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Integrating Bipolar Fuzzy Mathematical Morphology in Description Logics for Spatial Reasoning

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Abstract. Bipolarity is an important feature of spatial information, involved in the expression of preferences and constraints about spatial positioning or in pairs of *opposite* spatial relations such as left and right. Another important feature is imprecision which has to be taken into account to model vagueness, inherent to many spatial relations (as for instance vague expressions such as *close to*, *to the right of*), and to gain in robustness in the representations. In previous works, we have shown that fuzzy sets and fuzzy mathematical morphology are appropriate frameworks, on the one hand, to represent bipolarity and imprecision of spatial relations and, on the other hand, to combine qualitative and quantitative reasoning in description logics extended with fuzzy concrete domains. The purpose of this paper is to integrate the bipolarity feature in the latter logical framework based on bipolar and fuzzy mathematical morphology and description logics with fuzzy concrete domains. Two important issues are addressed in this paper: the modeling of the bipolarity of spatial relations at the terminological level and the integration of bipolar notions in fuzzy description logics. At last, we illustrate the potential of the proposed formalism for spatial reasoning on a simple example in brain imaging.

1 Introduction

In image interpretation and computer vision, spatial relations between objects and spatial reasoning are of prime importance. Nevertheless, although spatial reasoning has been largely studied in artificial intelligence mainly using qualitative representations based on logical formalisms, there is still a gap with the quantitative representations used in image interpretation and computer vision. Description logics (DL) with concrete domains [22] are a widely accepted way to integrate *concrete and quantitative qualities* of real world objects with conceptual knowledge and as a consequence to combine qualitative and quantitative reasoning useful for real-world applications. In [19], extending our previous work on a fuzzy spatial ontology operational for image interpretation [20], we proposed to merge the mathematical morphology setting with description logics with fuzzy concrete domains. To our knowledge, this association of two frameworks developed in two different communities was novel and mathematical morphology was never exploited in this context before. The resulting framework enabled to provide new mechanisms to derive useful concrete representations of concepts and new qualitative and quantitative reasoning tools. Moreover, it also enables to take into account imprecision to model vagueness, inherent to many spatial relations and to gain in robustness in the representations [7]. In this paper, we consider both imprecision and bipolarity which are two important features of spatial information. Indeed, bipolarity is important to distinguish between (i) positive information, which represents what is guaranteed to be possible, for instance because it has already been observed or experienced, and (ii) negative information, which represents what is impossible or forbidden, or surely false [15]. The intersection of the positive information and the negative information has to be empty in order to achieve consistency to the representation, and their union does not necessarily cover the whole underlying space. To our knowledge, bipolarity, which has motivated works in several directions and many domains, has not been much exploited in the spatial domain. Nevertheless, bipolarity implicitly occurs when dealing with spatial information (see Section 3). In [8], it has been shown that mathematical morphology, extended to the case of bipolar fuzzy sets, is a useful formalism to manage spatial bipolar information. In this paper, we first briefly review works related to bipolarity and imprecision, particularly for spatial information (Section 2). Then, we propose both (i) to model explicitly the bipolarity of spatial relations in our spatial relation ontology (Section 3) and (ii) to add the bipolarity feature to description logics with fuzzy concrete domains using bipolar mathematical morphology (Section 4). The resulting framework enables to provide new mechanisms to derive useful bipolar concrete representations of concepts and new qualitative and quantitative reasoning tools to manage bipolar spatial information. The potential of the proposed formalism for spatial reasoning is illustrated on a simple example in brain imaging (Section 5).

2 Related works and background

Bipolarity. Bipolarity is important to distinguish between positive information and negative information [15]. This domain has recently motivated work in several directions, for instance for applications in knowledge representation, preference modeling, argumentation, multi-criteria decision analysis, cooperative games, among others [1, 23, 6, 17, 16]. In particular, fuzzy and possibilistic formalisms for bipolar information have been proposed [15, 5]. To the best of our knowledge, some extensions of description logics have been also proposed to handle preferences [24] but these approaches often deal with a unique scale of preferences and do not handle the bipolar nature of information. Bipolarity has not been much exploited in the spatial domain either. A few works used intuitionistic fuzzy sets or interval valued fuzzy sets [2, 12] but not really asymmetric bipolarity. As shown in [8], mathematical morphology operations on bipolar fuzzy sets is a useful formalism to manage spatial bipolar information together with its imprecision. Let $\mathcal{S}$ be the underlying space (the spatial domain for spatial information processing), that is supposed to be bounded and finite. A bipolar fuzzy set on $\mathcal{S}$ is defined by a

