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Characterization of Positive and Negative Information in Comparative Preference Representation

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Abstract. In the last decade, AI researchers have pointed out the existence of two types of information: positive information and negative information. This distinction has also been asserted in cognitive psychology. Distinguishing between these two types of information may be useful in both knowledge and preference representation. In the first case, one distinguishes between situations which are not impossible because they are not ruled out by the available knowledge, and what is possible for sure. In the second case, one distinguishes between what is not rejected and what is really desired. Besides it has been shown that possibility theory is a convenient tool to model and distinguish between these two types of information. Knowledge/Preference representation languages have also been extended to cope with this particular kind of information. Nevertheless despite solid theoretical advances in this topic, the crucial question of "which reading (negative or positive) one should have" remains a real bottleneck. In this paper, we focus on comparative statements. We present a set of postulates describing different situations one may encounter. Then we provide a representation theorem describing which sets of postulates are satisfied by which kind of information (negative or positive) and conversely. One can then decide which reading to apply depending on which postulates she privileges.

1 INTRODUCTION

It is commonly acknowledged that a simple distinction between "good" worlds and "bad" worlds, as in classical logic, is not informative enough and can be refined. In fact some bad (resp. good) worlds may be considered better than some other bad (resp. good) worlds. So generally we need to deal with a complete preorder encoding the satisfaction or acceptability of each world. Depending on information we deal with, this complete preorder is sometimes represented by a numerical function (e.g. possibility distribution, kappa function, etc). However dealing with a complete preorder over the whole set of worlds appears to be problematic. This is because the number of variables used to describe worlds may be large which leads to an exponential set of worlds. Consequently, the direct assessment of a complete preorder (or a numerical function) over the whole set of worlds becomes infeasible. Fortunately in practice we do not need to explicitly elicit this preorder because generally we have at hand pieces of information concerning partial descriptions of worlds, pervaded with implicit or explicit priorities. The problem then consists in a "good" handling of such information in order to derive the associated complete preorder.

We distinguish between two possible readings of a set of pieces of information. The first view agrees with classical logic in the sense that each piece of information declares some worlds impossible/unacceptable. Pieces of information are then conjunctively combined which means that the set of impossible/unacceptable worlds is all the more large as the number of violated pieces of information is large. The complete preorder describes fully possible/acceptable worlds (as they are not excluded by any piece of information) and gives intermediary levels between impossible/unacceptable worlds. The second view behaves in an opposite way. Each piece of information declares some outcomes possible/satisfactory. Pieces of information are then disjunctively combined which means that the set of possible/satisfactory worlds is all the more large as the number of satisfied pieces of information is large. The complete preorder describes fully impossible/unsatisfactory worlds (because they do not satisfy any piece of information) and gives intermediary levels between possible/satisfactory worlds. In the light of the previous presentation of the two views, we can say that the first view offers a negative reading of information (as we penalize worlds which violate pieces of information) while the second view offers a positive reading (as we reward worlds which satisfy pieces of information).

Recently, it has been acknowledged both in AI and cognitive psychology communities that information could be bipolar [6, 1, 5, 11]. On the one hand, individuals express what they consider as impossible/unacceptable and on the other hand, they express what they consider as possible/satisfactory. The first type of information are called *negative* information (knowledge/constraints or rejections) and obey a negative reading while the second type are called *positive* information (facts/wishes) and obey a positive reading.

Possibility theory appears to be a convenient tool to handle the two kinds of information. In particular negative information is modeled in this setting by possibility and necessity measures while positive information is modeled by guaranteed possibility measures [1, 2]. In the first case, the minimal specificity principle is used. It is based on the assumption that what is not explicitly ruled out (i.e., rejected) is fully possible/acceptable. In fact this corresponds to the standard handling of knowledge/constraints or rejections. In the second case, the maximal specificity principle is used. It is based on a close world assumption, that is only information explicitly reported is true. In fact this corresponds to the meaning behind facts/wishes.
The main ingredient in possibility theory is a possibility distribution which associates an possibility/satisfaction degree with each world. However in practice, we do not explicitly deal with such a distribution. Generally it is compactly represented in different formats, namely weighted logical formulas, comparative statements and Bayesian-like formats.

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Whatever information being bipolar or not (i.e., the two kinds of information are simultaneously present or not), given a set of pieces of information one has to decide which reading (negative or positive) is more convenient. However deciding whether an information is negative or positive is not an easy task and represents a serious bottleneck in knowledge/preference representation. This is particularly true in preference representation because it is not obvious to clearly define the borders between what is rejected and what is desired. In order to overcome this shortcoming, we give in this paper a postulate-based comparison of these two kinds of information. We first provide a set of postulates proposed in literature describing different situations one may encounter. Then on the basis of which postulates are satisfied by which kind of information (negative or positive) we provide a representation theorem that characterizes subsets of postulates which are satisfied by each kind of information. This theorem is important as it goes a step further in handling negative and positive information and help to decide which reading one would apply. Our work focuses on comparative statements to compactly represent a possibility distribution.

