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On the Maximalization of the Witness sets in Independent Set readings

Livio Robaldo [1]

1 Introduction

This paper is about 'Independent Set (IS) readings'. Those are readings where two or more witness sets are independent of one another, and so they need to be evaluated in parallel. Four kinds of IS readings have been identified in the literature, starting from [10].

(1)
a. **Branching Quantifier readings**, e.g. *Two students of mine have seen three drug dealers in front of the school*.
b. **Collective readings**, e.g. *Three boys made a chair yesterday*.
c. **Cumulative readings**, e.g. *Three boys invited four girls*.
d. **Cover readings**, e.g. *Three children ate five pizzas*.

The preferred reading of (1.a) is the one where there are exactly two[2] students and exacly three drug dealers and each of the students saw each of the drug dealers. (1.b) may be true in case three boys cooperated in the construction of a single chair. In the preferred reading of (1.c), there are three boys and four girls such that each of the boys invited at least one girl, and each of the girls was invited by at least one boy. Finally, (1.d) allows for any sharing of five pizzas among three children. E.g., it is satisfied by the following extension of ate':

(2) $\|ate'\|^M \equiv \{\langle c_1{\oplus}c_2{\oplus}c_3,\, p_1{\oplus}p_2\rangle,\, \langle c_2{\oplus}c_3,\, p_3{\oplus}p_4\rangle,\, \langle c_3,\, p_5\rangle\}$

In (2), children c_1, c_2, and c_3 (cut into slices and) share pizzas p_1 and p_2, c_2 and c_3 share p_3 and p_4, and c_3 also ate pizza p_5 on his own.

This paper assumes, following [12], that Cover readings are *the* IS readings, of which the three kinds exemplified in (1.a-c) are merely special cases. The name "Cover readings" comes from the fact that their truth values are traditionally captured in terms of Covers. In [12], Covers are denoted by 2-order variables called "Cover variables". We may then define a meta-predicate $Cover$ that, taken a Cover variable C and two unary predicates P_1 and P_2, asserts that the extension of the former is a Cover of the extensions of the latter:

(3)
$$\begin{array}{l} Cover(C, P_1, P_2) \Leftrightarrow \\ \quad \forall_{X_1X_2}[C(X_1, X_2) \rightarrow \\ \quad\quad \forall_{x_1x_2}[((x_1 \subset X_1) \wedge (x_2 \subset X_2)) \rightarrow (P_1(x_1) \wedge P_2(x_2))]] \wedge \\ \quad \forall_{x_1}[\, P_1(x_1) \rightarrow \exists_{X_1X_2}[\, (x_1 \subset X_1) \wedge C(X_1, X_2)\,]\,] \wedge \\ \quad \forall_{x_2}[\, P_2(x_2) \rightarrow \exists_{X_1X_2}[\, (x_2 \subset X_2) \wedge C(X_1, X_2)\,]\,] \end{array}$$

Thus, it is possible to decouple the quantifications from the predications. We introduce two relational variables whose extensions include the *atomic* individuals involved. Another relational variable that covers them describes how the actions are actually done. For instance, in (2), in order to evaluate as true the variant of (1.d), we may introduce three variables P_1, P_2, and C such that:

$$\|P_1\|^M = \{c_1, c_2, c_3\} \qquad \|P_2\|^M = \{p_1, p_2, p_3, p_4, p_5\}$$
$$\|C\|^M = \{\ \langle c_1{\oplus}c_2{\oplus}c_3,\ p_1{\oplus}p_2\rangle,\ \langle c_2{\oplus}c_3,\ p_3{\oplus}p_4\rangle,\ \langle c_3,\ p_5\rangle\ \}$$

The above extensions of P_1, P_2, and C satisfy $Cover(C, P_1, P_2)$.

2 The Maximality requirement

In order to represent IS readings, it is necessary to reify the witness sets into relational variables as P_1 and P_2. Separately, the elements of these sets are combined as described by the Cover variables, in order to assert the predicates on the correct pairs of (possibly plural) individuals. As argued by [13], [6], [9], and others the relational variables must, however, be *Maximized* in order to achieve the proper truth values with any quantifier, regardless to its monotonicity.

