbb{S} \not\subseteq \mathbb{S}^+$ under *pref*.

Proposition 13. $\Sigma_{naive} = \{\mathbb{S} \neq \emptyset \mid \mathbb{S} = \mathbb{S}^+\}$; $\Sigma_{stb} = \{\mathbb{S} \mid \mathbb{S} \subseteq \mathbb{S}^+\}$; $\Sigma_{stage} = \{\mathbb{S} \neq \emptyset \mid \mathbb{S} \subseteq \mathbb{S}^+\}$.

Let us now turn to signatures for compact AFs.

Definition 5. The *c-signature* Σ^c_σ of a semantics σ is defined as $\Sigma^c_\sigma = \{\sigma(F) \mid F \in CAF_\sigma\}$.

It is clear that $\Sigma^c_\sigma \subseteq \Sigma_\sigma$ holds for any semantics. The following result is mainly by the fact that the canonical AF

$$F^{cf}_{\mathbb{S}} = (A^{cf}_{\mathbb{S}}, R^{cf}_{\mathbb{S}}) = (Arg_{\mathbb{S}}, (Arg_{\mathbb{S}} \times Arg_{\mathbb{S}}) \setminus Pairs_{\mathbb{S}})$$

has $\mathbb{S}^+$ as extensions under all semantics under consideration and by extension-sets obtained from non-compact AFs which definitely cannot be transformed to equivalent compact AFs.

Proposition 14. *It holds that (1) $\Sigma^c_{naive} = \Sigma_{naive}$; and (2) $\Sigma^c_\sigma \subset \Sigma_\sigma$ for $\sigma \in \{stb, stage, sem, pref\}$.*

For ordinary signatures it holds that $\Sigma_{naive} \subset \Sigma_{stage} = (\Sigma_{stb} \setminus \{\emptyset\}) \subset \Sigma_{sem} = \Sigma_{pref}$ [13]. This picture changes when considering the relationship of c-signatures.

Proposition 15. *$\Sigma^c_{pref} \not\subseteq \Sigma^c_{stb}$; $\Sigma^c_{pref} \not\subseteq \Sigma^c_{stage}$; $\Sigma^c_{pref} \not\subseteq \Sigma^c_{sem}$; $\Sigma^c_{naive} \subset \Sigma^c_\sigma$ for $\sigma \in \{stb, stage, sem\}$; $\Sigma^c_{stb} \subseteq \Sigma^c_{sem}$; $\Sigma^c_{stb} \subseteq \Sigma^c_{stage}$.*

Proof. $\Sigma^c_{pref} \not\subseteq \Sigma^c_{stb}$, $\Sigma^c_{pref} \not\subseteq \Sigma^c_{stage}$: For the extension-set $\mathbb{S} = \{\{a,b\}, \{a,x_1,s_1\}, \{a,y_1,s_2\}, \{a,z_1,s_3\}, \{b,x_2,s_1\}, \{b,y_2,s_2\}, \{b,z_2,s_3\}\}$ it does not hold that $\mathbb{S} \subseteq \mathbb{S}^+$ (as $\{a,b,s_1\}$, $\{a,b,s_2\}$, $\{a,b,s_3\} \in \mathbb{S}^{cf}$, hence $\{a,b\} \notin \mathbb{S}^+$), but there is a compact AF F realizing $\mathbb{S}$ under the preferred semantics, namely the one depicted in Figure 2. Hence $\Sigma^c_{pref} \not\subseteq \Sigma^c_{stb}$ and $\Sigma^c_{pref} \not\subseteq \Sigma^c_{stage}$.

$\Sigma^c_{pref} \not\subseteq \Sigma^c_{sem}$: Let $\mathbb{T} = (\mathbb{S} \cup \{\{x_1,x_2,s_1\}, \{y_1,y_2,s_2\}, \{z_1,z_2,s_3\}\})$ and assume there is some $F = (Arg_{\mathbb{T}}, R)$ compactly realizing $\mathbb{T}$ under *sem*. Let $S = \{a,x_1,s_1\}$, $T = \{x_1,x_2,s_1\}$, and $U = \{a,b\}$. There must be a conflict between a and x_2, otherwise $(S \cup T) \in sem(F)$. Since each T and U must defend itself, necessarily both $(x_2,a), (a,x_2) \in R$. By symmetry we get $\{(a,\alpha_1), (\alpha_1,a), (b,\alpha_2), (\alpha_2,b) \mid \alpha \in \{x,y,z\}\} \subseteq R$. Now as U must not be in conflict with any of s_1, s_2, and s_3, each s_i must have an attacker which is not attacked by U or s_i. Hence wlog. $\{(s_1,s_2), (s_2,s_3), (s_3,s_1)\} \subseteq R$. Now observe that S must defend s_1 from s_3, therefore $(x_1,s_3) \in R$. Since now $S^+_F \supseteq (Arg_{\mathbb{T}} \setminus \{y_1,z_1\})$, S has to attack both y_1 and z_1, a contradiction to $U \in sem(F)$, as $U^+_F \subset S^+_F$. $\Sigma^c_{pref} \not\subseteq \Sigma^c_{sem}$ now follows from the fact that $pref(F') = \mathbb{T}$ for $F' = (A_F, R_F \setminus \{(\alpha_1,\alpha_2), (\alpha_2,\alpha_1) \mid \alpha \in \{x,y,z\}\})$ where F is the AF depicted in Figure 2.