The remainder of this paper is organized as follows. After providing necessary background in Section 2, Section 3 first presents comparative statements and their semantics. Then it presents our framework, namely possibility theory, followed by an encoding of comparative statements and their associated semantics in this framework. A semantics represents whether information is negative or positive. In Section 4 we expose the problem we address in this paper. In Section 5 we give a postulate-based comparison of the semantics. In particular we provide a representation theorem which is intended to help in the choice of the semantics to employ. Lastly we conclude.

2 BACKGROUND

Let V be a set of variables each taking its values in a domain. A possible world, denoted by ω, is the result of assigning a value to each variable in its associated domain. Ω is the set of all possible worlds. We suppose that this set is fixed and finite. Let $\mathcal{L}$ be a language based on V. $Mod(\alpha)$ denotes the set of worlds that make the formula α (built on $\mathcal{L}$) true. It is also called α-worlds.
A preference relation $\succeq$ over Ω is a reflexive and transitive binary relation such that $\omega \succeq \omega'$ stands for "ω is at least as preferred (i.e., possible/satisfactory) as ω'". $x \approx y$ means that both $x \succeq y$ and $y \succeq x$ hold i.e., x and y are equally preferred. The notation $x \succ y$ means that x is strictly preferred to y. We have $x \succ y$ if $x \succeq y$ holds but $y \succeq x$ does not.
For convenience, a complete preorder $\succeq$ can also be represented by a well ordered partition of Ω. A sequence of sets of worlds of the form $(E_1, \ldots, E_n)$ is a partition of Ω if and only if (i) $\forall i$, $E_i \neq \emptyset$, (ii) $E_1 \cup \cdots \cup E_n = \Omega$, and (iii) $\forall i, j$, $E_i \cap E_j = \emptyset$ for $i \neq j$.
A partition of Ω is ordered if and only if it is associated with a preorder $\succeq$ over Ω such that ($\forall \omega, \omega' \in \Omega$ with $\omega \in E_i, \omega' \in E_j$ we have $i \leq j$ iff $\omega \succeq \omega'$).

3 COMPARATIVE STATEMENTS

The central notion of this paper is that of comparative statements which we denote by $\alpha \rhd \beta$ to mean that "α is preferable to β". It is worth noticing that the term "preferable" is generic. In preference representation it means that one likes α more than β while in knowledge (i.e., belief) representation, it means that one believes α more than β.

3.1 Semantics

At the semantic level, a statement $\alpha \rhd \beta$ means that α-worlds are preferable to β-worlds. This preference relation is easy to establish when both sets of α-worlds and β-worlds refer to single worlds. Let ω and ω' be these worlds respectively. We have that

$$\alpha \rhd \beta \text{ iff } \rho(\omega) > \rho(\omega'), \tag{1}$$

where ρ is a numerical function encoding a preference relation $\succeq$ over Ω. More precisely, $\rho(\omega) > \rho(\omega')$ iff $\omega \succ \omega'$. The function ρ refers to a utility function if preferences are encoded and to possibility (among other functions) if beliefs are encoded.

However the problem of comparing α-worlds and β-worlds turns to be more complex when these two sets are not single outcomes and not mutually exclusive i.e., their intersection is not empty. In order to avoid the second situation, we follow von Wright principle [14] and interpret $\alpha \rhd \beta$ as a choice problem between "having α and not β" and "having β and not α". Consequently, at the semantic level, we say that $\alpha \wedge \neg\beta$-worlds are preferable to $\beta \wedge \neg\alpha$-worlds. Particular situations are those when $\alpha \wedge \neg\beta$ (resp. $\beta \wedge \neg\alpha$) is a contradiction or is not feasible in which case it is replaced with α (resp. β). We refer the reader to [14, 8] for further details. For simplicity we suppose that both $\alpha \wedge \neg\beta$ and $\beta \wedge \neg\alpha$ are consistent and feasible.
We generalize Equation (1) in the following way:

$$\alpha \rhd \beta \text{ iff } \oplus\{\rho(\omega)|\omega \models \alpha \wedge \neg\beta\} > \odot\{\rho(\omega')|\omega' \models \beta \wedge \neg\alpha\}. \tag{2}$$

This is to say that we aggregate values associated with $\alpha \wedge \neg\beta$-worlds (resp. $\beta \wedge \neg\alpha$-worlds) in order to compute a global evaluation of α (resp. β). For the purpose of this paper, $\oplus$ and $\odot$ are $\min$ and $\max$ aggregation operators. This is because these operators recover existing widely used semantics to interpret statements of the form $\alpha \rhd \beta$. Moreover they encode negative and positive information.

