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Contextual Reasoning in Context-Aware Systems

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Abstract. Adding reasoning abilities to context-aware systems has been a focus of research in pervasive computing for several years and a broad range of approaches has been suggested. In particular, the well-known trade-off between expressivity and inferential power has been discussed as a major concern, as dimensions of context include well-known hard domains, such as spatial, temporal, and causal knowledge. In practice however, experiments report acceptable run-times and complexity of the used reasoning mechanism does not seem to be an issue at all. Two questions are addressed in this paper: why this is the case and whether these positive results will scale up as context-aware systems are leaving their experimental environments, are extended and modified by application developers, and employed in everyday life by millions of end-users.

The paper presents an analysis of results from pervasive computing, qualitative spatial and temporal reasoning, and logic-based contextual reasoning. The goal was to carve out the theoretical core of fast contextual reasoning reported from experimental context-aware systems, and to discuss how these good properties can be made to scale up. The findings suggest that partial order reasoning is the core of tractable contextual reasoning. Examples illustrate the surprisingly high expressiveness and inferential power, and serve to emphasize the interrelations between tractable reasoning in pervasive computing, qualitative spatial and temporal reasoning, and logic-based contextual reasoning.

Keywords. context, context-awareness, contextual reasoning, context model, granularity

Introduction

Adding reasoning abilities to context-aware systems has been a focus of research in pervasive computing for several years and a broad range of approaches has been suggested. In particular, the well-known trade-off between expressivity and inferential power has been a major concern. Context-aware systems are expected to produce a reaction within milliseconds of detecting a context, they run on light-weight systems such as mobile phones or sensor nodes, and dimensions of context include well-known hard domains, such as spatial, temporal, and causal knowledge. Recent theoretical studies [35,7] consequently suggest that we need to abandon the idea of using an integrated contextual reasoning framework. On the experimental side, however, issues with run-time have rarely been reported, even when using semi-decidable [30] or even undecidable [41] logics.

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Contextual reasoning, as a field within the research area of large-scale knowledge representation and reasoning systems [8,5], studies reasoning mechanisms enabled to reproduce the human capability to separate information that is relevant for a task from irrelevant information. Three types of contextual reasoning can be distinguished [5]: *partial reasoning* focusses on relevant parts, *approximate reasoning* leaves out unnecessary detail, *perspectival reasoning* can move the point of view on a problem.

A wealth of research exists concerning formal logic frameworks for reasoning about spatial and temporal structures, including natural language constructs, such as *here*, *now*, or *yesterday*, whose meaning depends on context [11,28,13,3,1]. Especially, the seminal research by Gabbay [12] should be highlighted here, forming a fundament on which non-classical formal logic and data structures for AI can be compared and made to work together.

Spatial and temporal reasoning has been a focus of research also in the area of constraint satisfaction methods. Qualitative spatial reasoning (QSR) is an active research area [9,10] studying qualitative spatial calculi and heuristics that make reasoning about spatial relations, such as the mereotopological region-connection-calculus (RCC) of [29] or the cardinal direction calculus of [25], tractable. Newer results in this area particularly important for this paper shed light on the complexity for combining calculi [34,17,33,32,27].

The paper aims to carve out the theoretical core of a class of particularly fast contextual reasoning mechanisms and to hint at possible approaches for retaining these properties as systems scale up. The rest of the paper is organized as follows. After a brief introduction of contextual reasoning approaches in context-aware computing, the theoretical framework of partial orders and lattice structures underlying hierarchical contextual reasoning mechanisms is discussed. The versatility of this approach is explored in a section on applications. Finally, the possibilities to realize partial reasoning and reasoning with a limited level of detail are discussed. We conclude that the hierarchical contextual reasoning frameworks proposed in the literature will scale well when context-aware systems enter everyday life.

1. Reasoning in Context-Aware Systems

From the earliest location-aware systems [36] to recent pervasive computing systems [19], automatic reasoning capabilities of the system have been a core component. The key idea of being able to react to a context is an essential component of intelligence. Representations that are *cued* by a perceptual trigger have been argued to have come early in the evolution of intelligence, and seem to require less computational power than *detached* representations that can be manipulated independently from any external stimulus [16]. Similarly, context-aware applications can be designed efficiently for the purpose of immediately triggering or adapting behavior in response to sensory input [19]. The objective of adequate representations for such immediate reactive procedures has been pursued in the field of context modeling.