$\Sigma^c_{naive} \subset \Sigma^c_\sigma$ for $\sigma \in \{stb, stage, sem\}$: First of all note that any extensions-set compactly realizable under *naive* is compactly realizable under σ (by making the AF symmetric). Now consider the extension-set $\mathbb{S} = \{\{a_1,b_2,b_3\}, \{a_2,b_1,b_3\}, \{a_3,b_1,b_2\}\}$. $\mathbb{S} \neq \mathbb{S}^+$ since $\{b_1,b_2,b_3\} \in \mathbb{S}^+$, hence $\mathbb{S} \notin \Sigma^c_{naive}$. $\Sigma^c_{naive} \subset \Sigma^c_\sigma$ follows from the fact that $\mathbb{S}$ is compactly realizable under σ [13].

$\Sigma^c_{stb} \subseteq \Sigma^c_{sem}$, $\Sigma^c_{stb} \subseteq \Sigma^c_{stage}$: Follow from the fact that $stage(F) = sem(F) = stb(F)$ for any $F \in CAF_{stb}$ [6]. □

5.2 The Explicit-Conflict Conjecture

So far we only have exactly characterized c-signatures for the naive semantics (Proposition 14). Deciding membership of an extension-set in the c-signature of the other semantics is more involved. In what follows we focus on stable semantics in order to illustrate difficulties and subtleties in this endeavor.

Although there are, as Proposition 1 showed, more compact AFs for *naive* than for *stb*, one can express a greater diversity of outcomes with the stable semantics, i.e. $\mathbb{S} = \mathbb{S}^+$ does not necessarily hold. Consider some AF F with $\mathbb{S} = stb(F)$. By Proposition 13 we know that $\mathbb{S} \subseteq \mathbb{S}^+$ must hold. Now we want to compactly realize extension-set $\mathbb{S}$ under *stb*. If $\mathbb{S} = \mathbb{S}^+$, then we can obviously find a compact AF realizing $\mathbb{S}$ under *stb*, since $F^{cf}_{\mathbb{S}}$ will do so. On the other hand, if $\mathbb{S} \neq \mathbb{S}^+$ we have to find a way to handle the argument-sets in $\mathbb{S}^- = \mathbb{S}^+ \setminus \mathbb{S}$. In words, each $S \in \mathbb{S}^-$ is a $\subseteq$-maximal set with evidence of no conflict, which is not contained in $\mathbb{S}$.

Now consider some AF $F' \in CAF_{stb}$ having $\mathbb{S} \subsetneq \mathbb{S}^+$ as its stable extensions. Further take some $S \in \mathbb{S}^-$. There cannot be a conflict within S in F', hence we must be able to map S to some argument $t \in (Arg_{\mathbb{S}} \setminus S)$ not attacked by S in F'. Still, the collection of these mappings must fulfill certain conditions in order to preserve a justification for all $S \in \mathbb{S}$ to be a stable extension and not to give rise to other stable extensions. We make these things more formal.

Definition 6. Given an extension-set $\mathbb{S}$, an *exclusion-mapping* is the set $\mathfrak{R}_{\mathbb{S}} = \bigcup_{S \in \mathbb{S}^-} \{(s, \mathfrak{f}_{\mathbb{S}}(S)) \mid s \in S \text{ s.t. } (s, \mathfrak{f}_{\mathbb{S}}(S)) \notin Pairs_{\mathbb{S}}\}$ where $\mathfrak{f}_{\mathbb{S}} : \mathbb{S}^- \to Arg_{\mathbb{S}}$ is a function with $\mathfrak{f}_{\mathbb{S}}(S) \in (Arg_{\mathbb{S}} \setminus S)$.

