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Election Attacks with Few Candidates

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Abstract. We investigate the parameterized complexity of strategic behaviors in generalized scoring rules. In particular, we prove that the manipulation, control (all the 22 standard types), and bribery problems are fixed-parameter tractable for many generalized scoring rules, with respect to the number of candidates. Our results imply that all these strategic problems are fixed-parameter tractable for many common voting rules, such as Plurality, r-Approval, Borda, Copeland, Maximin, Bucklin, Ranked pairs, Schulze, etc., with respect to the number of candidates.

1 Introduction

In this paper, we study the strategic voting problems from the parameterized complexity perspective. Our main result is that the manipulation, (all the 22 standard types) control and bribery problems are fixed-parameter tractable ($\mathcal{FPT}$) for many generalized scoring rules, with respect to the number of candidates. Since many common voting rules fall into the category of the generalized scoring rules, these tractability results hold for these voting rules, among which are all the positional scoring rules (e.g., Borda, r-Approval, Veto, Plurality), Copeland$^{\alpha}$, Maximin, Bucklin, Ranked pairs, Schulze, Nanson's and Baldwin's. Recall that an instance of a *parameterized problem* consists of a main part I and a parameter t. A parameterized problem is $\mathcal{FPT}$ if it is solvable in $O(h(t) \cdot |I|^{O(1)})$ time, where h is a computable function that depends only on the parameter t. Due to space limitations, all proofs are deferred to the full version which is available at http://arxiv.org/abs/1405.6562.

Preliminaries. Let $\mathcal{C} = \{c_1, ..., c_m\}$ be a set of candidates. A *linear order* on $\mathcal{C}$ is a transitive, antisymmetric, and total relation on $\mathcal{C}$. The set of all linear orders on $\mathcal{C}$ is denoted by $L(\mathcal{C})$. An *n-voter profile* P on $\mathcal{C}$ consists of n votes defined as linear orders on $\mathcal{C}$. That is, $P = (V_1, ..., V_n)$, where for every $i \leq n, V_i \in L(\mathcal{C})$. A *voting rule* is a function that maps a voting profile to a single candidate, the winner. In the remainder of the paper, m denotes the number of candidates.

In the following, we give the definition of the class of the generalized scoring rules which was introduced by Xia and Conitzer [11].

Let $K = \{1, ..., k\}$. For any $\vec{a}, \vec{b} \in \mathbb{R}^k$, we say that $\vec{a}$ and $\vec{b}$ are *equivalent* with respect to K, denoted by $\vec{a} \sim_K \vec{b}$, if for any $i, j \in K, \vec{a}[i] > \vec{a}[j] \Leftrightarrow \vec{b}[i] > \vec{b}[j]$ and $\vec{a}[i] < \vec{a}[j] \Leftrightarrow \vec{b}[i] < \vec{b}[j]$ (where $\vec{a}[i]$ denotes the i-th component of the vector $\vec{a}$, etc.).

A function $g : \mathbb{R}^k \to \mathcal{C}$ is *compatible* with K if for any $\vec{a}, \vec{b} \in \mathbb{R}^k, \vec{a} \sim_K \vec{b} \Rightarrow g(\vec{a}) = g(\vec{b})$.

A generalized scoring rule is defined by a generalized scoring function f which maps votes to score vectors, and a decision function g which maps total score vectors to candidates. Given a profile of votes, the generalized scoring rule selects the winner as follows. First, the generalized scoring function f maps every vote to a score vector with each component a rational number. Here, all the score vectors have the same dimension which is called the *order* of the generalized scoring rule. Then, these score vectors are added up to a total score vector. Finally, the decision function g maps the total vector to a candidate, the winner. Here, the decision function g is compatible with $K = \{1, 2, ..., k\}$, where k is the order of the generalized scoring rule.

In the following, we briefly introduce the strategic voting problems discussed in this paper. We refer to [5, 7] for all the detailed definitions, including the manipulation, bribery and the 22 standard control problems. In all these problems, we have as input a set $\mathcal{C} \cup \{p\}$ of candidates where p is a distinguished candidate, and a profile $P = (V_1, ..., V_n)$ of votes. The question is whether the distinguished candidate p can become a winner (in this case, p is not the winner in the initial election) or become a loser (in this case, p is the winner in the initial election) by imposing a specific strategic behavior on the voting. The former case of making p a winner is called a *constructive* strategic behavior, and the latter case is called a *destructive* strategic behavior. Observe that if the problem of a specific constructive strategic behavior is $\mathcal{FPT}$ with respect to the number of candidates, so is the corresponding destructive case. To check this, suppose that we have an $\mathcal{FPT}$ algorithm $Algo$ for a specific constructive strategic behavior problem. Then, we can guess a candidate $p' \in \mathcal{C}$ and run the algorithm $Algo$ but with the distinguished candidate being p'. Since we have at most m guesses, the destructive case is solved in $\mathcal{FPT}$-time. Due to this fact, we consider only the problems of constructive strategic behaviors.