3.2 Function ρ

Generally we deal with several comparative statements, put $\Gamma = \{\alpha_i \rhd \beta_i\}$, in which case Equation (2) is generalized as follows: $\forall \alpha_i \rhd \beta_i \in \Gamma$,

$$\alpha_i \rhd \beta_i \text{ iff } \oplus\{\rho(\omega)|\omega \models \alpha_i \wedge \neg\beta_i\} > \odot\{\rho(\omega')|\omega' \models \beta_i \wedge \neg\alpha_i\}. \tag{3}$$

A different meaning is associated with each sense of the above equivalence. Consider first "$\forall \alpha_i \rhd \beta_i \in \Gamma$, if $\oplus\{\rho(\omega)|\omega \models \alpha_i \wedge \neg\beta_i\} > \odot\{\rho(\omega')|\omega' \models \beta_i \wedge \neg\alpha_i\}$ then $\alpha_i \rhd \beta_i$". This implication means that ρ, $\oplus$ and $\odot$ are given. Then we can say that α_i is preferable to β_i i.e., $\alpha_i \rhd \beta_i$ is satisfied, only when the strict comparison is true.
The other sense of the implication i.e., $\forall \alpha_i \rhd \beta_i \in \Gamma$, if $\alpha_i \rhd \beta_i$ then $\oplus\{\rho(\omega)|\omega \models \alpha_i \wedge \neg\beta_i\} > \odot\{\rho(\omega')|\omega' \models \beta_i \wedge \neg\alpha_i\}$ is more complex. In fact this case means that we have at hand $\Gamma = \{\alpha_i \rhd \beta_i\}$ and aim to define ρ which satisfies every $\alpha_i \rhd \beta_i$ in Γ, given $\oplus$ and $\odot$. We say that ρ satisfies Γ. Clearly this case is more interesting because generally we have Γ and aim to rank-order worlds i.e., compute ρ. This is the focus of this paper.

Given $\oplus$ and $\odot$ we say that Γ is inconsistent when ρ does not exist, that is there is no ρ which satisfies Γ. On the other hand, when Γ is consistent we may have different ρ which satisfy Γ. However note that ρ encodes a complete preorder over Ω. On the other hand, it is commonly acknowledged that the existence of different preorders may not be informative from decision point of view [4]. Indeed two worlds may not be identically rank-ordered w.r.t. all these preorders which results in declaring these worlds incomparable. In the worst case this may result in an incomparability between all possible worlds. In order to overcome this undesirable situation, researchers look for unique preorders. In the next section, we fix the framework and show how to characterize unique ρ given $\oplus$ and $\odot$.

3.3 The framework

We encode ρ in possibility theory. It is a function from Ω to a finite ordered scale with bottom and top elements. For simplicity and without loss of generality, we consider this scale the unit interval $[0, 1]$. ρ is called possibility distribution. $\rho(\omega)$ evaluates the possibility/satisfaction degree of ω. Originally developed to encode uncertain knowledge, possibility theory appeared to be also a convenient way to encode preferences. In particular, it helps understand the semantics underpinning the choice of $\oplus$ and $\odot$ operators.
Before we go further in the exposition of our approach, let us give necessary background on possibility theory.

Given a possibility distribution ρ and a formula ϕ, we distinguish between two measures:

- The possibility measure of ϕ, denoted $\Pi(\phi)$, is defined by:

$$\Pi(\phi) = \max\{\rho(\omega)|\omega \in \Omega, \omega \models \phi\}. \quad (4)$$

  $\Pi(\phi)$ evaluates the maximal extent to which ϕ is consistent with information encoded by ρ.

- The guaranteed satisfaction measure of a formula ϕ, denoted $\Delta(\phi)$, is defined by:

$$\Delta(\phi) = \min\{\rho(\omega)|\omega \in \Omega, \omega \models \phi\}. \quad (5)$$

  $\Delta(\phi)$ evaluates the minimal extent to which ϕ is consistent with information encoded by ρ.

Note that Π is a maxitive set function (increasing in the wide sense with set inclusion) i.e.,

$$\Pi(\alpha \vee \beta) = max(\Pi(\alpha), \Pi(\beta)),$$

while guaranteed possibility functions are decreasing set functions such that

$$\Delta(\alpha \vee \beta) = min(\Delta(\alpha), \Delta(\beta)).$$

This means that interpretations covered by α or β are guaranteed to be possible/satisfactory if and only if both the interpretations satisfying α and those satisfying β are guaranteed to be possible/satisfactory.
We also have the following properties:

$$\Pi(\alpha \wedge \beta) \leq \min(\Pi(\alpha), \Pi(\beta))$$

and

$$\Delta(\alpha \wedge \beta) \geq \max(\Delta(\alpha), \Delta(\beta)).$$

Due to the lack of space, we will not provide proofs of the results presented in Section 5. However keep in mind that the above properties are useful to write the proofs.
Possibility distributions can be compared on the basis of specificity principle [15].