Definition 7. A set $\mathbb{S} \subseteq 2^{\mathfrak{A}}$ is called *independent* if there exists an antisymmetric exclusion-mapping $\mathfrak{R}_{\mathbb{S}}$ such that it holds that $\forall S \in \mathbb{S}\, \forall a \in (Arg_{\mathbb{S}} \setminus S) : \exists s \in S : (s, a) \notin (\mathfrak{R}_{\mathbb{S}} \cup Pairs_{\mathbb{S}})$.

The concept of independence suggests that the more separate the elements of some extension-set $\mathbb{S}$ are, the less critical is $\mathbb{S}^-$. An independent $\mathbb{S}$ allows to find the required orientation of attacks to exclude sets from $\mathbb{S}^-$ from the stable extensions without interferences.

Theorem 16. *For every independent extension-set* $\mathbb{S}$ *with* $\mathbb{S} \subseteq \mathbb{S}^+$ *it holds that* $\mathbb{S} \in \Sigma^c_{stb}$.

Proof. Consider, given an independent extension-set $\mathbb{S}$ and an antisymmetric exclusion-mapping $\mathfrak{R}_{\mathbb{S}}$ fulfilling the independence-condition (cf. Definition 7), the AF $F^{stb}_{\mathbb{S}} = (Args_{\mathbb{S}}, R^{stb}_{\mathbb{S}})$ with $R^{stb}_{\mathbb{S}} = (R^{cf}_{\mathbb{S}} \setminus \mathfrak{R}_{\mathbb{S}})$. We show that $stb(F^{stb}_{\mathbb{S}}) = \mathbb{S}$. First note that $stb(F^{cf}_{\mathbb{S}}) = \mathbb{S}^+ \supseteq \mathbb{S}$. As $\mathfrak{R}_{\mathbb{S}}$ is antisymmetric, one direction of each symmetric attack of $F^{cf}_{\mathbb{S}}$ is still in $F^{stb}_{\mathbb{S}}$. Hence $stb(F^{stb}_{\mathbb{S}}) \subseteq \mathbb{S}^+$.
$stb(F^{stb}_{\mathbb{S}}) \subseteq \mathbb{S}$: Consider some $S \in stb(F^{stb}_{\mathbb{S}})$ and assume that $S \notin \mathbb{S}$, i.e. $S \in \mathbb{S}^-$. Since $\mathfrak{R}_{\mathbb{S}}$ is an exclusion-mapping fulfilling the independence-condition by assumption, there is an argument $\mathfrak{f}_{\mathbb{S}}(S) \in (Arg_{\mathbb{S}} \setminus S)$ such that $\{(s, \mathfrak{f}_{\mathbb{S}}(S)) \mid s \in S, (s, \mathfrak{f}_{\mathbb{S}}(S)) \notin Pairs_{\mathbb{S}}\} \subseteq \mathfrak{R}_{\mathbb{S}}$. But then, by construction of $F^{stb}_{\mathbb{S}}$, there is no $a \in S$ such that $(a, \mathfrak{f}_{\mathbb{S}}(S)) \in R^{stb}_{\mathbb{S}}$, a contradiction to $S \in stb(F^{stb}_{\mathbb{S}})$.
$stb(F^{stb}_{\mathbb{S}}) \supseteq \mathbb{S}$: Consider some $S \in \mathbb{S}$ and assume that $S \notin stb(F^{stb}_{\mathbb{S}})$. We know that S is conflict-free in $F^{stb}_{\mathbb{S}}$. Therefore there must be some $t \in (Arg_{\mathbb{S}} \setminus S)$ with $S \not\mapsto_{F^{stb}_{\mathbb{S}}} t$. Hence $\forall s \in S : (s, t) \in (Pairs_{\mathbb{S}} \cup \mathfrak{R}_{\mathbb{S}})$, a contradiction to the assumption that $\mathbb{S}$ is independent. $\square$

Corollary 17. *For every* $\mathbb{S} \in \Sigma_{stb}$, *with* $|\mathbb{S}| \leq 3$, $\mathbb{S} \in \Sigma^c_{stb}$.

Theorem 16 gives a sufficient condition for an extension-set to be contained in Σ^c_{stb}. Section 4 provided necessary conditions with respect to numbers of extensions. As these conditions do not match, we have not arrived at an exact characterization of the c-signature for stable semantics yet. In what follows, we identify the missing step which has to be left open but, as we will see, results in an interesting problem of its own. Let us first define a further class of frameworks.

Definition 8. We call an AF $F = (A, R)$ conflict-explicit under semantics σ iff for each $a, b \in A$ such that $(a, b) \notin Pairs_{\sigma(F)}$, we find $(a, b) \in R$ or $(b, a) \in R$ (or both).

