Workshop Proceedings of the 9th International Conference on Intelligent Environments
J.A. Botía and D. Charitos (Eds.)

doi:10.3233/978-1-61499-286-8-4

Multilevel Wavelet Transform Based Sparsity Reduction for Compressive Sensing

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Abstract. Compressive sensing has become a popular technique in broad areas of science and engineering for data analysis, which leads to numerous applications in signal and image processing. It exploits the sparseness and compressibility of the data in order to reduce the size. Wavelet analysis is one of leading techniques for compressive sensing. In 2D discrete wavelet transform, the digital image is decomposed with a set of basis functions. At each level, wavelet transform is applied to compute the lowpass outcome (approximation) and highpass outcomes (three details), each with a quarter size of the source image. For the subsequent levels, the lower level outcomes turn out to be the inputs of the higher level to conduct further wavelet decompositions recursively, so that another set of approximation and detail components is generated. Discrete wavelet transform and discrete wavelet packet transform differ in higher levels other than the first level of decomposition. From the second level, discrete wavelet transform applies the transform to the lowpass outcomes exclusively, while wavelet packet transform applies the transform to lowpass and highpass outcomes simultaneously. As the more comprehensive approach, wavelet packet transform is selected for scene image compression on cases of both the lower and higher dynamic range images. Quantitative measures are then introduced to compare the outcomes of two cases.

Keywords. Compressive Sensing, Discrete Wavelet Transform, Wavelet Packet Transform, Sparsity Reduction, Thresholding

1. Introduction

Wavelet analysis has a wide range of applications in math, engineering, computer science, and so on. The related topics cover continuous wavelet transform and discrete wavelet transform, decimated wavelet transform and non-decimated wavelet transform, as well as the multiresolution analysis. The notions of sparsity and thresholding are always emphasized. Different algorithms have been designed and applied, such as the famous Haar wavelets and Haar-Fisz transformation [1-3]. Some fundamental applications have appeared in literatures. An algorithm based on the wavelet-packet transform has been used for the analysis of harmonics in the power systems. It decomposes the voltage and current waveforms into frequency bands corresponding to the odd-harmonic components of the signal, so that spectral leakage due to the

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imperfect frequency response of the existing wavelet filter bank is reduced. By comparing with the harmonic group, it is shown that wavelet analysis is a potential alternative for harmonic estimation in the power systems [4]. Fractional wavelet transform has been applied to extract largest analytical information from spectral bands. Absorption spectra of the pharmaceutical samples are processed by fractional wavelet transform. The coefficients obtained can be applied to construct principal component regression and partial least squares calibrations [5]. Wavelet transform has been also implemented on biometric pattern recognition and medical diagnosis successfully. In addition, by means of soft thresholding, discrete wavelet transform can be employed for image fusion of still and moving pictures [6-8]. In subband and wavelet image coding, size-limited subband decompositions is used to limit the number of samples. To reduce coding distortions at borders, the symmetric extension filter banks are introduced in the cyclic frequency domain framework. Enhancement to the filter bank is made at a tree-structured system level. The new filter banks can implement FIR (Finite Impulse Response) and IIR (Infinite Impulse Response) filters, even with irrational transfer functions. The condensed wavelet packet transform has superior compression performance over those existing biorthogonal wavelets and block transforms [9].

A novel compression scheme has been proposed with a tunable complexity-rate-distortion as the trade-off. As the images increase in size and resolution, better compression schemes with low complexity are required on-board. Satellite mission specifications expect higher performance in terms of rate-distortion. To comply with existing on-board devices, the wavelet transform is applied in association with a linear post processing. The post transform decomposes a small block of wavelet coefficients on a particular basis. The basis can be adaptively selected by the rate-distortion optimization scheme [10]. Multiresolution synthetic aperture radar (SAR) signal processing traditionally carried out in the Fourier domain has inherent limitations in context of the image formation at hierarchical scales. A generalized approach is presented to form multiresolution SAR images using biorthogonal shift invariant discrete wavelet transform. The inherent subband decomposition of wavelet packet transform is introduced to produce multiscale filtering without any approximations. Analytical results and sample imagery of diffuse backscatter are presented to validate the efficiency [11]. A compressive sensing coding paradigm is proposed for high packet loss transmission. 2D discrete wavelet transform (DWT) is applied for sparse representation. By fully exploiting the intra-scale and inter-scale correlation of multiscale DWT, two different recovery algorithms are developed for the low-frequency subband and high-frequency subbands of the decoder. It is more robust against lossy channels, while achieving higher rate-distortion, compared with conventional wavelet-based methods and coding schemes [12]. A study of the role of wavelet packet transform in sparsity reduction is conducted in this research to enhance the compressive sensing. Quantitative results are introduced and computed in order to compare the two cases [13].