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pair of functions (μ, ν) such that $\forall x \in \mathcal{S}, \mu(x) + \nu(x) \leq 1$. For each point x, $\mu(x)$ defines the membership degree of x (positive information) and $\nu(x)$ its non-membership degree (negative information), while $1 - \mu(x) - \nu(x)$ encodes a degree of neutrality, indifference or indetermination. This formalism allows representing both bipolarity and fuzziness. Concerning semantics, it should be noted that a bipolar fuzzy set does not necessarily represent one physical object or spatial entity, but rather more complex information, potentially issued from different sources (called asymmetric bipolarity in [15]). Let us consider the set $\mathcal{L}$ of pairs of numbers (a, b) in $[0, 1]$ such that $a + b \leq 1$. It is a complete lattice, for the partial order defined as [13]: $(a_1, b_1) \preceq (a_2, b_2)$ iff $a_1 \leq a_2$ and $b_1 \geq b_2$ (Pareto ordering). The greatest element is $(1, 0)$ and the smallest element is $(0, 1)$. The supremum and infimum are respectively defined as: $(a_1, b_1) \vee (a_2, b_2) = (\max(a_1, a_2), \min(b_1, b_2))$, $(a_1, b_1) \wedge (a_2, b_2) = (\min(a_1, a_2), \max(b_1, b_2))$. The partial order $\preceq$ induces a partial order on the set of bipolar fuzzy sets: $(\mu_1, \nu_1) \preceq (\mu_2, \nu_2)$ iff $\forall x \in \mathcal{S}, \mu_1(x) \leq \mu_2(x)$ and $\nu_1(x) \geq \nu_2(x)$, and infimum and supremum are defined accordingly. It follows that, if $\mathcal{B}$ denotes the set of bipolar fuzzy sets on $\mathcal{S}$, $(\mathcal{B}, \preceq)$ is a complete lattice and hence the appropriate framework for defining bipolar fuzzy mathematical morphology operators [8]. In Section 4, we propose to integrate bipolar fuzzy sets and mathematical morphology into description logics extended by fuzzy concrete domains in order to combine both qualitative and quantitative spatial reasoning.

Spatial relations. In [20], we proposed an ontology of spatial relations (in DL) whose main concepts are represented in Figure 1. This ontology is intended to guide image interpretation and the recognition of the structures it contains using structural information on the spatial arrangement of these structures. An important feature of our ontology is that we have used a process of reification of spatial relations to carry the double nature of spatial relations, i.e. concepts with their own properties but also links between concepts. A spatial relation is not considered in our ontology as a role (property) between two spatial objects but as a concept on its own (***SpatialRelation***). Another important concept of our ontology is the concept ***SpatialObject***.

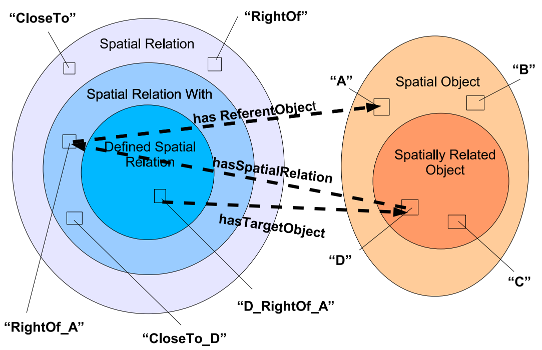

Figure 1. Representation of the main concepts of the spatial relation ontology, as a Venn diagram.

Another important feature of this ontology is that it is enriched by fuzzy representations of concepts, which define their semantics, and allow establishing the link between these concepts (which are often expressed in linguistic terms) and the information that can be extracted from images. We make use of fuzzy concrete domains towards this aim. Indeed, fuzzy representations of spatial relations combine two important features: (i) modeling of imprecision using fuzzy sets, and (ii) computation through mathematical morphology operators. Due to limited size of the paper, we do not recall mathematical models of spatial relations based on fuzzy mathematical morphology but the reader can find an overview in [7]. Contrary to [28] which proposes a spatial fuzzy description logics to reason on both RCC (Region Connection Calculus) relations and directional relations, we consider only fuzzy relations in the concrete domain (i.e. the image space). Moreover, our work is focused on the integration of fuzzy mathematical morphology in DL in order to guide scene segmentation and recognition in images which is quite different from the task of image classification illustrated in [28].

3 Bipolarity of spatial information

In this section, we propose an approach to model the bipolarity of spatial information and spatial relations at the terminological level. Indeed, as explained in the introduction, bipolarity implicitly occurs when dealing with spatial information. For instance, the position of a spatial object in the space can be evaluated in terms of *positive* (e.g. set of possible places) and *negative* (e.g. set of forbidden places which are already occupied by other objects) aspects. A semantics of *constraints* (whose negation defines what is forbidden or unacceptable) and *preferences* (which represent what is satisfactory or what is desired) also expresses bipolarity. For instance, in a brain imaging application aiming at finding the right thalamus, constraints can be all the image space corresponding to the left hemisphere and known objects, and preferences can be the spatial locations derived from known anatomical structures having spatial relations with the right thalamus. Moreover, the positive and the negative part of a concept can be issued from different sources. For instance in the case of medical images, negative information could represent anatomical constraints, which have always to be satisfied, while positive information could represent what is actually seen in the images, for a specific case. Another issue concerns the modeling of the bipolarity of spatial relations, which has not been addressed so far. Indeed, spatial relations often go by pairs. For instance, we often consider *left* and *right* as opposite relations while they are not the contrary of each other. The semantics of *opposite* captures a notion of symmetry (with respect to some axis or plane) rather than a strict complementation. In particular, there may be positions which are considered neither to the right nor to the left of some reference object, thus leaving room for some indifference or neutrality. This corresponds to the idea that the union of positive and negative information does not cover all the space.