As a simple example consider the AF $F = (\{a, b, c, d\}, \{(a, b), (b, a), (a, c), (b, d)\})$ which has $\mathbb{S} = stb(F) = \{\{a, d\}, \{b, c\}\}$. Note that $(c, d) \notin Pairs_{\mathbb{S}}$ but $(c, d) \notin R$ as well as $(d, c) \notin R$. Thus F is not conflict-explicit under stable semantics. However, if we add attacks (c, d) or (d, c) we obtain an equivalent (under stable semantics) conflict-explicit (under stable semantics) AF.

Theorem 18. *For each compact* AF F *which is conflict-explicit under stb, it holds that stb*(F) *is independent.*

Proof. Consider some $F \in CAF_{stb}$ which is conflict-explicit under *stb* and let $\mathbb{E} = stb(F)$. Observe that $\mathbb{E} \subseteq \mathbb{E}^+$. Further let $\mathfrak{R}_{\mathbb{E}} = \{(b, a) \in R \mid (a, b) \in R\}$ and consider the AF $F^s = (A_F, R_F \cup \mathfrak{R}_{\mathbb{E}})$ being the symmetric version of F. Now let $E \in \mathbb{E}^-$. Note that $E \in cf(F) = cf(F^s)$. But as $E \notin \mathbb{E}$ there must be some $t \in (A \setminus E)$ such that for all $e \in E$, $(e, t) \notin R_F$. For all such $e \in E$ with $(e, t) \notin Pairs_{\mathbb{E}}$ it holds, as F is conflict-explicit under *stb*, that $(t, e) \in R_F$, hence $(e, t) \in \mathfrak{R}_{\mathbb{E}}$, showing that $\mathfrak{R}_{\mathbb{E}}$ is an exclusion-mapping.

It remains to show that $\mathfrak{R}_{\mathbb{E}}$ is antisymmetric and $\forall E \in \mathbb{E} \forall a \in Arg_{\mathbb{S}} \setminus E : \exists e \in E : (e, a) \notin (\mathfrak{R}_{\mathbb{E}} \cup Pairs_{\mathbb{E}})$ holds. As some pair (b, a) is in $\mathfrak{R}_{\mathbb{E}}$ iff $(a, b) \in R$ and $(b, a) \notin R$, $\mathfrak{R}_{\mathbb{E}}$ is antisymmetric. Finally consider some $E \in \mathbb{E}$ and $a \in Arg_{\mathbb{S}} \setminus E$ and assume that $\forall e \in E : (e, a) \in \mathfrak{R}_{\mathbb{E}} \vee (e, a) \in Pairs_{\mathbb{E}}$. This means that $e \not\mapsto_F a$, a contradiction to E being a stable extension of F. $\square$

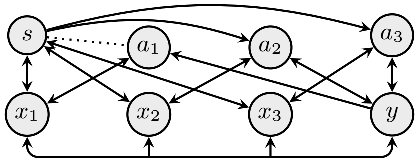
Since our characterizations of signatures completely abstract away from the actual structure of AFs but only focus on the set of extensions, our problem would be solved if the following was true.

EC-Conjecture. For each AF $F = (A, R)$ there exists an AF $F' = (A, R')$ which is conflict-explicit under the stable semantics such that $stb(F) = stb(F')$.

Theorem 19. *Under the assumption that the EC-conjecture holds,* $\Sigma^c_{stb} = \{\mathbb{S} \mid \mathbb{S} \subseteq \mathbb{S}^+ \wedge \mathbb{S}$ *is independent*$\}$.

Unfortunately, the question whether an equivalent conflict-explicit AF exists is not as simple as the example above suggests. We provide a few examples showing that proving the conjecture includes some subtle issues. Our first example shows that for adding missing attacks, the orientation of the attack needs to be carefully chosen.

Example 4. Consider AF F below and observe $stb(F) = \{\{a_1, a_2, x_3\}, \{a_1, a_3, x_2\}, \{a_2, a_3, x_1\}, \{s, y\}\}$.