Definition 1 (Minimal/Maximal specificity principle) *Let ρ and ρ' be two possibility distributions. We say that ρ is less (resp. more) specific than ρ' if and only if $\forall\omega \in \Omega$, $\rho(\omega) \geq \rho(\omega')$ (resp. $\rho(\omega) \leq \rho(\omega')$). ρ is strictly less (resp. more) specific than ρ' if and only if ρ is less (resp. more) specific than ρ' and $\exists\omega, \omega' \in \Omega$ such that $\rho(\omega) > \rho(\omega')$ (resp. $\rho(\omega) < \rho(\omega')$). We say that ρ belongs to the set of minimally (resp. maximally) specific possibility distributions, among a set of possibility distributions, if and only if there is no possibility distribution in the set that is strictly less (resp. more) specific than ρ. If ρ is the unique minimally (resp. maximally) specific possibility distribution then it is called the least (resp. most) specific possibility distribution.*

In a qualitative setting, a possibility distribution can be represented by the well ordered partition of its associated complete preorder.

3.4 Encoding comparative statements in possibility theory

As we previously said, given a set of comparative statements $\Gamma = \{\alpha_i \rhd \beta_i\}$ we aim to define ρ such that $\forall\alpha_i \rhd \beta_i \in \Gamma$, $\alpha_i \rhd \beta_i$ iff $\oplus\{\rho(\omega)|\omega \models \alpha_i \wedge \neg\beta_i\} > \odot\{\rho(\omega')|\omega' \models \beta_i \wedge \neg\alpha_i\}$. We consider $\oplus$ and $\odot$ as $\max$ and $\min$ operators. Therefore we distinguish between four types of comparative statements [4, 3, 1, 9]:

Definition 2 (Preference semantics) *Let α be preferable to β iff $\oplus\{\rho(\omega)|\omega \models \alpha \wedge \neg\beta\} > \odot\{\rho(\omega')|\omega' \models \beta \wedge \neg\alpha\}$.*

- *α is* strongly *preferable to β, denoted by $\alpha \rhd_{st} \beta$, iff $\oplus = \min$ and $\odot = \max$.*
- *α is* optimistically *preferable to β, denoted by $\alpha \rhd_{opt} \beta$, iff $\oplus = \max$ and $\odot = \max$.*
- *α is* pessimistically *preferable to β, denoted by $\alpha \rhd_{pes} \beta$, iff $\oplus = \min$ and $\odot = \min$.*
- *α is* opportunistically *preferable to β, denoted by $\alpha \rhd_{opp} \beta$, iff $\oplus = \max$ and $\odot = \min$.*

Therefore the use of $\max$ and $\min$ operators helps to recover the most well-known semantics studied in the literature. Moreover it allows to link these semantics to negative and positive information as explained in the next section. More precisely, making the parallel with Π and Δ measures we have that

- $\alpha \rhd_{st} \beta$ iff $\Delta(\alpha \wedge \neg\beta) > \Pi(\beta \wedge \neg\alpha)$
- $\alpha \rhd_{opt} \beta$ iff $\Pi(\alpha \wedge \neg\beta) > \Pi(\beta \wedge \neg\alpha)$
- $\alpha \rhd_{pes} \beta$ iff $\Delta(\alpha \wedge \neg\beta) > \Delta(\beta \wedge \neg\alpha)$
- $\alpha \rhd_{opp} \beta$ iff $\Pi(\alpha \wedge \neg\beta) > \Delta(\beta \wedge \neg\alpha)$

The following proposition states the existence (or not) of the least/most specific distribution ρ for each case:

Proposition 1 *[12, 4, 3, 9]*

- *The least specific possibility distribution satisfying $\Gamma = \{\alpha_i \rhd_{opt} \beta_i\}$ (resp. $\Gamma = \{\alpha_i \rhd_{st} \beta_i\}$) exists.*
- *The most specific possibility distribution satisfying $\Gamma = \{\alpha_i \rhd_{pes} \beta_i\}$ (resp. $\Gamma = \{\alpha_i \rhd_{st} \beta_i\}$) exists.*
- *The least (resp. most) specific possibility distribution satisfying $\Gamma = \{\alpha_i \rhd_{opp} \beta_i\}$ does not exist.*

Due to the lack of space we do not present algorithms which compute the least and most specific possibility distributions. For a formal exposition of these algorithms, we refer the reader to [12, 4, 3, 9].