Modeling bipolarity in description logics. Hereafter, we assume the reader be familiar with DL syntax and semantics and usual notations are used. The reader can point to [3] for a complete overview. We define a **bipolar spatial concept** C_{bip} as a pair (Pos_C, Neg_C) where Pos_C and Neg_C are concepts that respectively represent preferences and constraints related to the spatial object C. A bipolar concept (Pos, Neg) is interpreted by: $\{Pos^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}, Neg^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}, Pos^{\mathcal{I}} \cap Neg^{\mathcal{I}} = \emptyset^{\mathcal{I}}\}$, expressing that what is possible or preferred (positive information) should be included in what is not forbidden (negative information) [15]. Note that we consider asymmetric bipolarity as defined in [17]. Thus, duality is not required between the positive and the negative part of a concept. Given a spatial knowledge base $\mathcal{K} = \langle \mathcal{T}, \mathcal{A} \rangle$ composed of a terminology $\mathcal{T}$ (i.e. Tbox) and assertions about individuals $\mathcal{A}$ (i.e. Abox), to satisfy the bipolar concept C with respect to $\mathcal{K}$ means to **strictly** satisfy $\neg Neg_C$ and to satisfy Pos_C **as much as possible**. We have the following important constructs :

- $C_{bip} \sqcap D_{bip} \equiv (Pos_C \sqcap Pos_D, Neg_C \sqcup Neg_D)$ interpreted as $\{Pos_C^{\mathcal{I}} \cap Pos_D^{\mathcal{I}}, Neg_C^{\mathcal{I}} \cup Neg_D^{\mathcal{I}}, (Pos_C^{\mathcal{I}} \cap Pos_D^{\mathcal{I}}) \cap (Neg_C^{\mathcal{I}} \cup Neg_D^{\mathcal{I}}) = \emptyset^{\mathcal{I}}\}$
- $C_{bip} \sqcup D_{bip} \equiv (Pos_C \sqcup Pos_D, Neg_C \sqcap Neg_D)$ interpreted as $\{Pos_C^{\mathcal{I}} \cup Pos_D^{\mathcal{I}}, Neg_C^{\mathcal{I}} \cap Neg_D^{\mathcal{I}}, (Pos_C^{\mathcal{I}} \cup Pos_D^{\mathcal{I}}) \cap (Neg_C^{\mathcal{I}} \cap Neg_D^{\mathcal{I}}) = \emptyset^{\mathcal{I}}\}$
- $\exists r.C_{bip} \equiv (\exists r.Pos_C, \exists r.Neg_C)$ interpreted as $\{x \in \Delta^{\mathcal{I}} \mid \exists y_+, y_- \in \Delta^{\mathcal{I}} : (x, y_+) \in R^{\mathcal{I}} \wedge y_+ \in Pos_C^{\mathcal{I}}, (x, y_-) \in R^{\mathcal{I}} \wedge y_- \in \Delta^{\mathcal{I}} \setminus Neg_C^{\mathcal{I}}\}$
- $\forall r.C_{bip} \equiv (\forall r.Pos_C, \forall r.Neg_C)$ interpreted as $\{x \in \Delta^{\mathcal{I}} \mid \forall y_+, y_- \in \Delta^{\mathcal{I}} : (x, y_+) \in R^{\mathcal{I}} \Rightarrow y_+ \in Pos_C^{\mathcal{I}}, (x, y_-) \in R^{\mathcal{I}} \Rightarrow y_- \in \Delta^{\mathcal{I}} \setminus Neg_C^{\mathcal{I}}\}$
- $C_{bip} \sqsubseteq D_{bip}$ interpreted as $\{Pos_C^{\mathcal{I}} \subseteq Pos_D^{\mathcal{I}}, Neg_D^{\mathcal{I}} \subseteq Neg_C^{\mathcal{I}}\}$

We should note that the underlying ordering is the Pareto ordering as defined in Section 2. Hence, we consider that the main concepts of our ontology are bipolar concepts.

Bipolar spatial relations. According to the definition of a bipolar concept in description logics, we propose a representation of several spatial relations with a bipolar perspective, which can be very useful in many applications. For instance, we will see in Section 5, that whereas we have a positive information related to the position of an anatomical structure (e.g. *to the left of the right ventricle*), we also want to restrict the search to the right hemisphere of the brain without being too strict. In our framework the forbidden area is automatically defined by the opposite relation (i.e. *to the right of*) but other constraints could be modeled as well. Our model includes distance, directional and topological relations, represented by bipolar concepts such as *(Close To, Very Far From)*, *(Right Of, Left Of)*, *(Top Of, Bottom Of)*, (*Below*, *In Front Of*), *(In a direction* α*, In a direction* $\alpha + \pi$*)*, (*In the interior of*, *Exterior to*).

4 Bipolar and morphological fuzzy description logics

The main objective of this section is to provide a foundation to reason about qualitative and quantitative bipolar spatial information using mathematical morphology operators for defining a specific description logic with bipolar fuzzy concrete domains. These operators provide concrete tools for modifying positive and negative information, for instance to reduce constraints, to extend preferences in order to reach a consensus in a group, or to model spatial relations for exploring an image.

4.1 Description of the formalism

The proposed framework is based on extensions of the basic description logics incorporating concrete domains $\mathcal{ALC}(\mathcal{D})$ [22] and enabling the management of uncertainty and vagueness [21]. In particular, we integrate **bipolar fuzzy concrete information** into description logic concepts using bipolar fuzzy concrete domains. We first briefly recall the definition of concrete domains and we introduce their use in description logics.

Definition [27] A *concrete domain* D is a pair (Δ_D, Φ_D) where Δ_D is a set and Φ_D a set of predicates names on Δ_D. A *fuzzy concrete domain* is a pair (Δ_D, Φ_D), where Δ_D is an interpretation domain and Φ_D a set of fuzzy predicates d with a predefined arity n and an interpretation $d^D : \Delta_D^n \rightarrow [0, 1]$, which is a $n-$ary fuzzy relation over Δ_D.