Among the first and most efficient context-modeling approaches were highly efficient tree-structures [36] and directed acyclic graphs (DAG) [23,43] for representing hierarchies of contexts, such as containment hierarchies of locations or class hierarchies of types of situations, possibly supplemented by additional numerical or coordinate information [23,22,4], which can be processed using numerical constraint processing or value comparison.

On the other end of the spectrum between fast inference and high expressiveness lie logic-based approaches that use standard ontology languages [35,42] or rule-based formalisms [26,31]. The knowledge necessary for operating context-aware systems can be distinguished from that for other computing systems as consisting of particularly many statements about individuals for describing the environment, e.g. locating a certain room in a certain building, a certain sensor in a certain room, etc. Class hierarchies for contextual domains such as space or time, are rather flat, comprising e.g. regions and points as possible classes, however, for handling applications and devices the sub-class relation is relevant.

For domains like space and time, relations, such as the well-known 13 temporal interval relations of [2] or the eight mereotopological relations between regions from [29] are highly relevant and modeled in many existing systems. The use of the language SROIQ for OWL 2 [18], featuring means for specifying relations as transitive, reflexive, irreflexive, symmetric, asymmetric, or mutually disjoint, is therefore an important step for context-aware computing [35]. A recent survey [7], however, emphasizes the growing concern that the limited reasoning performance of expressive, integrated reasoning mechanisms could lead to a lack of scalability. The question then arises whether the more light-weight hierarchy-based approaches face similar concerns, and whether their expressiveness is sufficient to enable interesting applications beyond simple location hierarchies.

In line with these observations, we identified in [39] a system of six partial ordering relations that describe context hierarchies in the hierarchical context model of [21]. In [38], we studied a system of 54 compound spatial relations including mereological relations, cardinal direction relations (such as to the North of, to the North-West of, etc.), and size comparisons (smaller than, same size), and found similarly good reasoning properties in a framework of constraint satisfaction. This paper further explains these findings and illustrates why complexity does not increase, even if arbitrary additional partial ordering relations are allowed.

2. Theoretical Foundations

We briefly outline the basic framework of pre-orders, partial orders, and lattice structures that underlies hierarchical reasoning in a first order language and demonstrate the theoretical foundations of its versatility. Here and elsewhere, we omit brackets, in particular outer brackets, if no ambiguity can arise; however, we enclose all atomic formulae in square brackets to support visual separation between terms and formulae.

2.1. Pre-Orders, Partial Orders and Equivalence Relations

A pre-order is a relation $\sqsubseteq_m$ that is reflexive (A1) and transitive (A2).

$$\forall x : [x \sqsubseteq_m x] \tag{A1}$$

$$\forall x, y, z : [x \sqsubseteq_m y] \wedge [y \sqsubseteq_m z] \rightarrow [x \sqsubseteq_m z] \tag{A2}$$

A pre-order $\sqsubseteq$ (A3), (A4) that is antisymmetric (A5) is called partial order.

$$\forall x : [x \sqsubseteq x] \tag{A3}$$

$$\forall x, y, z : [x \sqsubseteq y] \wedge [y \sqsubseteq z] \rightarrow [x \sqsubseteq z] \tag{A4}$$

$$\forall x, y : [x \sqsubseteq y] \wedge [y \sqsubseteq x] \rightarrow [x = y] \tag{A5}$$

Every pre-order $\sqsubseteq_m$ gives rise to an equivalence relation $=_m$, that is, a pre-order that is symmetric (1).

$$\forall x, y : [x =_m y] \overset{def}{\Leftrightarrow} [x \sqsubseteq_m y] \wedge [y \sqsubseteq_m x] \tag{D1}$$

$$\forall x, y : [x =_m y] \rightarrow [y =_m x] \tag{1}$$

$$\forall x : [x =_m x] \tag{2}$$

$$\forall x, y, z : [x =_m y] \wedge [y =_m z] \rightarrow [x =_m z] \tag{3}$$

$$\forall x, y : [x \sqsubseteq_m y] \wedge [y \sqsubseteq_m x] \rightarrow [x =_m y] \tag{4}$$

Moreover, a pre-order $\sqsubseteq_m$ behaves like a partial order on the equivalence classes of its corresponding equivalence relation$=_m$ (4).

