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Restarts and Nogood Recording in Qualitative Constraint-based Reasoning

Matthias Westphal [1] and **Stefan Wölfl** [1] and **Jason Jingshi Li** [2]

Abstract. This paper introduces restart and nogood recording techniques in the domain of qualitative spatial and temporal reasoning. Nogoods and restarts can be applied orthogonally to usual methods for solving qualitative constraint satisfaction problems. In particular, we propose a more general definition of nogoods that allows for exploiting information about nogoods and tractable subclasses during backtracking search. First evaluations of the proposed techniques show promising results.

1 INTRODUCTION

Qualitative Spatial and Temporal Reasoning (QSTR) is a knowledge representation discipline that deals with information about relations between objects defined on infinite domains, such as time and space. A typical reasoning task considered in QSTR is to solve constraint satisfaction problems with constraints from a fixed finite set of relations. To this end, reasoning is conducted by employing constraint inference techniques to tighten given constraints.

Constraint solving in QSTR has benefited from the development of large tractable subclasses, i.e., sets of relations for which (polynomial) inference techniques are refutation-complete. For general sets of relations a backtracking search is used, where the branching factor can be drastically reduced by using these tractable subclasses [4]. Recently, encodings of constraint satisfaction problems considered in QSTR into SAT-formulae have attracted considerable interest. However, these encodings lack a usable integration of information about tractable subclasses. Results presented in [5] indicate that while the prevalent QSTR methods often result in good runtime, the runtime distribution exhibits heavy tails, in contrast to SAT solvers employing restarts, which suffer less from this phenomenon. In this paper we study the effect of restarts with nogood recording in QSTR, based on a concept of nogoods that takes into account tractable subclasses and thus achieves a considerable improvement over state-of-the-art solvers. Our work is based on previous work on restarts and nogood recording in the CSP field and closely follows [3], where nogoods are recorded in a global constraint, which is used during search to enforce generalized arc-consistency on the enriched problem.

2 QUALITATIVE REASONING

In the CSP domain constraint satisfaction problems are defined on *finite* domains. Solutions or refutations are generated by explicitly assigning values to variables. In contrast, QSTR considers constraints on infinite domains, that is, proving (or disproving) the existence of a solution is done without assigning values to variables. Instead, constraints are represented as symbol set and local consistency methods are applied to manipulate and tighten these symbol sets. Hence, depending on the domain and the used relations, local consistency methods allow to prove or disprove the existence of solutions.

To explain the most important concepts, consider a fixed nonempty set $\mathcal{D}$ and a finite set of jointly exhaustive, pairwise disjoint binary relations $\mathcal{B}$ defined on $\mathcal{D}$ (called *base relations*). The set $\mathcal{B}$ must include the equality relation on $\mathcal{D}$ and be closed under converses. In qualitative constraint networks unions of such base relations may be used to express constraints between variables. Such unions of base relations will be written as sets of base relations. For reasoning purposes, one uses an approximation of the composition of relations (called *weak composition*): $B \circ_w B' := \{ B'' \mid B'' \cap (B \circ B') \neq \emptyset \}$, where $\circ$ denotes the composition of relations in the set-theoretical sense. For arbitrary relation in $2^{\mathcal{B}}$, the weak composition is given by $R \circ_w R' := \bigcup_{B \in R, B' \in R'} B \circ_w B'$. Together with the converse operation $R^{\smile} := \{ B^{-1} \mid B \in R \}$, this defines a *qualitative calculus*.