Role and concept terms. Let C, R_a, R_c, I_a, I_c be non empty and pair-wise disjoint sets of *concept names* (A), *abstract role names* (R), *concrete role names* (T), *abstract individual names* (a), *concrete individual names* (c). R_a also contains a non-empty set F_a of *abstract features names* (r) and R_c contains a non-empty set F_c of *concrete features names* (t). These features are functional roles. A composition of features (denoted $f_1, f_2, ...$) is called a feature chain. In addition to the basic concept and term constructors of DL, we have the following constructs with $P \in \Phi_D$ a predicate name with an arity n and $u_1, ..., u_n, v_1, ..., v_m$ are features chains:

• Predicate exists restriction: $\exists u_1, ..., u_n.P$ interpreted by : $\{a \in \Delta^{\mathcal{I}} \mid \exists x_1, ...x_n \in \Delta^{\mathcal{D}} : (u_1^{\mathcal{I}}(a) = x_1) \wedge ... \wedge (u_n^{\mathcal{I}}(a) = x_n) \wedge (x_1, ...x_n) \in P^{\mathcal{D}}\}$;

• Role forming predicate restriction [18]:
$\exists (u_1, ..., u_n)(v_1, ..., v_m).P$ interpreted by:
$\{(x, y) \in \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}} \mid \exists r_1, \ldots, r_n, s_1, \ldots, s_m \in \Delta^{\mathcal{D}} :$
$u_1^{\mathcal{I}}(x) = r_1, \ldots, u_n^{\mathcal{I}}(x) = r_n, v_1^{\mathcal{I}}(y) = s_1, \ldots, v_m^{\mathcal{I}}(y) = s_m$
and $(r_1, \ldots, r_n, s_1, \ldots, s_m) \in P^{\mathcal{D}}\}$.

Terminology and assertions. A Tbox $\mathcal{T}$ is a finite set of terminological axioms ($A \doteq D$ and $A \sqsubseteq D$) and an Abox $\mathcal{A}$ is an finite set of assertions ($a : C$ (concept membership), $(a, b) : R$ (role filler), $(a, x) : f$ (feature filler) and $(x_1, ..., x_n) : P$ (concrete domain predicate membership)).

In our framework, we instantiate the description logics $\mathcal{ALC}(\mathcal{D})$ with the concrete domain $S = (\Delta_{\mathcal{S}}, \Phi_{\mathcal{S}})$. $\Delta_{\mathcal{S}} = \mathcal{S}$ is a 3D space (the image space) where bipolar fuzzy concrete objects are defined. $\mathcal{S}$ is typically $\mathbb{Z}^2$ or $\mathbb{Z}^3$ for 2D or 3D images. Let $\mathcal{B}$ the set of bipolar fuzzy sets defined over the spatial domain $\mathcal{S}$. In our framework, $\Phi_{\mathcal{S}}$ contains:

- The unary predicates $\perp_S$ and $\top_S$ denoting ($\emptyset$, Δ_S) and (Δ_S,$\emptyset$).
- The names of two unary fuzzy predicates μ and ν which associate to a bipolar spatial object concept X and to a bipolar spatial relation concept R the interpretations (μ_X, ν_X) and (μ_R, ν_R) in $\mathcal{S}$. For each point $x \in \mathcal{S}$, $\mu_X(x)$ represents the degree to which x belongs to the spatial representation of the object X in the image (positive information) and $\nu_X(x)$ represents its non-membership degree to the spatial representation of X. (μ_R, ν_R) represents the bipolar fuzzy structuring element defined on $\mathcal{S}$ which represents the bipolar fuzzy spatial relation R in the image space.
- The names of two binary fuzzy predicates δ and ε with $\delta_{(\mu_R, \nu_R)}^{(\mu_X, \nu_X)}$ the fuzzy bipolar dilation and $\varepsilon_{(\mu_R, \nu_R)}^{(\mu_X, \nu_X)}$ the fuzzy bipolar erosion of the bipolar spatial fuzzy set (μ_X, ν_X) by the bipolar structuring element (μ_R, ν_R). Possible definitions of the fuzzy bipolar dilation and erosion [8] are, $\forall x \in \mathcal{S}$:
  - $\varepsilon_{(\mu_R, \nu_R)}(\mu_X, \nu_X)(x) = \varepsilon_{(\mu_R, \nu_R)}^{(\mu_X, \nu_X)}(x) = \Big(\inf_{y \in \mathcal{S}} T\big(\nu_R(y-x), \mu_X(y)\big), \sup_{y \in \mathcal{S}} t\big(\mu_R(y-x), \nu_X(y)\big)\Big)$ where T is a t-conorm (fuzzy union) and t is a t-norm (fuzzy intersection) [14].
  - $\delta_{(\mu_R, \nu_R)}(\mu_X, \nu_X)(x) = \delta_{(\mu_R, \nu_R)}^{(\mu_X, \nu_X)}(x) = \Big(\sup_{y \in \mathcal{S}} t\big(\mu_R(x-y), \mu_X(y)\big), \inf_{y \in \mathcal{S}} T\big(\nu_R(x-y), \nu_X(y)\big)\Big)$

  Here dual definitions of these operators are chosen for their properties, as will be seen later. Here duality is intended with respect to the complementation c defined as $c(\mu, \nu) = (\nu, \mu)$ but other complementations can be used as well, under the condition to be defined according to the underlying ordering.
- Names for composite fuzzy predicates consisting of composition of elementary bipolar binary predicates.