An important property of pre-orders is that they have sub-relations that are also pre-orders. In particular, we can define a pre-order $\sqsubseteq_{m,e}$ from a pre-order $\sqsubseteq_m$ and a given element e as the relation that compares for two elements x and y elements x', which are in $\sqsubseteq_m$ both below e and below x, with y. More formally: $\sqsubseteq_{m,e}$ holds for x and y, iff for all x' that are below both x and e, x' is below y in $\sqsubseteq_m$.

$$\forall x, y : [x \sqsubseteq_{m,e} y] \overset{def}{\Leftrightarrow} \forall x' : [x' \sqsubseteq_m x] \wedge [x' \sqsubseteq_m e] \rightarrow [x' \sqsubseteq_m y] \tag{D2}$$

$$\forall x : [x \sqsubseteq_{m,e} x] \tag{5}$$

$$\forall x, y, z : [x \sqsubseteq_{m,e} y] \wedge [y \sqsubseteq_{m,e} z] \rightarrow [x \sqsubseteq_{m,e} z] \tag{6}$$

To show that $\sqsubseteq_{m,e}$ is a pre-order, we need to show that it is reflexive and transitive. It is clear that $\sqsubseteq_{m,e}$ is reflexive (5), since $\forall x' : [x' \sqsubseteq_m x] \wedge [x' \sqsubseteq_m e] \rightarrow [x' \sqsubseteq_m x]$ is a tautology. For transitivity, assume $[x \sqsubseteq_{m,e} y]$ and $[y \sqsubseteq_{m,e} z]$ hold, and consider any arbitrary x' that is below x and e in $\sqsubseteq_m$. From the assumption $[x \sqsubseteq_{m,e} y]$, we know that x' must be below y in $\sqsubseteq_m$, by transitivity of $\sqsubseteq_m$ and (D2). Any such x' is an element that is both below y and below e. By the assumption $[y \sqsubseteq_{m,e} z]$, we know any such x' must be below z, again by transitivity of $\sqsubseteq_m$ and (D2), and therefore $[x \sqsubseteq_{m,e} z]$.

Since $\sqsubseteq_{m,e}$ is a pre-order, an equivalence relation $\equiv_{m,e}$ can be defined following (D1).

$$\forall x, y : [x \sqsubseteq_m y] \rightarrow [x \sqsubseteq_{m,e} y] \tag{7}$$

We can show that $\sqsubseteq_{m,e}$ extends $\sqsubseteq_m$, that is, it behaves like $\sqsubseteq_m$ for all pairs of elements in $\sqsubseteq_m$ (7). Assume $[x \sqsubseteq_m y]$ holds. With respect to (D2), we only need to check elements x' below x in $\sqsubseteq_m$ that are below e – if there are no such x', $[x \sqsubseteq_{m,e} y]$ holds trivially. For any given such x', it holds by transitivity of $\sqsubseteq_m$ that $[x' \sqsubseteq_m y]$, and thus $[x \sqsubseteq_{m,e} y]$.

It thus follows, that a single pre-order (or partial order relation) $\sqsubseteq$ suffices to give rise to a range of pre-orders as extensions. In particular, this entails that a framework for reasoning with context hierarchies need not encode the dimensions of context into the given logical language. Rather system developers can define required relations as needed using the construction (D2). The widely used hierarchical context modeling approach is more versatile than assumed.

2.2. Lattice Structures

If a partial order $\sqsubseteq$ has a unique *least upper bound* (A6) and *greatest lower bound* (A7) for any two elements x and y, it can be extended into a lattice structure.

$$\forall x,y : \exists z : [x \sqsubseteq z] \wedge [y \sqsubseteq z] \wedge \forall z' : [x \sqsubseteq z'] \wedge [y \sqsubseteq z'] \rightarrow [z \sqsubseteq z'] \tag{A6}$$

$$\forall x,y : \exists z : [z \sqsubseteq x] \wedge [z \sqsubseteq y] \wedge \forall z' : [z' \sqsubseteq x] \wedge [z' \sqsubseteq y] \rightarrow [z' \sqsubseteq z] \tag{A7}$$

A lattice structure for $\sqsubseteq$ comprises two binary operations $\sqcup$ (*join*), yielding the least upper bound (D3) of two elements, and $\sqcap$ (*meet*), yielding the greatest lower bound (D4). Furthermore, we characterize two elements $\top$ (top) and $\bot$ (bottom) as the upper bound and lower bound of $\sqsubseteq$ to obtain a bounded lattice (A8).