As we are looking for the least/most specific distributions ρ we do no longer consider $\Pi(.) > \Delta(.)$ comparatives. Now $\Delta(.) > \Pi(.)$ are in between $\Pi(.) > \Pi(.)$ and $\Delta(.) > \Delta(.)$. This can be easily checked by noticing that $\Delta(\alpha \wedge \neg\beta) > \Pi(\beta \wedge \neg\alpha)$ is equivalent to $\{\Pi(\omega_j) > \Pi(\beta \wedge \neg\alpha) | \omega_j \models \alpha \wedge \neg\beta\}$ (resp. $\{\Delta(\alpha \wedge \neg\beta) > \Delta(\omega_j) | \omega_j \models \beta \wedge \neg\alpha\}$) [9]. Accordingly, (Δ, Π) comparatives obey minimal specificity principle when encoded by $\Pi(.) > \Pi(.)$ comparatives and maximal specificity principle when encoded by $\Delta(.) > \Delta(.)$ comparatives. Consequently, we can simply focus on $\Pi(.) > \Pi(.)$ and $\Delta(.) > \Delta(.)$ comparatives.

4 THE PROBLEM

Besides the technical definition of $\Pi(.) > \Pi(.)$ and $\Delta(.) > \Delta(.)$ comparatives, what makes difference between these two types of comparatives is the way "non-rejected" worlds are handled. In fact consider $\alpha \rhd \beta$. We know that this statement is interpreted as "$\alpha \wedge \neg\beta$-worlds are preferable to $\beta \wedge \neg\alpha$-worlds". So in some sense $\beta \wedge \neg\alpha$-worlds are rejected while $\alpha \wedge \neg\beta$-worlds are desired. Nothing is said about $\neg(\beta \wedge \neg\alpha) \wedge \neg(\alpha \wedge \neg\beta)$-worlds i.e., those which are neither rejected nor desired. In the computation of the least specific possibility distribution satisfying $\alpha \rhd_{opt} \beta$ these worlds get the same possibility degree as $\alpha \wedge \neg\beta$-worlds. Therefore $\Pi(.) > \Pi(.)$ behave as constraints: what is not rejected is accepted. In fact $\Pi(.) > \Pi(.)$ encode negative information [1, 2]. On the other hand, when computing the most specific possibility distribution satisfying $\alpha \rhd_{pes} \beta$, $\neg(\beta \wedge \neg\alpha) \wedge \neg(\alpha \wedge \neg\beta)$-worlds get the same possibility degree as $\beta \wedge \neg\alpha$-worlds. This means that only $\alpha \wedge \neg\beta$-worlds are rewarded. Therefore $\Delta(.) > \Delta(.)$ behave as facts/wishes. In fact $\Delta(.) > \Delta(.)$ encode positive information [6, 1, 2].

According to the principles underpinning the computation of the least and most possibility distributions associated to $\Pi(.) > \Pi(.)$ and $\Delta(.) > \Delta(.)$ comparatives respectively, we see that these comparisons are intuitively meaningful. Moreover they have a solid theoretical foundation both in AI and cognitive psychology [6, 1, 2, 9, 5, 11]. However in spite of these advances, existing works do not give an indication on which comparative type one should choose in which case! This is particularly true when comparative statements encode preferences. In fact the distinction between a positive preference and a negative preference is not always, maybe never, easy. In this paper we give a basis to help in this choice by means of postulates. The latter describe different situations one may have, given one or two comparative statements.

In the sequel we call $\Pi(.) > \Pi(.)$ and $\Delta(.) > \Delta(.)$ optimistic and pessimistic semantics respectively (therefore $\Delta(.) > \Pi(.)$ is called strong semantics) referring to Definition 2 and its encoding in possibility theory.

5 POSTULATE-BASED ANALYSIS OF OPTIMISTIC AND PESSIMISTIC SEMANTICS

Mind that optimistic and pessimistic semantics encode negative and positive information respectively. As we previously said, we offer a postulate-based comparison of these semantics. We show that there is no semantics which satisfies the entire set of postulates at hand. Therefore the choice of a semantics can be done on the basis of postulates one wishes to satisfy.

5.1 Postulates

In this section we present our postulates. We do not have any innovative intention regarding postulates. In fact groups of these postulates have been independently studied in literature [10, 13, 7]. Putting them together in this paper will have the benefit to cover all situations one may encounter and allows a comparison of optimistic and pessimistic semantics on a complete ground. Roughly our postulates cover three principles: coherence (P1), deduction of preferable or less preferable formulas (P2,P2',P3,P3') and decomposition or composition of these formulas (P4,$\cdots$, P9). It is important to understand the meaning of "if-then" of the postulates. In fact it should be interpreted as follows: given a semantics $\rhd_*$, if ρ is the least/most specific possibility distribution satisfying comparative statements appearing in the antecedent of "if-then" then this distribution satisfies comparative statements appearing in the consequence of "if-then". It is worth noticing that we only require that the least/most possibility distribution satisfies the consequence and not be necessary the least/most possibility distribution satisfying these statements (i.e., those appearing in the consequence of "if-then"). We now present our postulates.