We now illustrate how these fuzzy concrete domain predicates are used to represent spatial relations and to support bipolar spatial inference. We assume that each abstract spatial relation concept and each abstract spatial object concept is associated with its bipolar fuzzy representation in the concrete domain by the concrete feature hasForFuzzyRepresentation, denoted hasFR (it is a concrete feature because each abstract concept has only one bipolar fuzzy spatial representation in the image space).

- SpatialObject $\doteq\exists$ hasFR.$\mathcal{B}$. It defines a SpatialObject as a concept which has a bipolar spatial existence in image represented by a bipolar spatial fuzzy set.
- In the same way, we have: SpatialRelation $\doteq$ Relation $\sqcap\exists$ hasFR.$\mathcal{B}$.

Then, the following constructors can be used to define the other concepts of the ontology:

- $\exists$ hasFR.(μ_X, ν_X) restricts the concrete region associated with the object X to the specific bipolar spatial fuzzy set (μ_X, ν_X),
- $\exists$ hasFR.(μ_R, ν_R) restricts the concrete region associated with the relation R to the specific bipolar fuzzy structuring element (μ_R, ν_R),
- $\exists$ hasFR.$\delta^{(\mu_X,\nu_X)}_{(\mu_R,\nu_R)}$ restricts the concrete region associated to the spatial relation R to a referent object X, denoted R_X, to the bipolar spatial fuzzy set obtained by the dilation of (μ_X, ν_X) by (μ_R, ν_R),
- each concept R_X can then be defined by:
R_X $\doteq$ SpatialRelation $\sqcap\exists$ (hasFR,hasRO.hasFR).λ where λ is a binary fuzzy predicate built with the bipolar mathematical fuzzy operators δ and ε. hasRO represents the relation has for referent object (see [19]). For a relation R which has a referent object X, we write:
(hasFR,hasRO.hasFR).$\delta \equiv$ hasFR. $\delta^{(\mu_X,\nu_X)}_{(\mu_R,\nu_R)}$.
- $C\doteq$SpatialObject$\sqcap$hasSR.R_X denotes the set of spatial objects which have a spatial relation of type R with the referent object X and we have the following axioms: $C \sqsubseteq \exists$relationTo.X and $C \sqsubseteq$ SpatiallyRelatedObject (c.f. Figure 1).

4.2 Examples for distance relations

To illustrate our approach, we take the example of distance relations. As in [11], we use a trapezoidal function $trz(x; a, b, c, d)$ to define the semantics of *close to* : $\mathbb{R}^+ \rightarrow [0, 1]$ which represents the degree of membership to the distance relation with $trz(t; a, b, c, d) = 0$ if $t \leq a$ or $t \geq d$; $(t-a)/(b-a)$ if $t \in]a, b[$; $(d-t)/(d-c)$ if $t \in]c, d[$. For the Close_To relation, $a = b = 0$. From this membership function, we can define a unipolar structuring element ν_C. This structuring element provides a representation in the spatial domain $\mathcal{S}$ [7]: $\forall x \in \mathcal{S}, \nu_C(x) = trz(d(x, O); a, b, c, d)$ where $d(x, O)$ is the distance from x to the origin O of $\mathcal{S}$ (Euclidean distance, or a digital distance when working on a discrete space). To define the bipolar structuring element associated to Close_To, we consider that its positive part is its unipolar structuring element and its negative part is the unipolar structuring element of its opposite relation Far_From. ν_F is derived by choosing a trapezoidal function expressing the semantics of this relation, i.e. b is chosen as the smallest distance for which the relation is satisfied with a non-zero degree, c is the largest distance for which the relation is not completely satisfied and $d = +\infty$. Figure 2(a) illustrates the obtained fuzzy structuring element of Close_To, (ν_C, ν_F), in the spatial domain. We can thus define the abstract bipolar spatial relation Close_to by its bipolar fuzzy representation in the concrete domain $\mathcal{S}$:
Close_to $\doteq$ DistanceRelation $\sqcap\exists$hasFR.(ν_C, ν_F).
Let X $\doteq\exists$ hasFR.(μ_X, ν_X), (μ_X, ν_X) being the bipolar spatial fuzzy set representing the information related to the spatial extent of the object X in the concrete domain (image space). Using the concept-forming predicate operator $\exists f.P$ (see [18]), we can define restrictions for the fuzzy representation of the abstract spatial concept Close_to_X using the bipolar dilation operator δ. As a consequence, we have:
Close_to_X $\doteq$ DistanceRelation $\sqcap\exists$hasFR.$\delta^{(\mu_X,\nu_X)}_{(\nu_C,\nu_F)}$
$\doteq$hasFR.$(\delta_{\nu_C}(\mu_X), \epsilon_{1-\nu_F}(\nu_X))$.
Other distance relations can be defined in a similar way, by adapting the parameters of the trapezoidal function and the definition of the interpretation in terms of dilation.
This construct has a nice interpretation, which well fits with intuition. Indeed, if we consider that (μ_X, ν_X) represents a spatial bipolar fuzzy set with μ_X the positive information for the location of X and ν_X the negative information for this location, dilating (μ_X, ν_X) by a bipolar structuring element (μ_B, ν_B) amounts to dilate μ by μ_B, i.e. the positive region is extended by an amount represented by the positive information encoded in the structuring element. On the contrary, the negative information is eroded by the complement of the negative information encoded in the structuring element.
As explained in [7], directional relations can also be defined using fuzzy structuring elements and thus with bipolar fuzzy structuring elements (Figure 2(b) illustrates the bipolar fuzzy structuring element $(\nu_{Left}, \nu_{Right})$, in the spatial domain) as well as adjacency relations which can be expressed using a distance relation, with a semantics of *Very close to*.