$Pairs_{stb(F)}$ yields one pair of arguments a_1 and s whose conflict is not explicit by F, i.e. $(a_1, s) \notin Pairs_{stb(F)}$, but $(a_1, s), (s, a_1) \notin R_F$. Now adding the attack $a_1 \mapsto_F s$ to F would reveal the additional stable extension $\{a_1, a_2, a_3\} \in (stb(F))^+$. On the other hand by

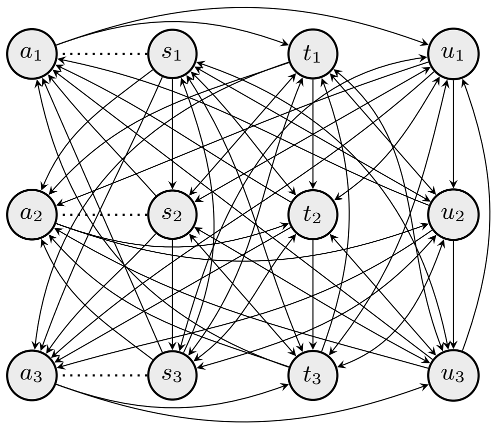


Figure 3. Guessing the orientation of non-explicit conflicts is not enough.

adding the attack $s \mapsto_F a_1$ we get the conflict-explicit AF F' with $stb(F) = stb(F')$.

Finally recall the role of the arguments x_1, x_2, and x_3. Each of these arguments enforces exactly one extension (being itself part of it) by attacking (and being attacked by) all arguments not in this extension. We will make use of this construction-concept in Example 5.

Even worse, it is sometimes necessary to not only add the missing conflicts but also change the orientation of existing attacks such that the missing attack "fits well".

Example 5. Let $X = \{x_{s,t,i}, x_{s,u,i}, x_{t,u,i} \mid 1 \leq i \leq 3\} \cup \{x_{a,1,2}, x_{a,1,3}, x_{a,2,3}\}$ and $\mathbb{S} = \{\{s_i, t_i, x_{s,t,i}\}, \{s_i, u_i, x_{s,u,i}\}, \{t_i, u_i, x_{t,u,i}\} \mid i \in \{1,2,3\}\} \cup \{\{a_1, a_2, x_{a,1,2}\}, \{a_1, a_3, x_{a,1,3}\}, \{a_2, a_3, x_{a,2,3}\}\}$. Consider the AF $F = (A' \cup X, R' \cup \bigcup_{x \in X} \{(x,b),(b,x) \mid b \in (A' \setminus \mathbb{S}_x)\} \cup \{(x,x') \mid x, x' \in X, x \neq x'\})$, where the essential part (A', R') is depicted in Figure 3 and $\mathbb{S}_x$ is the unique set $S \in \mathbb{S}$ with $x \in S$. We have $stb(F) = \mathbb{S}$. Observe that F contains three non-explicit conflicts under the stable semantics, namely the argument-pairs (a_1, s_1), (a_2, s_2), and (a_3, s_3). Adding any of (s_i, a_i) to R_F would turn $\{s_i, t_i, u_i\}$ into a stable extension; adding all (a_i, s_i) to R_F would yield $\{a_1, a_2, a_3\}$ as additional stable extension. Hence there is no way of making the conflicts explicit without changing other parts of F and still getting a stable-equivalent AF. Still, we can realize $stb(F)$ by a compact and conflict-explicit AF, for example by $G = (A_F, (R_F \cup \{(a_1, s_1), (a_2, s_2), (a_3, s_3)\}) \setminus \{(a_1, x_{a,2,3}), (a_2, x_{a,1,3}), (a_3, x_{a,1,2})\})$.

This is another indicator, yet far from a proof, that the EC-conjecture holds and by that Theorem 19 describes the exact characterization of the c-signature under stable semantics.

6 Discussion

We introduced and studied the novel class of σ-compact argumentation frameworks for σ among naive, stable, stage, semi-stable and preferred semantics. We provided the full relationships between these classes, and showed that the extension verification problem is still coNP-hard for stage, semi-stable and preferred semantics. We next addressed the question of compact realizability: Given a set of extensions, is there a compact AF with this set of extensions under semantics σ? Towards this end, we first used and extended recent results on maximal numbers of extensions to provide shortcuts for showing non-realizability. Lastly we studied signatures, sets of compactly realizable extension-sets, and provided sufficient conditions for compact realizability. This culminated in the explicit-conflict conjecture, a deep and interesting question in its own right: Given an AF, can all implicit conflicts be made explicit?

Our work bears considerable potential for further research. First and foremost, the explicit-conflict conjecture is an interesting research question. But the EC-conjecture (and compact AFs in general) should not be mistaken for a mere theoretical exercise. There is a fundamental computational significance to compactness: When searching for extensions, arguments span the search space, since extensions are to be found among the subsets of the set of all arguments. Hence the more arguments, the larger the search space. Compact AFs are argument-minimal since none of the arguments can be removed without changing the outcome, thus leading to a minimal search space. The explicit-conflict conjecture plays a further important role in this game: implicit conflicts are something that AF solvers have to deduce on their own, paying mostly with computation time. If there are no implicit conflicts in the sense that all of them have been made explicit, solvers have maximal information to guide search.

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