Manipulation. In addition to the aforementioned input, we have a set $\mathcal{V}'$ of voters who did not cast their votes yet. We call these voters *manipulators*. The question is whether the manipulators can cast their votes in a way so that p becomes the winner.

Bribery. The bribery problem asks whether we can change at most κ votes (in any way but still linear orders over the candidates) so that p becomes the winner, where $\kappa \in \mathbb{N}$ is also a part of the input.

Control. There are 11 standard constructive control behaviors in total. Among them 7 are imposed on the candidate set and 4 are imposed on the vote set. We first discuss the candidate control cases. In these scenarios, we either add some candidates (limited or unlimited), or delete some candidates, or partition the candidate set into two sets (runoff or non-runoff partitions with ties-promote or ties-eliminate models). Since the number of the candidates is bounded by the parameter m, we can enumerate all the possible ways of performing the control strategic behaviors in $\mathcal{FPT}$-time with respect to m. Thus, if the winner is computable in $\mathcal{FPT}$ time with respect to m (which holds for all the common voting rules studied in this paper), the candidate control problems are $\mathcal{FPT}$.

Deleting votes: The problem of control by deleting votes asks whether we can remove at most κ votes from the given profile so

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that p becomes the winner, where $\kappa \in \mathbb{N}$ is also a part of the input.

Partition votes: In the control by partitioning of votes, we are asked the following question: is there a partition of P into P_1 and P_2 such that p is the winner of the two-stage election where the winners of election $(\mathcal{C} \cup \{p\}, P_1)$ compete against the winners of $(\mathcal{C} \cup \{p\}, P_2)$? We distinguish between ties-promote model and ties-eliminate model. In the ties-promote model, all the candidates which are tied as winners in the first-stage election are promoted to the second stage election. In the ties-eliminate model, if there is more than one winner, then all these winners will not be moved to the second stage election. We remark that in the ties-promote model, we should adopt the multiwinner variant of the generalized scoring rules (which is easily to get as mentioned in [11]) as a tool.

Adding votes: In addition to the aforementioned input, we have another list P' of *unregistered votes*, and are asked whether we can add at most κ votes in P' to P so that the distinguished candidate p becomes the winner. Here, κ is also a part of the input.

2 The General Framework

The main result of this paper is summarized in the following theorem.

Theorem 1 *For any generalized scoring rule defined by a generalized scoring function f and a decision function g, if the order of the generalized scoring rule is bounded by a function of the number of the candidates, and f and g are computable in $\mathcal{FPT}$-time with respect to the number of candidates, then the manipulation, bribery and all the 22 standard control problems are $\mathcal{FPT}$ with respect to the number of candidates.*

To use the framework of Theorem 1, the order of the generalized scoring rule must be bounded by a function of the number of the candidates. Furthermore, the generalized scoring function and the decision function must be computable in $\mathcal{FPT}$ time with respect to the number of candidates. The following lemma summarizes the common voting rules which fulfill these conditions.

Lemma 2 *For the positional scoring rules, Copeland$^\alpha$ for every $0 \leq \alpha \leq 1$, Maximin, STV, Baldwin's, Nanson's, Ranked pairs, Schulze, and Bucklin, the orders of the corresponding generalized scoring rules are bounded by functions of the number of candidates, and the decision and generalized scoring functions of the corresponding generalized scoring rules are computable in $\mathcal{FPT}$-time with respect to the number of candidates.*

Due to Theorem 1 and Lemma 2, we have the following corollary.

Corollary 3 *The manipulation, bribery and the 22 standard control problems for the following voting rules are $\mathcal{FPT}$ with respect to the number of candidates: all the positional scoring rules, Copeland$^\alpha$ for every $0 \leq \alpha \leq 1$, Maximin, STV, Baldwin's, Nanson's, Ranked pairs, Schulze and Bucklin.*

3 Discussion and Related Work

In this paper, we extend the application of the generalized scoring rules by exploring the parameterized complexity of strategic voting problems. In particular, we show that from the viewpoint of parameterized complexity, the manipulation, bribery and control problems which are $\mathcal{NP}$-hard in many voting systems turned out to be fixed-parameter tractable ($\mathcal{FPT}$), with respect to the number of candidates. Several related works are summarized as follows.