- P1: if $\alpha \rhd_* \beta$ then $not(\beta \rhd_* \alpha)$
  This postulate expresses *coherence*. If α is preferable to β then β cannot be preferable to α.
- P2: if $\alpha \rhd_* \beta$ and $\alpha \vdash \gamma$ then $\gamma \rhd_* \beta$
  α cannot be preferable to β unless every consequence of α is also preferable to β.
- P2': if $\alpha \rhd_* \beta$ and $\gamma \vdash \alpha$ then $\gamma \rhd_* \beta$
  α cannot be preferable to β unless every formula that permits to have α is also preferable to β.
- P3: if $\alpha \rhd_* \beta$ and $\beta \vdash \gamma$ then $\alpha \rhd_* \gamma$
  α cannot be preferable to β unless it is preferable to any consequence of β.
- P3': if $\alpha \rhd_* \beta$ and $\gamma \vdash \beta$ then $\alpha \rhd_* \gamma$
  α cannot be preferable to β unless it is preferable to any formula that permits to have β.
- P4: if $\alpha \rhd_* \gamma$ and $\beta \rhd_* \gamma$ then $\alpha \vee \beta \rhd_* \gamma$
  This is a *left-hand or-composition*. If both α and β are preferable to γ then situations in which α or β is true are preferable to γ.
- P4': if $\alpha \rhd_* \gamma$ and $\beta \rhd_* \gamma$ then $\alpha \wedge \beta \rhd_* \gamma$
  This is a *left-hand and-composition*. If both α and β are preferable to γ then situations in which both α and β are true are preferable to γ.
- P5: if $\alpha \vee \beta \rhd_* \gamma$ then ($\alpha \rhd_* \gamma$ and $\beta \rhd_* \gamma$)
  This is a *left-hand or-and-decomposition*. If α or β is preferable to γ then α and β taken separately are preferred to γ.
- P5': if $\alpha \vee \beta \rhd_* \gamma$ then ($\alpha \rhd_* \gamma$ or $\beta \rhd_* \gamma$)
  This is a *left-hand or-or-decomposition*. If α or β is preferable to γ then α is preferable to γ or β is preferable to γ.
- P6: if $\alpha \rhd_* \beta$ and $\alpha \rhd_* \gamma$ then $\alpha \rhd_* \beta \vee \gamma$
  This a *right-hand or-composition*. If α is preferable to both β and γ taken separately then it is also preferable to situations in which β or γ is true.

- P6': if $\alpha \rhd_* \beta$ and $\alpha \rhd_* \gamma$ then $\alpha \rhd_* \beta \wedge \gamma$
  This is *right-hand and-composition*. If α is preferable to both β and γ taken separately then it is also preferable to situations in which both β and γ are true.

- P7: if $\alpha \rhd_* \beta \vee \gamma$ then ($\alpha \rhd_* \beta$ and $\alpha \rhd_* \gamma$)
  This is a *right-hand or-and-decomposition*. If α is preferable to β or γ then it is preferable to β and γ taken separately.

- P7': if $\alpha \rhd_* \beta \vee \gamma$ then ($\alpha \rhd_* \beta$ or $\alpha \rhd_* \gamma$)
  This is a *right-hand or-or-decomposition*. If α is preferable to β or γ then it is preferable to β or γ taken separately.

- P8: if $\alpha \wedge \beta \rhd_* \gamma$ then ($\alpha \rhd_* \gamma$ and $\beta \rhd_* \gamma$)
  If α and β are preferable to γ when taken together then they are still preferable when considered separately.

- P9: if $\alpha \rhd_* \beta \wedge \gamma$ then ($\alpha \rhd_* \beta$ and $\alpha \rhd_* \gamma$)
  If α is preferable to β and γ taken together than it is preferable to them when considered separately.

We didn't give "or" counterpart of postulates P4, P4', P6 and P6' because a preference set is a conjunction of comparative statements (and not their disjunction). Notice also that for all comparative statements given in the above postulates we suppose that the set of $\alpha \wedge \neg\beta$-worlds (resp. $\beta \wedge \neg\alpha$-worlds), for any $\alpha \rhd_* \beta$, is not empty.

5.2 Representation theorem

It is important to keep in mind that our aim is to study optimistic and pessimistic semantics in their "context", namely when the corresponding specificity principle is applied. Recall that optimistic semantics obeys minimal specificity principle while pessimistic semantics obeys maximal specificity principle. Accordingly, as we previously said, comparative statements in the antecedent of "if-then" of the postulates are interpreted following a given semantics and the corresponding least/most specific possibility distribution is computed. Then the idea is not to check whether this possibility distribution is the least/most specific distribution of comparative statements appearing in the consequence of "if-then" but just to check whether these statements are true in this possibility distribution. This reasoning means that once a least/most specific possibility distribution is computed for a given set of postulates, one would like to know which comparative statements can be deduced.
Table 1 gives a complete picture of which postulate is satisfied by which semantics. We ask the reader to focus on "Optimistic" and "Pessimistic" columns. However we provided the results regarding strong optimistic when both minimal and maximal specificity principles are applied, called Strong-Min-Sp and Strong-Max-Sp respectively. One can easily check that Strong-Min-Sp (resp. Strong-Max-Sp) semantics gives identical results as Optimistic (resp. Pessimistic) semantics. This supports existing results which state that strong semantics behaves like optimistic (resp. pessimistic) semantics when minimal (resp. maximal) specificity principle is applied [9].