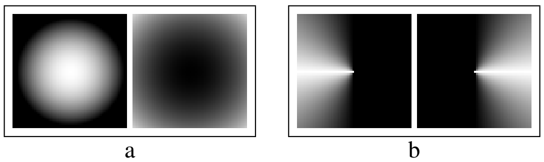

Figure 2. Bipolar fuzzy structuring element defining the semantics of the Close_To (a) and Left_of (b) relations in the spatial domain. Grey levels encode the membership or non-membership degrees, from 0 (black) to 1 (white).

4.3 Properties

As before we denote in a general way by R a bipolar spatial relation concept, X a bipolar spatial object concept, and R_X the bipolar concept *Relation R to X*. In the following, we consider the order $\preceq$ defined in Section 2, and the complete lattice $(\mathcal{B}, \preceq)$ on which bipolar and fuzzy mathematical morphology operators are defined [8]. The interpretation in the concrete domain of $X_1\sqcap X_2$ is then the bipolar fuzzy set $((\mu_{X_1} \wedge \mu_{X_2}), (\nu_{X_1} \vee \nu_{X_2}))$ and the one of $X_1\sqcup X_2$ is $((\mu_{X_1} \vee \mu_{X_2}), (\nu_{X_1} \wedge \nu_{X_2}))$. Several interesting properties of description logics can be derived from properties of mathematical morphology (for properties of mathematical morphology see [26],[10, 9] for the fuzzy case and [8] for the bipolar case). We summarize here the most important ones:

- dilation commutes with the supremum: $\delta_{(\mu,\nu)}((\mu_{X_1}, \nu_{X_1})) \vee \delta_{(\mu,\nu)}((\mu_{X_2}, \nu_{X_2})) = \delta_{(\mu,\nu)}((\mu_{X_1}, \nu_{X_1}) \vee (\mu_{X_2}, \nu_{X_2}))$ and $\delta_{(\mu_1,\nu_1)}((\mu_X, \nu_X)) \vee \delta_{(\mu_2,\nu_2)}((\mu_X, \nu_X)) = \delta_{(\mu_1\vee\mu_2,\nu_1\wedge\nu_2)}((\mu_X, \nu_X))$, and therefore we have the following equivalences between concepts: R_$X_1\sqcup$ R_$X_2 \equiv$ R_($X_1\sqcup X_2$) and R1_X$\sqcup$ R2_X $\equiv$ R12_X where R12 has for fuzzy representation $(\mu_1 \vee \mu_2, \nu_1 \wedge \nu_2)$;
- for the infimum, we only have: $\delta_{(\mu,\nu)}((\mu_{X_1}, \nu_{X_1})) \wedge \delta_{(\mu,\nu)}((\mu_{X_2}, \nu_{X_2})) \succeq \delta_{(\mu,\nu)}((\mu_{X_1}, \nu_{X_1}) \wedge (\mu_{X_2}, \nu_{X_2}))$ hence R_($X_1\sqcap X_2$) $\sqsubseteq$ R_$X_1\sqcap$ R_X_2 ;
- increasingness: $(\mu_{X_1}, \nu_{X_1}) \preceq (\mu_{X_2}, \nu_{X_2}) \Rightarrow \forall(\mu, \nu) \in \mathcal{B}, \delta_{(\mu,\nu)}((\mu_{X_1}, \nu_{X_1})) \preceq \delta_{(\mu,\nu)}((\mu_{X_2}, \nu_{X_2}))$ and $(\mu_1, \nu_1) \preceq (\mu_2, \nu_2) \Rightarrow \forall(\mu_X, \nu_X) \in \mathcal{B}, \delta_{(\mu_1,\nu_1)}((\mu_X, \nu_X)) \preceq$

$\delta_{(\mu_2,\nu_2)}((\mu_X,\nu_X))$ hence $\mathsf{X}_1 \sqsubseteq \mathsf{X}_2 \Rightarrow \forall$ R, R_X$_1 \sqsubseteq$ R_X$_2$ and R$_1 \sqsubseteq$ R$_2 \Rightarrow \forall$ X, R$_1$_X $\sqsubseteq$ R$_2$_X;

- iterativity property: $\delta_{(\mu_1,\nu_1)}(\delta_{(\mu_2,\nu_2)}((\mu_X,\nu_X)) = \delta_{(\delta_{(\mu_1,\nu_1)}((\mu_2,\nu_2)))}((\mu_X,\nu_X))$ hence R$_1$_(R$_2$_X) $\equiv$ (R$_1$_R$_2$)_X, where R$_1$_R$_2$ is the relation having as fuzzy representation $\delta_{(\mu_1,\nu_1)}((\mu_2,\nu_2))$;
- extensivity: $(\mu_B,\nu_B)(O) = (1,0) \Rightarrow \forall(\mu_X,\nu_X) \in \mathcal{B}, (\mu_X,\nu_X) \preceq \delta_{(\mu_B,\nu_B)}((\mu_X,\nu_X))$ hence X $\sqsubseteq$ R_X for any relation defined by a dilation with a bipolar structuring element containing the origin O of $\mathcal{S}$;
- duality: for the chosen definition of fuzzy dilation and erosion and the complementation c, we have $\varepsilon_{(\mu,\nu)}((\mu_X,\nu_X)) = c\left[\delta_{(\mu,\nu)}(c((\mu_X,\nu_X)))\right]$, which induces relations between some spatial relations.