$$\forall x,y,z : [z = x \sqcup y] \overset{def}{\Leftrightarrow} [x \sqsubseteq z] \wedge [y \sqsubseteq z] \wedge \forall z' : [x \sqsubseteq z'] \wedge [y \sqsubseteq z'] \rightarrow [z \sqsubseteq z'] \tag{D3}$$

$$\forall x,y,z : [z = x \sqcap y] \overset{def}{\Leftrightarrow} [z \sqsubseteq x] \wedge [z \sqsubseteq y] \wedge \forall z' : [z' \sqsubseteq x] \wedge [z' \sqsubseteq y] \rightarrow [z' \sqsubseteq z] \tag{D4}$$

$$\forall x : [x \sqsubseteq \top] \wedge [\bot \sqsubseteq x] \tag{A8}$$

With the operations $\sqcup$ and $\sqcap$ we can define further notions. The relation $\bigcirc$ (overlap) holds between two elements x and y iff the meet of x and y is not the bottom element. Two elements underlap (U) iff the join of x and y is not the top element.

$$\forall x,y : [x \bigcirc y] \overset{def}{\Leftrightarrow} \neg[x \sqcap y \sqsubseteq \bot] \tag{D5}$$

$$\forall x,y : [x \mathsf{U} y] \overset{def}{\Leftrightarrow} \neg[\top \sqsubseteq x \sqcup y] \tag{D6}$$

2.3. Linearizations

We call a pre-order $\leqslant_m$ *linear extension* or *linearization* of a pre-order $\sqsubseteq_m$, if and only if $\leqslant_m$ extends $\sqsubseteq_m$ (A11) and is linear (A12):

$$\forall x : [x \leqslant_m x] \tag{A9}$$

$$\forall x,y,z : [x \leqslant_m y] \wedge [y \leqslant_m z] \rightarrow [x \leqslant_m z] \tag{A10}$$

$$\forall x,y : [x \sqsubseteq_m y] \rightarrow [x \leqslant_m y] \tag{A11}$$

$$\forall x,y : [x \leqslant_m y] \vee [y \leqslant_m x] \tag{A12}$$

$$\forall x,y : [x \equiv_m y] \overset{def}{\Leftrightarrow} [x \leqslant_m y] \wedge [y \leqslant_m x] \tag{D7}$$

We again obtain a relation $\equiv_m$ (D7), as equivalence relation for $\leqslant_m$.

Table 1. Pre-orders and their linear extensions in QSR

relation system	relation	symbol	context	type
spatial mereology	part-of	$\sqsubseteq_{m_p}$	m_p	pre-order
	smaller-than	$\leqslant_{m_l}$	m_l	linear extension of part-of
cardinal directions	directly-to-the-North-of	$\sqsubseteq_{n_p}$	n_p	pre-order
	more-to-the-North-of	$\leqslant_{n_l}$	n_l	linear extension of directly-to-the-North-of
	directly-to-the-East-of	$\sqsubseteq_{e_p}$	e_p	pre-order (locally[2])
	more-to-the-East-of	$\leqslant_{e_l}$	e_l	linear extension of directly-to-the-East-of
temporal intervals	during-or-equal	$\sqsubseteq_{i_p}$	i_p	pre-order
	shorter-duration-than	$\leqslant_{i_l}$	i_l	linear extension of during-or-equal
temporal order	before-or-same-time (branching time)	$\sqsubseteq_{t_p}$	t_p	pre-order
	before-or-same-time (linear time)	$\leqslant_{t_l}$	t_l	linear extension of branching before-or-same-time
classes	sub-class	$\sqsubseteq_{c_p}$	c_p	pre-order
	smaller-cardinality	$\leqslant_{c_l}$	c_l	linear extension of sub-class

Many relations that are interesting for reasoning are pre-orders, partial orders, strict partial orders, or linearizations. Table 1 lists a range of relations that have been discussed in the area of qualitative reasoning [2,14,6,25,24,29,9,15] and that can be expressed in terms of pre-orders and their linear extensions.

3. Applications

We can now illustrate how the above framework can be used for modeling context hierarchies. We show examples how specific pre-orders $\sqsubseteq_m$ can be derived as extensions of a partial order $\sqsubseteq$ and how linear extensions $\leqslant_m$ can be defined from pre-orders $\sqsubseteq_m$.