Now one may wonder whether the set of postulates is minimal. Indeed the following propositions state that some postulates are redundant depending on the semantics:

Proposition 2 (Optimistic semantics)

- *If P2 (resp. P4') is satisfied then P4 is satisfied.*
- *If P3' (resp. P6) is satisfied then P6' is satisfied.*

Proposition 3 (Pessimistic semantics)

- *If P2' (resp. P4) is satisfied then P4' is satisfied.*
- *If P3 (resp. P6') is satisfied then P6 is satisfied.*

Moreover P7' (resp. P5') trivially follows from P7 (resp. P5) for optimistic (resp. pessimistic) semantics.

Propositions 2 and 3 highlight a coherent behavior of the two semantics. In fact the former proposition induces that postulates P4' and P6 are not redundant given optimistic semantics while the latter induces that postulates P4 and P6' are not redundant given pessimistic semantics. Indeed P4' and P6 state that preferred formulas[2] are conjunctively combined while rejected formulas are disjunctively combined. This is an interesting result as this reasoning is usually applied with negative information i.e., knowledge or constraints. This is also coherent with the fact that optimistic semantics encodes negative information [6, 1, 2]. On the other hand, P4 and P6' state that preferred formulas are disjunctively combined while rejected formulas are conjunctively combined. Again this is an interesting result as this reasoning is usually applied with positive information i.e., facts or wishes. This is also coherent with the fact that pessimistic semantics encodes positive information [6, 1, 2]. This distinction is also reflected in P8 and P9. In fact since optimistic semantics encodes negative information in which preferred formulas are conjunctively combined then they are conjunctively decomposed (see P8). On the other hand, pessimistic semantics encodes positive information in which rejected formulas are conjunctively combined then they are also conjunctively decomposed (see P9). This observation also holds for P5 and P7. The duality between optimistic and pessimistic semantics is also reflected in P2 and P3 (resp. P2' and P3').
Consequently we have that each time a postulate is satisfied by a given semantics (optimistic or pessimistic), it appears that the postulate describes a coherent behavior with information (negative or positive) encoded by the semantics.
Lastly (but not least) P1 is not trivial as it may not be satisfied. Consider three propositional variables p, q and r. Then the possibility distribution ρ represented by its well ordered partition $(\{\neg p\neg q\neg r, \neg p\neg qr, pq\neg r, pqr, p\neg q\neg r, \neg pqr\}, \{p\neg qr, \neg pq\neg r\})$ satisfies both $p \rhd_{opp} q$ and $q \rhd_{opp} p$ (regardless the fact that the least/most specific possibility distribution for opportunistic semantics does not exist).

From Table 1 we also have that there is no semantics which satisfies the entire set of postulates. This is not because these postulates are inconsistent together or because the semantics have bad theoretical foundations. The reason is that each postulate describes a particular behavior which is satisfied or not by optimistic and pessimistic semantics. As the latter exhibit a dual behavior, it is just not surprising that none of them satisfies all the postulates. This also supports existing claims about negative and positive information. Indeed these two kinds of information are not complementary but dual.

In the light of the above results we are now able to write the following representation theorem:

[2] Given $\alpha \rhd \beta$, we say that α is the preferred formula and β is the rejected one.

Table 1. Postulates satisfaction.