A *qualitative constraint network*, then, can be defined as a tuple $G = (V, l)$, where V is a set of variables and $l \colon V \times V \to 2^{\mathcal{B}}$ is a function that assigns to each constraint scope (x, y) a relation in $2^{\mathcal{B}}$. A *solution* of such a network is a function that assigns to each $v \in V$ an object in $\mathcal{D}$ such that all constraints $l(x, y)$ are satisfied (when interpreted over $\mathcal{D}$). A constraint network (V, l) is said to be *path-consistent* (or: *algebraically closed*) if (a) no label is empty and (b) $l(x, y) \subseteq l(x, z) \circ_w l(z, y)$ for each triple of variables x, y, z in V. Note that path consistency in this sense does not imply that each two-variable assignment consistent with the constraint network can be extended to a consistent three-variable assignment. Iteratively refining a constraint network into a path-consistent one (or one with some empty labels) is called the *path consistency method*. A subset $\mathcal{B}'$ of $2^{\mathcal{B}}$ is called a *tractable subclass* if the path consistency method applied to constraint networks with relation labels only from $\mathcal{B}'$ decides satisfiability (i.e., the path-consistency method is refutation-complete). A constraint network (V, l) is a *refinement* of a network (V, l') if $l(x, y) \subseteq l'(x, y)$ for each pair of variables from V. By applying backtracking search methods, one can systematically try out refinements of a given constraint graph in which only relations from a tractable subclass occur and check them for satisfiability. The reasoning time can be reduced by using large tractable subclasses, since on average fewer refinements have to be considered and thus the branching factor of the search tree is considerably reduced [4]. To adapt nogood techniques from the CSP domain, it is worth mentioning that the path consistency method used in QSTR roughly corresponds to the generalized arc-consistency method used in CSP (e.g., [5]).

[1] University of Freiburg, Germany, {westpham,woelfl}@informatik.uni-freiburg.de. We acknowledge support by DFG (*SFB/TR 8 Spatial Cognition*, project R4-[LogoSpace]) and DAAD (D/08/13855).
[2] Australian National University, Canberra, jason.li@anu.edu.au

3 NOGOODS AND THEIR USE IN QSTR

To consider nogoods in conjunction with tractable subclasses, the usual definition of nogood as used in CSP needs to be extended to cover not only assignments ($v = d$) and non-assignments ($v \neq d$) [2, 3], but sets of base relations ($l(x, y) \subseteq \mathcal{B}$).

Definition. Let (V, l) be a qualitative constraint network. A set of constraints $((x_i, y_i), N_i)$ $(1 \leq i \leq n)$ with variables from V and $N_i \in 2^{\mathcal{B}}$ is a *nogood* for (V, l) if (V, l) has no path-consistent refinement to base relations, l', such that $l'(x_i, y_i) \subseteq N_i$ for all $1 \leq i \leq n$.

Recording such nogoods does not differ from recording nogoods in CSP: a nogood can simply be recorded from the past decisions whenever a series of backtracks has ended or is interrupted by a restart (for details see [3]). Nogoods can immediately be used to recognize dead-ends. A search node representing some intermediate partial refinement l' of the network is a dead-end if there is a nogood $((x_i, y_i), N_i)_{1\leq i\leq n}$ such that $l'(x_i, y_i) \subseteq N_i$ for all $1 \leq i \leq n$.

At first glance it seems that recording and enforcing consistency based on nogoods is quite time intensive. However, as shown in [3], this is not the case if the recorded nogoods are checked using a "watched literals" scheme as introduced in the SAT domain. We can employ this technique even though we consider subsets instead of assignments. Nogood information from singleton set nogoods can be directly propagated. For any other nogood $((x_i, y_i), N_i)_{1\leq i\leq n}$ (with $n > 1$) we associate this nogood with a choice of two base relations n_{x_i,y_i} and n_{x_j,y_j}, where $i \neq j \in \{1, \ldots, n\}$ such that $n_{x_i,y_i} \notin N_i$ and $n_{x_j,y_j} \notin N_j$. As long as at least one of the watched base relations is included in the corresponding label during search, the nogood is not present in the network. Thus, during search, we only have to check nogoods where one of the watched base relations has been removed by constraint propagation. In such cases we analyze for which constraints N_i of the nogood, $l(x_i, y_i) \subseteq N_i$ holds. If it holds for all, we have found a dead-end. If all but one hold, we can again propagate information from the nogood (similar to the generalized arc-consistency scheme used in CSP). Note that two watched base relations guarantee correctness, since any nogood for which both are still included in the label is neither present in the network, nor would be used for propagation. We further note that watched base relations are constantly rearranged during search to further reduce the number of checked nogoods, as described in [3].

In practice, the average number of nogoods that have to be checked per search node is drastically reduced such that it has only minor effects on execution time. An analysis of theoretical complexities as given in [3] is beyond the scope of this short paper.

The proposed enhancements have been implemented based on the constraint-solving tool for qualitative constraint networks, GQR [1]. We never delete learnt nogoods (as in [3]) and do not minimize any of the nogoods to be recorded. We use restarts based on the number of decision failures, starting with 10 and increasing by a geometric scheme with the factor 1.5. GQR's heuristic is based on $dom/wdeg$ and should provide sufficient exploration when restarts occur.