Two kinds of Maximalization has been proposed in the literature. In this paper, they are termed as 'Local' and 'Global' Maximalization. In Local Maximalization, it is required the non-existence of a superset of either $\|P_1\|^M$ or $\|P_2\|^M$ such that the corresponding Cover is a superset of $\|C\|^{M,g}$ that is also included in the main predicate's extension[3]. Accordingly, (1.d) is represented as ($\forall_{P'_1}[\ \dots\ ]$ and $\forall_{P'_2}[\ \dots\ ]$ are the two Local Maximality conditions.):

(4)
$$\begin{array}{l} \exists P_1P_2[\ {}^{=}3_x(\text{child'}(x), P_1(x)) \wedge {}^{=}5_y(\text{pizza'}(y), P_2(y)) \wedge \\ \qquad Cover(C, P_1, P_2) \wedge \forall_{xy}[C(x, y)) \rightarrow \text{ate'}(x, y)] \wedge \\ \quad \forall_{P'_1}[\ (\forall_x[P_1(x) \rightarrow P'_1(x)] \wedge \\ \qquad \exists_{C'}[Cover(C', P'_1, P_2) \wedge \forall_{xy}[C(x, y) \rightarrow C'(x, y)] \wedge \\ \qquad \forall_{xy}[C'(x, y) \rightarrow \text{ate'}(x, y)]]\ ) \rightarrow \forall_x[P'_1(x) \rightarrow P_1(x)]\ ] \wedge \\ \quad \forall_{P'_2}[\ (\forall_y[P_2(y) \rightarrow P'_2(y)] \wedge \\ \qquad \exists_{C'}[Cover(C', P_1, P'_2) \wedge \forall_{xy}[C(x, y) \rightarrow C'(x, y)] \wedge \\ \qquad \forall_{xy}[C'(x, y) \rightarrow \text{ate'}(x, y)]]\ ) \rightarrow \forall_y[P'_2(y) \rightarrow P_2(y)]\ ] \end{array}$$

The other kind of Maximalization, termed here as 'Global Maximalization' has been advocated by [11], and formalized in most formal theories of Cumulativity, e.g. [7], [5], [3], and [2]. With respect to IS readings involving two witness sets $\|P_1\|^M$ and $\|P_2\|^M$, Global Maximalization requires the non-existence of other two witness sets that also satisfy the predication but *that do not necessarily include* $\|P_1\|^M$ *and* $\|P_2\|^M$. For instance, the event-based logical framework defined by [7] represents the Cumulative reading of (1.c) as:

[1] University of Turin, Italy, email: robaldo@di.unito.it

[2] In (1.a-d) "two/three/etc." are interpreted as "*exactly* two/three/etc." as in [10]. That is actually a pragmatic implicature, as noted in [7], pp.224-238.

[3] Without going down into formal details, we assume that quantifiers are Conservative, i.e. that for every quantifier Q_x, $\|P^B_x\|^M$ is a subset of $\|P^R_x\|^M$.

(5) $\exists e \in {}^*$INVITE: $\exists x \in {}^*$BOY: $|x|$=3 $\wedge {}^*$Ag(e)=x $\wedge$
$\exists y \in {}^*$GIRL: $|y|$=4 $\wedge {}^*$Th(e)=y $\wedge$
$|^*$**Ag**$(\bigcup\{e \in$ **INVITE: Ag**$(e)\in$**BOY** $\wedge$ **Th**$(e)\in$**GIRL**$\})|$ **= 3** $\wedge$
$|^*$**Th**$(\bigcup\{e \in$ **INVITE: Ag**$(e)\in$**BOY** $\wedge$ **Th**$(e)\in$**GIRL**$\})|$ **= 4**

Formula in (5) asserts the existence of a plural event e whose Agent is a plural individual made up of three boys and whose Theme is a plural individual made up of four girls. The two final conjuncts, in boldface, are Maximality conditions. If e_x is the plural sum of all inviting events having a boy as agent and a girl as theme, i.e. e_x=$\bigcup\{e \in$INVITE: Ag$(e)\in$BOY $\wedge$ Th$(e)\in$GIRL$\}$, the cardinality of its agent *Ag(e_x) is exactly three while the cardinality of its theme *Th(e_x) is exactly four. Therefore, Landman's Maximality conditions in (5) do not refer to the same events and actors quantified in the first row. Rather, they require that the number of the boys who invited a girl *in the whole model* is exactly three and the number of girls who were invited by a boy *in the whole model* is exactly four.