Postulates	Strong-Min-Sp	Strong-Max-Sp	Optimistic	Pessimistic
P1: if $\alpha \rhd_* \beta$ then $not(\beta \rhd_* \alpha)$	YES	YES	YES	YES
P2: if $\alpha \rhd_* \beta$ and $\alpha \vdash \gamma$ then $\gamma \rhd_* \beta$	YES	NO	YES	NO
P2': if $\alpha \rhd_* \beta$ and $\gamma \vdash \alpha$ then $\gamma \rhd_* \beta$	NO	YES	NO	YES
P3: if $\alpha \rhd_* \beta$ and $\beta \vdash \gamma$ then $\alpha \rhd_* \gamma$	NO	YES	NO	YES
P3': if $\alpha \rhd_* \beta$ and $\gamma \vdash \beta$ then $\alpha \rhd_* \gamma$	YES	NO	YES	NO
P4: if $\alpha \rhd_* \gamma$ and $\beta \rhd_* \gamma$ then $\alpha \vee \beta \rhd_* \gamma$	YES	YES	YES	YES
P4': if $\alpha \rhd_* \gamma$ and $\beta \rhd_* \gamma$ then $\alpha \wedge \beta \rhd_* \gamma$	YES	YES	YES	YES
P5: if $\alpha \vee \beta \rhd_* \gamma$ then ($\alpha \rhd_* \gamma$ and $\beta \rhd_* \gamma$)	NO	YES	NO	YES
P5': if $\alpha \vee \beta \rhd_* \gamma$ then ($\alpha \rhd_* \gamma$ or $\beta \rhd_* \gamma$)	NO	YES	NO	YES
P6: if $\alpha \rhd_* \beta$ and $\alpha \rhd_* \gamma$ then $\alpha \rhd_* \beta \vee \gamma$	YES	YES	YES	YES
P6': if $\alpha \rhd_* \beta$ and $\alpha \rhd_* \gamma$ then $\alpha \rhd_* \beta \wedge \gamma$	YES	YES	YES	YES
P7: if $\alpha \rhd_* \beta \vee \gamma$ then ($\alpha \rhd_* \beta$ and $\alpha \rhd_* \gamma$)	YES	NO	YES	NO
P7': if $\alpha \rhd_* \beta \vee \gamma$ then ($\alpha \rhd_* \beta$ or $\alpha \rhd_* \gamma$)	YES	NO	YES	NO
P8: if $\alpha \wedge \beta \rhd_* \gamma$ then ($\alpha \rhd_* \gamma$ and $\beta \rhd_* \gamma$)	YES	NO	YES	NO
P9: if $\alpha \rhd_* \beta \wedge \gamma$ then ($\alpha \rhd_* \beta$ and $\alpha \rhd_* \gamma$)	NO	YES	NO	YES

Theorem 1

- *P1, P2, P3', P4', P6, P7 and P8 are satisfied all together iff $\rhd$ is optimistic semantics.*
- *P1, P2', P3, P4, P5, P6' and P9 are satisfied all together iff $\rhd$ is pessimistic semantics.*

Theorem 1 states that optimistic and pessimistic semantics can be characterized by two dual subsets of postulates, namely (P2, P3', P4', P6, P7, P8) and (P2', P3, P4, P5, P6', P9) respectively. P1 is not a characterization postulate but only ensures that the two semantics behave in a coherent way. Due to the lack of space we do not provide the proof of the above theorem. However let us sketch the proof of the first item. For example optimistic semantics satisfying P4'. Suppose that $\alpha \rhd_{opt} \gamma$ and $\beta \rhd_{opt} \gamma$. The least specific possibility distribution associated to these statements is (E_1, E_2) with $E_1 = Mod(\neg\alpha \wedge \neg\beta \wedge \neg\gamma) \cup Mod(\neg\alpha \wedge \beta \wedge \neg\gamma) \cup Mod(\alpha \wedge \neg\beta \wedge \neg\gamma) \cup Mod(\alpha \wedge \beta \wedge \neg\gamma) \cup Mod(\alpha \wedge \beta \wedge \gamma)$ and $E_2 = Mod(\neg\alpha \wedge \neg\beta \wedge \gamma) \cup Mod(\neg\alpha \wedge \beta \wedge \gamma) \cup Mod(\alpha \wedge \neg\beta \wedge \gamma)$ which satisfies $\alpha \wedge \beta \rhd_{opt} \gamma$. Now following Table 1 we can check that only optimistic semantics satisfies the postulates P1, P2, P3', P4', P6, P7 and P8 all together.

6 CONCLUSION

In our daily-life we may express what we dislike or judge impossible. We may also express what we like or judge possible. It is worth noticing that these ways do not complement each other. That is, what we like or judge possible does not simply mirror what we do not dislike or judge impossible. The first kind of information is called negative while the second kind is called positive.

In practice we may need to handle one or both kinds of information. In the latter case we speak about bipolar information. In any case, the main problem is to put each available piece of information in the right category. However it appears that this problem is far from being easy because linguistic terms are complex and do not give a clear distinction between positive and negative information. It has been shown that these two kinds of information can be conveniently modeled in possibility theory by means of constraints on possibility and guaranteed possibility measures. Roughly we distinguish between optimistic semantics and pessimistic semantics which respectively model negative and positive information.

In this paper we focused on comparative statements. We offered a first step to clarify the borders between negative and positive information and help decide which of them is suitable. We provided a set of postulates describing different situations to compose/decompose or strengthen/weaken comparative statements. Then we gave a representation theorem characterizing two subsets of postulates each satisfied by a unique semantics (optimistic or pessimistic).

Our postulates are not new. Subsets of them have been separately proposed in the literature. However put together, they give a clue to distinguish between the two semantics and the choice of a semantics should be done on the basis of postulates one wishes to get satisfied.

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