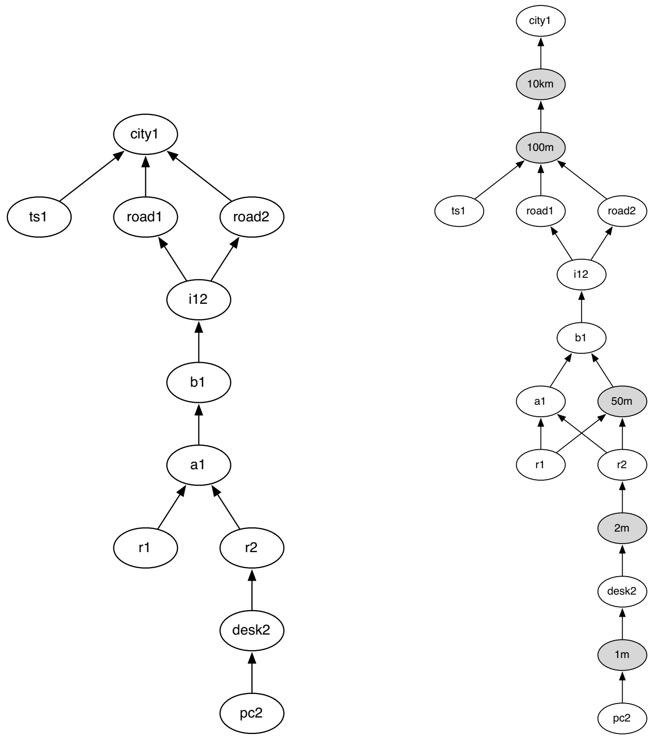
3.1. Modeling Context Hierarchies

We can introduce arbitrary sets of partial ordering relations, such as the six relations of [39], or the detailed modeling of the spatial domain discussed in [38], by deriving extensions $\sqsubseteq_m$ from $\sqsubseteq$ using elements m representing the domains. Here and in the following, we use Greek letters ξ, χ, κ to indicate schema variables ranging over contexts.

$$[\xi \sqsubseteq_\kappa \chi] \overset{def}{\Leftrightarrow} [\xi \sqcap \kappa \sqsubseteq \chi] \tag{D8}$$

$$[\xi \bigcirc_\kappa \chi] \overset{def}{\Leftrightarrow} \neg[\xi \sqcap \chi \sqsubseteq_\kappa \bot] \tag{D9}$$

$$[\xi =_\kappa \chi] \overset{def}{\Leftrightarrow} [\xi \sqsubseteq_\kappa \chi] \wedge [\chi \sqsubseteq_\kappa \xi] \tag{D10}$$



(a) (b)

Figure 1. A location hierarchy (a) and size hierarchy (b) specifying the locations (a: relation part-of) and respective sizes (b: relation smaller-than) of a PC (pc2) on a desk (desk2) in a room (r2). In turn, r2 and room r1 are in an office space (a1) in a building (b1) on the intersection i12 of roads road1 and road2. The roads road1 and road2 and the train station ts1 are in a city city1. Arrows indicate the relation described by a spatial containment relation $\sqsubseteq_{m_p}$, not the relation $\sqsubseteq$ itself. The hierarchy of the sizes of locations $\leqslant_{m_p}$ in (b) was obtained by extending the location hierarchy of (a) with additional information about sizes.

Any relation $\sqsubseteq_\kappa$ thus gives rise to relations $\bigcirc_\kappa$ (D9) and $=_\kappa$ (D10).

In general, we retain as a consequence of (D8), that for arbitrary x and y: if x is an m-sub-context of y and m' is a sub-context of m, then x is also an m'-sub-context of y.

$$\forall x, y, m, m' : [x \sqsubseteq_m y] \land [m' \sqsubseteq m] \rightarrow [x \sqsubseteq_{m'} y] \tag{8}$$

Hierarchical context models store knowledge given in the form $[a \sqsubseteq_m b]$ in hierarchical data structures that facilitate transitive inference. An example is shown in Figure 1a.