4 EVALUATION

We evaluated reasoning with nogoods on the basis of random networks from the phase transition region in the so-called $A(n, d, l)$-model [4], where n is the number of variables, d the average degree, and l the average number of base relations per relation. As qualitative calculus we used Allen's Interval Algebra and its ORD-Horn tractable subclass (IA-$\mathcal{H}$) [4]. We evaluated backtracking search using (a) the base relations as refinements ($\mathcal{B}$), (b) IA-$\mathcal{H}$, and (c) IA-$\mathcal{H}$ in combination with nogoods and restarts. All experiments were conducted on a 2.6 GHz Intel Xeon CPU with a time limit of 2 hours.

Table 1. Average execution times and visited search nodes for 1 000 instances of Allen's Interval Algebra from $A(100, 10.5, 6.5)$.

	$\mathcal{B}$	IA-$\mathcal{H}$	IA-$\mathcal{H}$ +NG +restarts
SAT	162 377.97 nodes	14 798.40 nodes	7 961.55 nodes
	335.36 s	41.58 s	27.58 s
UNSAT	73 158.06 nodes	7 148.71 nodes	5 614.49 nodes
	147.41 s	18.80 s	18.46 s
Total	99 896.75 nodes	9 458.91 nodes	6 323.30 nodes
	203.74 s	25.68 s	21.21 s

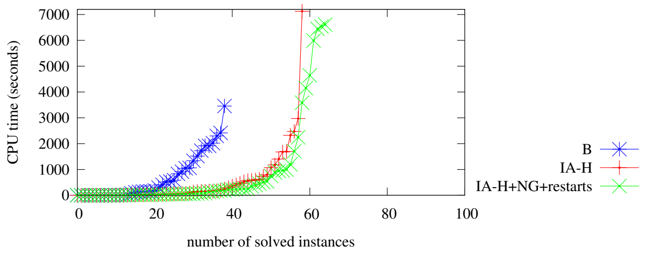

Figure 1. Average execution times for 100 instances of Allen's Interval Algebra from $A(200, 11.5, 6.5)$.

Our results for 100 nodes (Table 1) indicate that restarts with nogoods indeed help considerably to find correct labellings, but do not seem to reduce execution time when refuting instances. For larger instances with 200 nodes, all of the considered approaches fail to solve all instances. In particular, all but two solved instances were unsatisfiable instances. Here, our results (Figure 1) show that restarts with nogoods can help to refute instances.

5 CONCLUSIONS AND FUTURE WORK

We have presented an approach to exploit state-of-the-art nogood techniques within qualitative constraint-based reasoning by extending the usual definition of nogoods to arbitrary relations. This extension is still capable of using efficient lookup structures [3]. The presented nogoods can be combined with tractable subclasses, and thus we achieve a considerable improvement over state-of-the-art solvers.

For applications of QSTR, nogoods can be used to blacklist unwanted scenarios. Further, our work opens several interesting topics for future work in QSTR, such as explanation generation, minimal nogoods, and stronger local consistency methods.

REFERENCES

[1] Zeno Gantner, Matthias Westphal, and Stefan Wölfl, 'GQR - A fast reasoner for binary qualitative constraint calculi', in *Proceedings of the AAAI'08 Workshop on Spatial and Temporal Reasoning*, (2008).

[2] George Katsirelos and Fahiem Bacchus, 'Generalized nogoods in CSPs', in *AAAI'05*, pp. 390–396, (2005).

[3] Christophe Lecoutre, Lakhdar Sais, Sébastien Tabary, and Vincent Vidal, 'Recording and minimizing nogoods from restarts', *JSAT*, **1**(3-4), 147–167, (2007).

[4] Bernhard Nebel, 'Solving hard qualitative temporal reasoning problems: Evaluating the efficiency of using the ORD-horn class', *Constraints*, **1**(3), 175–190, (1997).

[5] Matthias Westphal and Stefan Wölfl, 'Qualitative CSP, finite CSP, and SAT: Comparing methods for qualitative constraint-based reasoning', in *IJCAI*, ed., Craig Boutilier, pp. 628–633, (2009).