2. Discrete Wavelet Packet Transform

Most digital images are essentially sparse, with zero and nearly zero components in the matrix form. After squeezing out zero and nearly zero elements, sparse representation of digital images are formulated. Discrete wavelet transform (DWT) has the tight affinity with compressive sensing in fields of image processing. Hence, two dimensional DWT has been proposed for data compression.

In 2D wavelet transform, the decomposition of an image matrix f(x, y) of the size M by N is calculated by (1) and (2):

$$w_{\varphi}(j_0,m,n)=\frac{1}{\sqrt{MN}}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)\varphi_{j_0,m,n}(x,y) \tag{1}$$

$$w^{i}_{\psi}(j,m,n)=\frac{1}{\sqrt{MN}}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)\psi^{i}_{j,m,n}(x,y) \tag{2}$$

where the $w_{\varphi}(j_0, m, n)$ function and $w^{i}_{\psi}(j, m, n)$ functions are calculated which represent the approximation component, three (horizontal, vertical and diagonal) detail components, respectively for scales $j >= j_0$; $j_0 = 0$, $j=0, 1, 2, \ldots, J-1$ and $m, n = 0, 1, 2, \ldots, 2^j - 1$, $N+M=2^J$; i is the directional index where $i=\{H, V, D\}$ and j_0 is starting scale. The scaling function and wavelet functions are expressed as (3) and (4).

$$\varphi_{j,m,n}(x, y) = 2^{j/2}\, \varphi(2^j x - m, 2^j y - n) \tag{3}$$

$$\psi^{i}_{j,m,n}(x, y) = 2^{j/2}\, \psi^{i} (2^j x - m, 2^j y - n) \tag{4}$$

Haar wavelet is chosen whose basis has been formulated as an increasing power of two of the source data set. Three-level wavelet decomposition is scheduled and the corresponding bases can be reached easily. At the level one, the data has a basis of two. At the level two, the data has a basis of four. At the level three, the data has a basis of eight, and so on for higher level decomposition. In this case, at the first level of 2D decomposition, the source digital image is decomposed into four components: approximation, horizontal detail, vertical detail and diagonal detail, each with a quarter size of the original image. The wavelets include the scaling function and three wavelet functions along with the variations in horizontal, vertical and diagonal directions, serving as the lowpass filter and highpass filters, respectively. Four resulting image matrices are computed by taking the inner products of the source image matrix with the scaling and three wavelet coefficients.

At the second and third levels, the resulting approximation is always decomposed recursively into another subset of four components (approximation, horizontal detail, vertical detail and diagonal detail). From the compressive sensing point of view, regular DWT deals with solely the approximation component while disregarding the three detail components. Its role in compression is relatively limited since no further information on the detail components can be provided. Thus, wavelet packet transform is applied to process detail components in the same way as that of the approximation component. The quaternary tree structure is thus generated, where the Haar wavelet packet transform is in charge of applying the transform to both the lowpass and the highpass components across decomposition from the level one to level three of DWT. This process can be repeated recursively to the higher levels until satisfactory results can be retrieved via inverse discrete wavelet transform (IDWT).

After decomposition, hard thresholding is introduced and applied to the detail coefficients in three orientations (horizontal, diagonal and vertical) at all levels related. These coefficients are in conjunction with the predefined thresholding function. It will then provide a smooth reconstruction process. Image reconstruction is made after two steps of three-level decomposition and thresholding. The approximation components at three levels are kept unchanged. However, the revised detail components at three levels are all subject to hard thresholding. Then the approximated f(x, y) is retrieved by the inverse discrete wavelet transform as (5).

$$\begin{aligned} f(x,y) = & \frac{1}{\sqrt{MN}} \sum_{m} \sum_{n} w_{\varphi}(j_0,m,n)\varphi_{j_0,m,n}(x,y) + \\ & \frac{1}{\sqrt{MN}} \sum_{i=H,V,D} \sum_{j=j0}^{\infty} \sum_{m} \sum_{n} w^{i}_{\psi}(j_0,m,n)\psi^{i}_{j_0,m,n}(x,y) \end{aligned} \tag{5}$$

Based on schemes of the multilevel discrete wavelet packet transform, two case studies have been conducted for compressive sensing. The lower dynamic range image is selected in the first case and the higher dynamic range image is selected in the second case, respectively. The corresponding results are described in the following section.