3.2. Modeling Linearizations in Context Hierarchies

Using sub-relations as in (8), we can also define linear relations $\leqslant_m$ that extend partial order relations $\sqsubseteq_m$. However, the linearity constraint (A12) demands that a complete linearization exists, which would in practice be hard to enforce. Moreover, in order to infer something from the linearity constraint a disjunction has to be checked. In the worst case, a complete linearization would have to be generated, with every pair of contexts, that is n^2 disjunctions, to be checked. This would lead to an explosion of computational effort that might only rarely be useful.

Similarly as in [38], we will therefore neglect the disjunctive restriction with respect to reasoning. The argument for this is practical rather than formal: linearizations, such as the size constraints, facilitate reasoning. Following this argument, we can assume that a benefit encourages the reasoning agent to collect this information so that linearity of the $\leqslant_m$ orders is approximated but not required.

We define the following schema exemplarily for the pair of the mereological pre-order $\sqsubseteq_{m_p}$ (part-of) and a linear extension $\leqslant_{m_l}$ (smaller-than) indicated by contexts m_p and m_l so that $\leqslant_{m_l}$ is an extension of $\sqsubseteq_{m_p}$ (D13):

$$[\xi \sqsubseteq_{m_p} \chi] \overset{def}{\Leftrightarrow} [\xi \sqcap m_p \sqsubseteq \chi] \tag{D11}$$

$$[\xi \leqslant_{m_l} \chi] \overset{def}{\Leftrightarrow} [\xi \sqcap m_l \sqsubseteq \chi] \tag{D12}$$

$$[m_p \sqsubseteq m_l] \tag{D13}$$

$$[\xi \equiv_{m_l} \chi] \overset{def}{\Leftrightarrow} [\xi \leqslant_{m_l} \chi] \wedge [\chi \leqslant_{m_l} \xi] \tag{D14}$$

Equivalence with respect to $\leqslant_{m_l}$ is expressed with $\equiv_{m_l}$ (D14). Like knowledge given in the form $[a \sqsubseteq_m b]$, knowledge of the type $[a \leqslant_m b]$ can be stored in hierarchical data structures that facilitate transitive inference. An example is shown in Fig. 1b. The hierarchy for $[a \leqslant_m b]$ can be obtained from that of $[a \sqsubseteq_m b]$ by adding size information. In actual applications, linear ordering can also be encoded numerically.

Large relation systems, such as the 54 combined spatial relations introduced in [38], can then be defined. There, the three spatial pre-orders listed in Table 1, each with linearizations and equivalence relations are used to represent, and reason about, spatial layouts. Relations, such as north-western part ($x\, M_P N_L E_L\, y$), can be defined as combinations of relations. We obtain the inferences from [38] as valid schemata, for instance (9).

$$[\xi\; M_P N_L E_L\; \chi] \overset{def}{\Leftrightarrow} [\xi \sqsubseteq_{m_p} \chi] \wedge [\xi \leqslant_{n_l} \chi] \wedge [\xi \leqslant_{e_l} \chi] \tag{D15}$$

$$[\xi\; M_P N_Q E_L\; \chi] \overset{def}{\Leftrightarrow} [\xi \sqsubseteq_{m_p} \chi] \wedge [\xi \equiv_{n_l} \chi] \wedge [\xi \leqslant_{e_l} \chi] \tag{D16}$$

$$[a\; M_P N_L E_L\; b] \wedge [b\; M_P N_Q E_L\; c] \rightarrow [a\; M_P N_L E_L\; c] \tag{9}$$

Class hierarchies are an important tool for inferring and specifying properties of types of contexts in a generic way. Since the subclass relation is a partial order over the domain of classes, it can be defined as a pre-order $\sqsubseteq_{c_p}$. This allows interesting constructions: for instance, we can define that a context is a class of locations iff it is only in the spatial and sub-class domain and actually overlaps the spatial domain, i.e. has a non-empty spatial extent (D17). If we want to ensure that locations are always given the corresponding type, we can define a relation *subLoc* as an external interface for the user, hiding the spatial mereological $\sqsubseteq_{m_p}$ and sub-class $\sqsubseteq_{c_p}$ as internal operators.

$$[\xi \sqsubseteq_{c_p} \mathit{location}] \overset{def}{\Leftrightarrow} [\xi \sqsubseteq m_p \sqcup c_p] \wedge [\xi \bigcirc m_p] \tag{D17}$$

$$[\xi\; \mathit{subLoc}\; \chi] \overset{def}{\Leftrightarrow} [\xi \sqsubseteq_{m_p} \chi] \wedge [\xi \sqsubseteq_{c_p} \mathit{location}] \tag{D18}$$