These properties provide the basis for inference processes. Other examples use simple operations, such as conjunction and disjunction of relations, in addition to these properties, to derive useful bipolar spatial representations of potential areas of target objects, based on knowledge about their relative positions to known reference objects.

5 Application to Spatial Reasoning for Image Interpretation

In this section, we illustrate the potential of our framework on a brain imaging application. Our aim is to segment and recognize anatomical structures progressively by using the spatial information between the different structures. The recognition is performed in 3D magnetic resonance images (MRI) obtained in routine clinical acquisitions. A slide of a typical 3D MRI is shown in Figure 3 with a few labeled structures of interest.

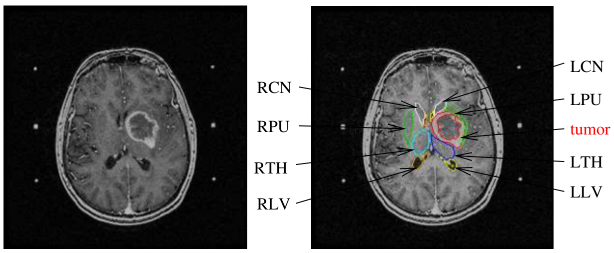

Figure 3. A slice of a 3D MRI brain image, with a few structures: left and right lateral ventricles (LLV and RLV), caudate nuclei (LCN and RCN), putamen (LPU and RPU) and thalamus (LTH and RTH). A ring-shaped tumor is present in the left hemisphere (the usual "left is right" convention is adopted for the visualization).

Let us consider the right hemisphere (i.e. the non-pathological one). We consider the problem of defining a region of interest for the RPU, based on a known segmentation of RLV and RTH. Anatomical knowledge, derived from existing medical ontologies, such as the FMA [25, 20] provides some information about the relative position of these structures:

• **directional information**: the RPU is exterior (left on the image) of the union of RLV and RTH (positive information) and cannot be interior (negative information);

• **distance information**: the RPU is quite close to the union of RLV and RTH (positive information) and cannot be very far (negative information).

This knowledge is converted in our formalism. We have the following excerpt of the TBox ($\mathcal{T}$) describing anatomical knowledge (for readability we denote by A the disjunction $(RLV \sqcup RTH)$ and by μ_A the set $\mu_{RLV} \vee \mu_{RTH}$):

AnatomicalStructure $\sqsubseteq$ SpatialObject
GN $\sqsubseteq$ AnatomicalStructure
RLV $\doteq$ AnatomicalStructure $\sqcap \exists$ hasFR.(μ_{RLV},ν_{RLV})
LLV $\doteq$ AnatomicalStructure $\sqcap \exists$ hasFR.(μ_{LLV},ν_{LLV})
LV $\equiv$ RLV $\sqcup$ LLV
RTH $\doteq$ GN $\sqcap \exists$ hasFR.(μ_{RTH},ν_{RTH})
LTH $\doteq$ GN $\sqcap \exists$ hasFR.(μ_{LTH},ν_{LTH})
Left_of $\doteq$ ($\exists$ hasFR.ν_L , Right_of)
Close_to $\doteq$ ($\exists$ hasFR.ν_C , Far_from)
Left_of_A $\doteq$ ($\exists$ hasFR.$\delta^{\mu_A}_{\nu_L}$,$\exists$ hasFR.$\delta^{\mu_A}_{\nu_R}$)
Close_to_A $\doteq$ ($\exists$ hasFR.$\delta^{\mu_A}_{\nu_C}$,$\exists$ hasFR.$\delta^{A}_{\nu_F}$)
RPU $\doteq$ GN $\sqcap\exists$ hasSR.(Left_of_A $\sqcap$ Close_To_A)

Given a known segmentation of RLV and RTH, the goal is to define a region of interest of the RPU in the image. From an ontological perspective, it means to find the bipolar spatial constraints and preferences on concrete domains to ensure the satisfiability of the following assertions c_3 : RCN , $(c_3,(\mu_{S_3},\nu_{S_3}))$: hasFR given the following ABox $\mathcal{A}$:

c_1 : RLV , (c_1,μ_{S_1}): hasFR
c_2 : RTH , (c_2,μ_{S_2}): hasFR
r_1 : Left_of, $(r_1,(\nu_L,\nu_R))$: hasFR
r_2 : Close_to, $(r_2,(\nu_C,\nu_F))$: hasFR

