

Application of a Fuzzy Model of Making a Decision to Choose a Medicine in the Case of Symptoms Prevailing after the Treatment

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Abstract. Fuzzy set theory has introduced many auxiliary methods into medical problems. One of the attempts was the fixing of the optimal level of the drug action in the case when clinical symptoms retreat completely after the treatment [1,5]. In many morbid processes, however, there occurs problem of symptoms prevailing to some extent after the course of the medication process improving too high or too low indices of the measurable symptom. Different medicines applied have many a time an effect on the same symptoms and it is sometimes difficult to choose uniquely that therapeutic remedy which brings the best results in the treatment of all the symptoms typical of the given diagnosis. A fuzzy model of decision making is to make it easier to choose a drug which acts optimally in the case of symptoms not retreating to the full after the treatment.

1. Presentation of the Problem

Assume that the patient's state S is described in the form of a fuzzy set

$$S = \mu_S(x_1)/x_1 + \mu_S(x_2)/x_2 + \dots + \mu_S(x_m)/x_m$$

where $\{x_1, x_2, \dots, x_m\}$ is a set of symptoms belonging to X .

In order to watch the process of taking a decision in the case of problem of drugs presented above, we consider a set $A = \{a_1, a_2, \dots, a_m\}$ of drugs which may be treated as decisions $a_1, a_2, \dots, a_m \in A$ with a view to have an effect on the symptoms representing certain states, characteristic of given morbid unit. Let us assume that each symptom $x_j \in X$ is understood as the result of the treatment of this symptom after the cure with the drugs $a_i \in A$, $i = 1, 2, \dots, n$ has been carried out. On the basis of earlier experiments, the physician knows how to define in words the effectiveness of the cure of these symptoms, which we shall describe in terms of the linguistic variable "the effectiveness of the cure of symptom" -- $\{\text{none, almost none, very little, little, medium, large, very large, almost complete, complete}\}$. Each notion from the list of terms of the linguistic variable is the name of a fuzzy variable. Assume that all the fuzzy variables are defined on the space $Z = [0,1]$ where the intervals of Z essential to these variables can be presented in a table.

Let us define a utility function U as a function $U : A \rightarrow [0, 1]$ such that " a_i is preferred to a_j , $i, j = 1, 2, \dots, n$, if and only $U(a_i) > U(a_j)$ ". The values of the utility function U are equal to the midpoints of the presented intervals, representing the verbal definitions. If we take a decision $a_i \in A$, $i = 1, 2, \dots, n$, concerning states (results) $x_j \in X$,

$j = 1, 2, \dots, m$, then the problem is reduced to the consideration of the triplet (X, A, U) . The utility matrix $U = X \times A$ expresses a relationship between the drug (decision) a_i and the effectiveness of the retreat of the symptom x_j . In the matrix U each element u_{ij} , $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$, is a value of the utility function, defining the utility following from the decision a_i with the result x_j .

The utility of matrix U is built by the physician basing himself on his experience. The matrix U can take, for instance, such a hypothetical form:

$$U = \begin{bmatrix} \text{medium} & \text{very large} & \text{medium} & \text{large} & \text{large} & \text{medium} \\ \text{medium} & \text{very large} & \text{large} & \text{large} & \text{almost complete} & \text{medium} \\ \text{large} & \text{complete} & \text{large} & \text{large} & \text{almost complete} & \text{large} \end{bmatrix}$$

After substituting the numerical representatives, one can write the matrix U down as

$$U = \begin{bmatrix} 0.5 & 0.85 & 0.5 & 0.7 & 0.7 & 0.5 \\ 0.5 & 0.85 & 0.7 & 0.7 & 0.95 & 0.5 \\ 0.7 & 1.0 & 0.7 & 0.7 & 0.95 & 0.7 \end{bmatrix}$$

The fuzzy utility [2,3] for each decision (drug) a_i , $i = 1, 2, \dots, n$, with the patient's fuzzy state $S \in X$ characterized by means of the membership function $\mu_s(x)$ is defined to be the fuzzy set

$$U_i = \mu_s(x_1)/u_{i1} + \mu_s(x_2)/u_{i2} + \dots + \mu_s(x_m)/u_{im}, \quad i = 1, 2, \dots, n.$$

2. Solution of the Problem

The problem of choosing an optimal decision is solved as follows [2,3]:

- 1) We form a non-fuzzy set Y which is the sum of the supports of the sets $U_1, \dots, U_n$.
- 2) We choose the maximal element of the set Y .
- 3) We define the fuzzy sets U'_i :

$$U'_i = \mu_{U'_i}(u_{i1})/u_{i1} + \mu_{U'_i}(u_{i2})/u_{i2} + \dots + \mu_{U'_i}(u_{im})/u_{im}$$

in which $\mu_{U'_i}(u_{ij}) = u_{ij}/u_{\max}$, $i = 1, 2, \dots, n$; $j = 1, 2, \dots, m$.

- 4) The next introduced fuzzy set has the form:

$$U_{i0} = \mu_{U_{i0}}(u_{i1})/u_{i1} + \mu_{U_{i0}}(u_{i2})/u_{i2} + \dots + \mu_{U_{i0}}(u_{im})/u_{im}$$

where $i = 1, 2, \dots, n$, and membership degree $\mu_{U_{i0}}(u_{ij})$ is calculated according to the formula

$$\mu_{U_{i0}}(u_{ij}) = \min(\mu_{U_i}(u_{ij}), \mu_{U'_i}(u_{ij}))$$

5) The fuzzy set A^* with the elements $a_1, a_2, \dots, a_n$ of the supports is assumed to be

$$A^* = \mu_{A^*}(a_1) / a_1 + \mu_{A^*}(a_2) / a_2 + \dots + \mu_{A^*}(a_n) / a_n$$

where $\mu_{A^*}(a_i) = \max(\mu_{U_{ij}}(u_{ij}))$, $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$.

Now taking maximum in the set A^* , we ascertain the optimal decision and, consequently, the application of the determined drug should yield the best effects in the process of the retreating of the symptoms.

References

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