Using this, a developer can enter that road intersections are sub-locations of roads (D19). Applying (D18), this would be expanded to mean: the sub-class relation holds between *roadIntersection* and *location* and that the spatial containment relation holds between between *roadIntersection* and *road* (10).

$$[\mathit{roadIntersection\ subLoc\ road}] \tag{D19}$$

$$[\mathit{roadIntersection} \sqsubseteq_{m_p} \mathit{road}] \wedge [\mathit{roadIntersection} \sqsubseteq_{c_p} \mathit{location}] \tag{10}$$

As contexts in this framework are portions of reality not individuals, we have defined classes of locations as classes and as locations.

The is-a-relation between an individual and a class is usually treated in a different manner than the sub-class relation. We can identify individuals as classes of cardinality 1 (D20), based on the comparison of cardinality as a linearization $\leqslant_{c_l}$ of the sub-class relation $\sqsubseteq_{c_p}$.

$$[\xi\ \mathit{instanceOf}\ \chi] \overset{def}{\Leftrightarrow} [\xi \sqsubseteq_{c_p} \chi] \wedge [\xi \equiv_{c_p} \mathit{card1}] \tag{D20}$$

$$[\mathit{roadIntersection12\ instanceOf\ roadIntersection}] \tag{D21}$$

With this definition we can specify that *roadIntersection12* is a sub-class of *roadIntersection* with cardinality 1 (D21).

4. Partial and Approximate Reasoning

Linearizations, ordering the whole domain, are a suitable tool for stratifying a domain into levels of granularity and thus for enforcing that reasoning does not go beyond a given level of detail for a certain task. This holds true even if the total ordering of a domain cannot be enforced (cf. Figure 1b). Size information, even if incomplete, can be used to delimit reasoning and allow for approximate representation.

Size-based granularity is a representational tool that can be used to handle differences in sizes and to model uncertainty [37]. The key idea is to formally specify what is a small, and therefore irrelevant, detail in a context [40]. Size-based granularity is built upon the concept of a grain-size for a context region: parts of an object that are smaller than the grain-size can be disregarded as unimportant details in the context. This concept forms the basis for level of detail reasoning or *approximate reasoning* in the terminology of [5]. Similarly, parts of an object outside the context region are irrelevant. This concept forms the basis for realizing *partial reasoning*[5] in context-aware systems.

We can realize both types of restricted reasoning in the framework. Instead of asking whether $CKB \cup \{\neg\phi(ans)\}$ is inconsistent for a contextual knowledge base CKB and query $\phi(ans)$, we can instead ask whether

$$CKB \cup \{[g_m \leqslant_{ml} ans], [ans \sqsubseteq_{m_p} c]\} \cup \{\neg\phi(ans)\},$$

is inconsistent, that is: whether query $\phi(ans)$ can be solved for *ans* larger than *g* for $\leqslant_{ml}$ and below context *c* for $\sqsubseteq_{m_p}$. In practice, this can be implemented directly into the reasoning mechanism. In a graph-based representation of the context hierarchy, for instance, by pruning the hierarchy below the point g_m and above the point c. The algorithm reasoning about $\phi(ans)$ is then limited to the resulting hierarchy between g_m and c. It follows that additions outside of this local hierarchy would not impede the reasoning process inside, and that growing hierarchical context models can retain their good runtime, a step towards realizing the idea of granularity as advocated by Hobbs [20], that is, towards a tool to make reasoning and representation tractable and scalable.

5. Outlook and Conclusion

Adding reasoning abilities to context-aware systems has been a focus of research in pervasive computing for several years and a broad range of approaches has been suggested. The well-known trade-off between expressivity and inferential power has raised considerable concerns in this area, as dimensions of context include well-known hard domains, such as spatial, temporal, and causal knowledge. We suggested that hierarchical context models, which have been used in numerous practical applications, can be described by pre-order relations and their linearizations. We showed that this framework, for which fast reasoning procedures exist, is expressive, and proposed a mechanism for limiting level of detail and relevant context.

We conjecture that the proposed means can ensure that reasoning with hierarchical context models will scale well. Future works need to include experimental validation especially regarding large-scale context-aware systems. Moreover, heuristics for calculating appropriate level detail and context of reasoning need to be determined.

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