We should note that we only consider the positive part for spatial concepts that are known (already segmented) but we consider bipolar spatial relations. Using the basics of description logics reasoning, it means that the ABox $\mathcal{A}\cup\{c_3$: RPU $\}$ is satisfiable. First, we replace the concept RPU by its definition: $\mathcal{A}\cup$ { c_3 : GN $\sqcap\exists$ hasSR.(Left_of_A $\sqcap$ Close_To_A)}. Then, completion rules of tableau calculus (currently used in description logics reasoning [4]) and the properties of our framework are used to transform the given ABox into more descendent ABoxes and to derive both constraints and preferences on the fuzzy representation of concepts in the concrete domain (in our case, the image domain). For instance, the completion rule adds the assertion c_3 : $\exists$ hasSR.(Left_of_A $\sqcap$ Close_To_A) in the resulting ABox. As a consequence, we have an individual named c_4 such that c_4 : Left_of_A $\sqcap$ Close_To_A, (c_3,c_4):hasSR, $(c_4,(\mu_{S_4},\nu_{S_4}))$:hasFR, fit$(\mu_{S_3},\mu_{S_4})$ **as much as possible** and **strictly** fit(ν_{S_3},ν_{S_4}). As c_4 is an instance of a conjunction of bipolar concepts, its bipolar fuzzy spatial representation in the image domain is $(\mu_{\text{Left\_of\_A}} \wedge \mu_{\text{Close\_To\_A}}, \mu_{\text{Right\_of\_A}} \vee \mu_{\text{Far\_From\_A}})$. Then, we have to consider assertions of the type $a : R_X$ with R_X a bipolar spatial relation and we can apply the constructs of Section 4. So, we have : $(\mu_{S_4},\nu_{S_4}) = (\delta^{\mu_A}_{\nu_C} \wedge \delta^{\mu_A}_{\nu_L}, 1 - (\delta^{\mu_A}_{1-\nu_F} \vee \delta^{\mu_A}_{\nu_R}))$. Indeed, we consider a conjunction of the positive parts and a disjunction of the negative parts. By considering that the fit function just consists in checking an inclusion relation, we derive the following spatial contraints in the image domain : **as much as possible** $\mu_{S_3} \le \delta^{\mu_A}_{\nu_L} \wedge \delta^{\mu_A}_{\nu_C}$ and **strictly** $\nu_{S_3} \le \delta^{\mu_A}_{\nu_R} \vee \delta^{\mu_A}_{1-\nu_F}$.

As shown in Figure 4, the RPU is well included in the bipolar fuzzy region of interest which is obtained using this procedure. This region can then be efficiently used to drive a segmentation and recognition technique of the RPU.

Let us now consider the left hemisphere, where a ring-shaped tumor is present. The tumor induces a deformation effect which strongly changes the shape of the normal structures, but also their spatial relations, to a less extent. In particular the LPU is pushed away from the inter-hemispheric plane, and the LTH is pushed towards the posterior part of the brain and compressed (see Figure 3). Applying the same procedure as for the right hemisphere does not lead to very satisfactory results in this case (see Figure 5, conjunctive function positive and negative parts). The default relations are here too strict and the resulting region of interest is not adequate: the LPU only satisfies with low degrees the positive part of the information, while it also slightly overlaps the negative part.

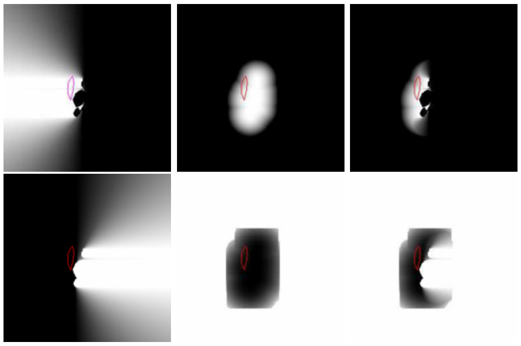

Figure 4. Bipolar fuzzy representations of spatial relations with respect to RLV and RTH. Top: positive information, bottom: negative information. From left to right: directional relation, distance relation, conjunctive fusion. The contours of the RPU are displayed to show the position of this structure with respect to the region of interest.

In such cases, some relations (in particular metric ones) should be considered with care. This means that they should be more permissive, so as to include a larger area in the possible region, accounting for the deformation induced by the tumor. This can be easily modeled by a bipolar fuzzy dilation of the region of interest with a structuring element (μ_{var}, ν_{var}) (Figure 2(a)): $(\mu'_{dist}, \nu'_{dist}) = \delta_{(\mu_{var},\nu_{var})}(\mu_{dist}, \nu_{dist})$ where (μ_{dist}, ν_{dist}) is defined as for the other hemisphere, i.e. $(\delta_{\nu_C}^{\mu_{\mu_{LLV}\cup\mu_{LTH}}}, \delta_{1-\nu_F}^{\mu_{\mu_{LLV}\cup\mu_{LTH}}})$. Now the obtained region is larger but includes the correct area and thus the corresponding spatial constraints are satisfied, as shown in the last column of Figure 5. This bipolar dilation amounts to dilate the positive part and to erode the negative part.

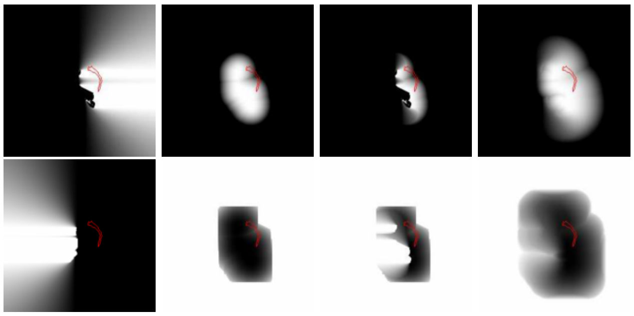

Figure 5. Bipolar fuzzy representations of spatial relations with respect to LLV and LTH. From left to right: directional relation, distance relation, conjunctive fusion, Bipolar fuzzy dilation. First line: positive parts, second line: negative parts. The contours of the LPU are displayed to show the position of this structure.

6 Conclusion

In this paper, we proposed a new formalism merging ontological reasoning and mathematical morphology reasoning, in the case of bipolar information, in order to handle both positive and negative information. The similarity between the underlying algebraic frameworks of description logics and mathematical morphology leads to interesting properties of the proposed extension of description logics, which are useful in particular for spatial reasoning. The new reasoning capabilities offered by this extension have been illustrated in this domain, on a brain imaging example: the proposed formalism allows us to manipulate both abstract bipolar concepts and their spatial concrete representations. Developing further such examples will be the aim of future work.

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