Hemaspaandra et al. [9] recently studied the manipulation, control and bribery problems in Schulze and Ranked pairs voting systems. They proved that all these strategic problems in Schulze and Ranked pairs voting systems are $\mathcal{FPT}$ with respect to the number of candidates. Gaspers et al. [8] proved that the manipulation problem in Schulze voting system is indeed polynomial-time solvable for any number of manipulators. Faliszewski et al. [7] studied Copeland$^\alpha$ control problems and achieved $\mathcal{FPT}$ results for most of the control problems in Copeland$^\alpha$ voting with respect to the number of candidates. Besides the manipulation, (22 standard) control and bribery problems, many other strategic voting problems were also studied from the parameterized complexity perspective by researchers. Faliszewski et al. [6] studied a multimode control problem (in this model, the strategy individuals are allowed to add votes, delete votes, add candidates, delete candidates, and change votes simultaneously) and proved that this problem is $\mathcal{FPT}$ with respect to the number of candidates for voting rules which are integer-linear-program implementable. Dorn and Schlotter [3] proved that the swap bribery problem is $\mathcal{FPT}$ with respect to the number of candidates for any voting system which is described by linear inequalities. Betzler et al. [2] proved that the possible winner problem is $\mathcal{FPT}$ with respect to the number of candidates for all positional scoring rules, Bucklin, Maximin, Copeland$^\alpha$ and Ranked pairs. Elkind et al. [4] devised a general framework for classifying the fixed-parameter tractability of the winner determination problem for voting rules which are "distance-rationalizable". For parameterized complexity of strategic voting problems with respect to other parameters than the number of candidates, we refer to [1] for a survey.

Similar results of this paper were independently announced by Xia [10]. However, there are several differences. First, our results apply to all the 22 standard control problems, while the results in [10] does not include the control by partition votes. Second, Xia [10] studied the winner determination problem which is not discussed in this paper.

REFERENCES

[1] N. Betzler, R. Bredereck, J. Chen, and R. Niedermeier, 'Studies in computational aspects of voting - a parameterized complexity perspective', in *The Multivariate Algorithmic Revolution and Beyond*, pp. 318–363, (2012).

[2] N. Betzler, S. Hemmann, and R. Niedermeier, 'A multivariate complexity analysis of determining possible winners given incomplete votes', in *IJCAI*, pp. 53–58, (2009).

[3] B. Dorn and I. Schlotter, 'Multivariate complexity analysis of swap bribery', *Algorithmica*, **64**(1), 126–151, (2012).

[4] E. Elkind, P. Faliszewski, and A. M. Slinko, 'On the role of distances in defining voting rules', in *AAMAS*, pp. 375–382, (2010).

[5] G. Erdélyi, M. Nowak, and J. Rothe, 'Sincere-strategy preference-based approval voting fully resists constructive control and broadly resists destructive control', *Math. Log. Q.*, **55**(4), 425–443, (2009).

[6] P. Faliszewski, E. Hemaspaandra, and L. A. Hemaspaandra, 'Multimode control attacks on elections', *J. Artif. Intell. Res. (JAIR)*, **40**, 305–351, (2011).

[7] P. Faliszewski, E. Hemaspaandra, L. A. Hemaspaandra, and J. Rothe, 'Llull and copeland voting computationally resist bribery and constructive control', *J. Artif. Intell. Res. (JAIR)*, **35**, 275–341, (2009).

[8] S. Gaspers, T. Kalinowski, N. Narodytska, and T. Walsh, 'Coalitional manipulation for Schulze's rule', in *AAMAS*, pp. 431–438, (2013).

[9] L. A. Hemaspaandra, R. Lavaee, and C. Menton, 'Schulze and ranked-pairs voting are fixed-parameter tractable to bribe, manipulate, and control', in *AAMAS*, pp. 1345–1346, (2013).

[10] L. Xia, 'Fixed-parameter tractability of integer generalized scoring rules', in *AAMAS*, pp. 1599–1600, (2014).

[11] L. Xia and V. Conitzer, 'Generalized scoring rules and the frequency of coalitional manipulability', in *ACM Conference on Electronic Commerce*, pp. 109–118, (2008).