3. Numerical Case Studies

Two digital images in the gray level have been selected with different dynamic ranges. The first one has a relatively lower dynamic range together with fewer objects in the scene (e.g. one dolphin in the scope). The second one has a relatively higher dynamic range together with more objects in the scene (e.g, plenty of vehicles and seagulls in the scope). Two opposite examples are chosen for a comparison purpose. From Figure 1 to Figure 5, some simulation results are illustrated. The decomposition results of the lower dynamic range image are shown in Figure 1 and Figure 2, respectively, where Figure 1 depicts the wavelet packet approximations at different decomposition levels as well as the source image; and Figure 2 depicts four decomposition components at the level one exclusively. The decomposition results of the higher dynamic range image are illustrated in Figure 3 and Figure 4, respectively, where Figure 3 depicts the wavelet packet approximations at different decomposition levels and the source image, and Figure 3 depicts the four decomposition components at the level one exclusively. Figure 5 depicts the two reconstructed images using compressive sensing based on the multilevel inverse wavelet transform.

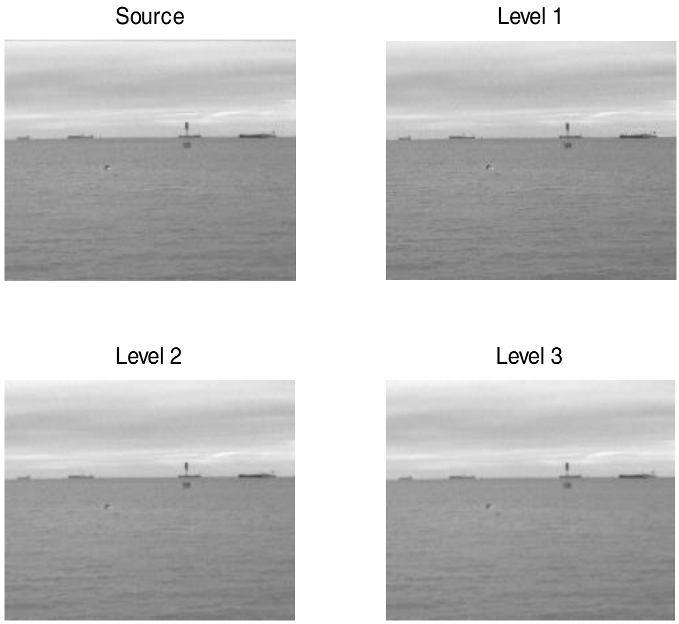

Figure 1. Wavelet Packet Approximations of Lower Dynamic Range Image

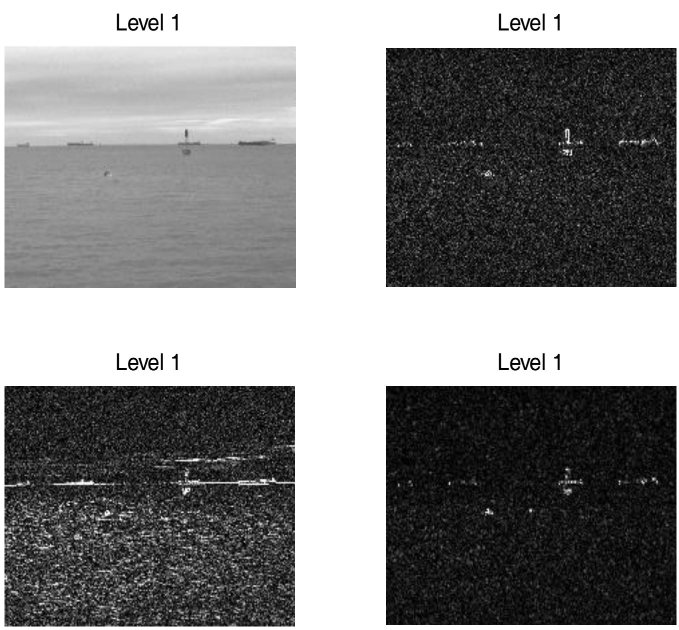

Figure 2. Decomposition at Level One of Lower Dynamic Range Image

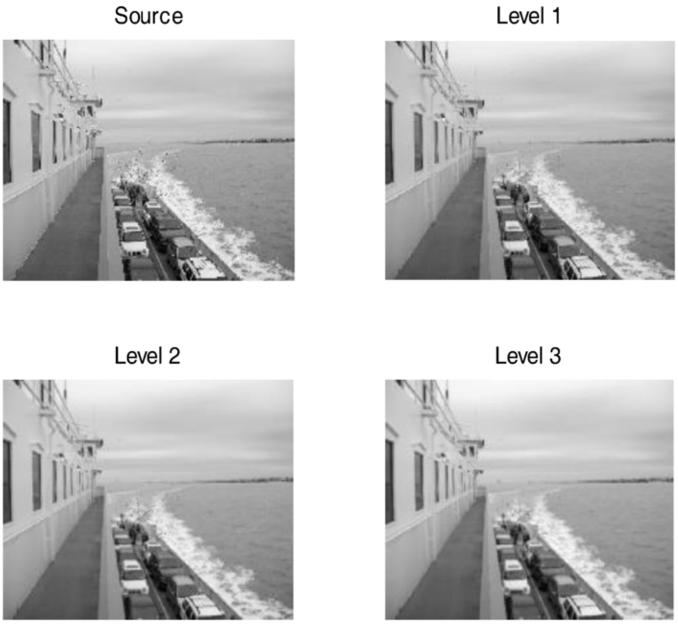

Figure 3. Wavelet Packet Approximations of Higher Dynamic Range Image

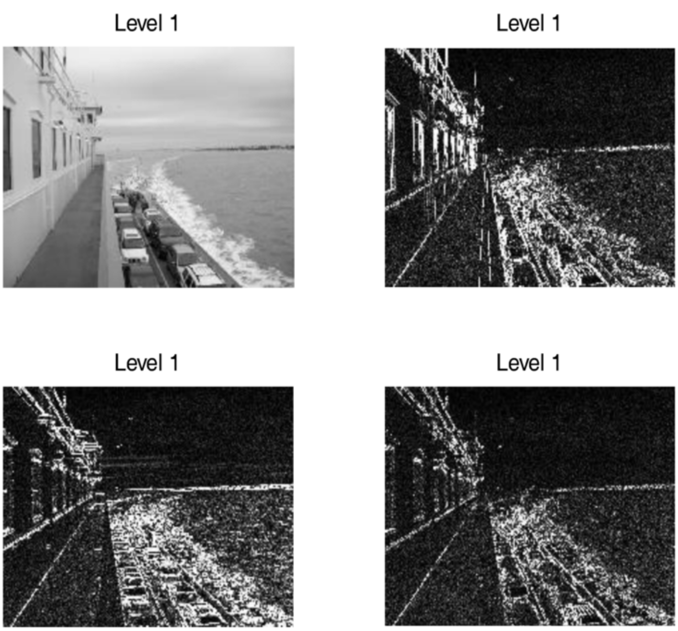

Figure 4. Decomposition at Level One of Higher Dynamic Range Image

Figure 5. Reconstruction Images after Wavelet Packet Decomposition

Comparing the reconstructed images with source images, there is almost no visual difference being observed, even though the lossy compression has been conducted in both cases. To further quantify the role of the discrete wavelet packet transform in scene image compression, define a couple of basic quantitative measures for the matter of simplicity: (1) Compression Ratio; and (2) Discrete Entropy; in order to evaluate the actual outcomes. Compression ratio is hereby defined as the ratio of the size of compressed image after reconstruction over that of the source image. On the other hand, for digital images, the occurrence of the gray level has been defined as co-occurrence matrix of relative frequencies, which is represented as the histogram. The occurrence probability distribution is thus computed based on the histogram. The discrete entropy is equal to the sum of products of the probability of the outcome times the logarithm of the inverse of probability of the outcome, with all the possible outcomes taking into account. It manifests the average uncertainty of the information source. Based on the proposed computation, the quantities are the two cases are obtained.