Global Maximalization appears to be more problematic than Local one. For instance, it seems that (6) is intuitively true in fig.1.

(6) Less than half of the dots are connected with exactly three stars.

Nevertheless, Global Maximalization predicts that it is false. The number of all dots in the model connected to a star is six, while the number of all stars in the model connected to a dot is five, not exactly three. On the contrary, once the witness sets have been identified as in fig.1, Local Maximalization predicts (6.b) as true, in that no other star is connected to a dot *occurring in* $\|P_1\|^M$, and no other dot is connected to a star *occurring in* $\|P_2\|^M$.

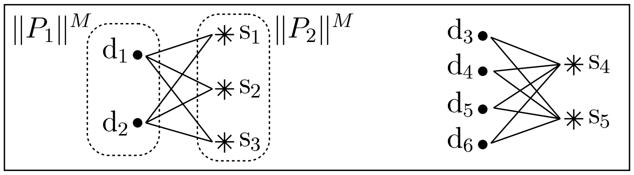

Figure 1. Identification of the witness sets for evaluating (6) as true.

A comparison between Local/Global Maximalization is found in [11], who reasonably argues that (7.a-b) are false in fig.2, while (7.c) is true. Local Maximalization wrongly predicts all (7.a-c) as true.

(7) a. Few dots are totally connected with few stars.
b. Exactly two dots are totally connected with exactly two stars.
c. At least two dots are totally connected with at least two stars.

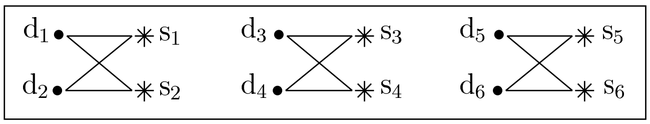

Figure 2. A model for sentences in (7).
(7.a-b) seem to be false in the model, or at least odd.

In the light of this, Schein concludes that [13]'s Local Maximalization, which is defined for any kind of quantifier, with any monotonicity, is incorrect. A proper semantics for NL quantification should instead stipulate two *different* semantics depending on monotonicity: one for M↑ quantifiers, e.g. *At least two*, and one for M↓ quantifiers, e.g. *Few*, and non-M quantifiers, e.g. *Exactly two*.

While I accept the truth values attested by Schein for sentences (7.a-c) in fig.2, I do not share his conclusions. The present paper suggests that such an oddity stems from Pragmatics. No English speaker would ever utter (7.a-b) in those contexts, as they would not be informative enough, and so they would violate a Gricean Maxim. From the examples above, it seems that sentences involving non-M↑ quantifiers sound odd in contexts where more pairs of witness sets are available. For instance, the reader gets confused when he tries to evaluate (7.a-b) in fig.2, as multiple pairs of (witness) sets of dots and stars are available, i.e. $\langle\{d_1, d_2\}, \{s_1, s_2\}\rangle$, $\langle\{d_3, d_4\}, \{s_3, s_4\}\rangle$, etc., and he does not have enough information to prefer one of them upon the others.
The multiple availability of witness sets does not seem to confuse the reader for sentences involving M↑ quantifiers, perhaps because they are simpler to interpret (cf. [4]). However, several cognitive experimental results showed that many other factors besides monotonicity, e.g. expressivity/computability, fuzzyness, the fact that quantifiers are cardinal rather than proportional, etc., may affect the accuracy and reaction time of the interpretation of IS readings involving these quantifiers. See [1], [8], and [14] to begin with.

In order to formally obtain this result, a final modification of the formulae is needed: it is necessary to pragmatically interpret the relational variables denoting the witness sets, besides the ones denoting the Covers. Accordingly, formula (4) is revised as in (8). Maximality conditions are omitted as they are the same shown in (4).

(8) $^=20_x$(child'(x), $P_1(x)$) $\wedge$ $^=10_y$(pizza'(y), $P_2(y)$) $\wedge$
$Cover(C, P_1, P_2) \wedge \forall_{xy}[C(x, y)) \rightarrow$ ate'$(x, y)] \wedge$
$\forall_{P'_1}[\ldots] \wedge \forall_{P'_2}[\ldots]$

The only difference between (8) and (4) is that the value of P_1 and P_2 is provided by an assignment g, as it is done for the Cover variable C. The assignment g must clearly obey to all (extra-)linguistic pragmatic constraints briefly listed above.

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