For the lower dynamic range image, the compression ratio is 70.07%. The discrete entropy of the source image is 6.4740 while the discrete entropy of the reconstructed image is 6.4014. For the higher dynamic range image, the compression ratio is 78.22%. The discrete entropy of the source image is 7.4105 while the discrete entropy of the reconstructed image is 7.3715. In terms of these results, it indicates that the larger compression ratio occurs along with the lower dynamic range image using multilevel wavelet packet transform. In each case, the information loss occurs using compressive sensing, since discrete entropies of reconstructed images are lower than those of source images. At the same time, lower discrete entropies are corresponding to smaller total number of the gray levels. Hence, less intrinsic information has been kept after wavelet packet transform and reconstruction. Sparsity reduction has been achieved in both cases.

4. Conclusions

Wavelet transform is powerful in image compression and restoration so as to reduce the size needed for the actual scene representation. It can also be applied to multiresolution analysis. In the special case studies for compressive sensing of two diverse sparsity level images, discrete wavelet packet transform has been introduced for multilevel decomposition. In wavelet packet transform, a digital image has passed through both lowpass and highpass filters at each level rather than the lowpass filter itself in discrete wavelet transform, which covers more information. Thresholding is applied for better compression. As a tradeoff, representation complexity has been slightly reduced with the decrement in resolution, giving rise to lossy compression. From outcomes of two compressed digital images with the higher and lower dynamic ranges, respectively, it has been illustrated that wavelet packet transform based image compression leads to virtually no visual difference after thresholding and reconstruction. The density of the images after wavelet decomposition will be reduced in both cases. The higher compression ratio occurs in the lower dynamic range case than the higher dynamic range case. Using discrete entropy analysis, similar conclusions are made as well. The wavelet packet transform is effective in image compression while slight information loss is observed. It indicates that multilevel wavelet packet transform is fairly suitable for the general applications on the multi-dimensional compressive sensing.

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