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A Combined Calculus on Orientation with Composition Based on Geometric Properties

Frank Dylla and **Jae Hee Lee** [1]

1 INTRODUCTION

Qualitative spatial reasoning (QSR) is an established field of research investigating qualitative representations of space that abstract from the details of the physical world together with reasoning techniques that allow predictions about spatial relations, even if precise quantitative information is not available [1]. Qualitative spatial (and temporal) calculi are a sophisticated means to deal with imprecise knowledge. A calculus is comprised of a set of relations, e.g., {*front,left,back,right*}, and a set of operations. Besides standard set-theoretic operations *composition* ($\circ$) is the most important to perform constraint reasoning. Simplified, if A is left of B and B is left of C, it can be derived that A is left of C ($r_{A,B} \circ r_{B,C} = r_{A,C}$).

If a calculus needs to be extended by a property or two calculi are combined, the composition operation for the compound calculus must be given in order to perform any constraint reasoning. Previous results, e.g., regarding the INDU Calculus [6], show that reasoning on the basis of the individiual compositions (bipath consistency) does not return the correct result in many cases. Therefore, general approaches to deal with relation dependencies in combined calculi have to be investigated. Wölfl et al. distinguish two different categories: tight and loose combinations [8]. A loose combination is given if the calculi are kept separately and specialized algorithms are developed for solving biconstraint networks. In case of tight combinations a new combined calculus is defined regarding the interdependencies of the calculi when determining the new base relations and the results of the operations, i.e., composition and converse. They evaluate tight combinations to be more expressive than loose combinations. Several examples for tight and loose combinations are given. For example, in [3] a biconstraint algorithm that works for a rather large class of biconstraint networks with topological and qualitative size information is developed. Similar, in [4] loose combinations of RCC-8 and Rectangle Algebra, Cardinal Direction Calculus respectively, are investigated. In case of the INDU Calculus a new composition table was derived by hand [6]. For details on the different calculi mentioned sofar we refer to [1].

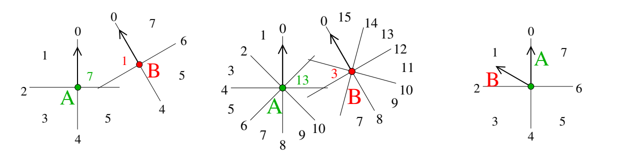
In this paper we take the position of tight combination of two calculi. We present a well founded approach to combine two orientation calculi based on their geometric dependencies. With this, we take a step in the direction of a general approach to deal with relation dependencies in combined calculi. We represent the dependencies by sets of linear equations and inequalities, which can be solved by standard methods, e.g., the Fourier-Motzkin elimination [7]. The set of equations and inequalities corresponds to the interdependency function in the definition of tight combination in [8].

[1] Transregional Collaborative Research Center SFB/TR 8 Spatial Cognition, University of Bremen, Germany, email: {dylla, jay}@sfbtr8.uni-bremen.de

In the next section we introduce the Oriented Point Relation Algebra ($\mathcal{OPRA}_m$) and the Alignment Calculus ($\mathcal{AC}$), which we exemplarily combine to a calculus called $\mathcal{OPRA}^*_m$.

2 BASIC CALCULI

The domain of the *Oriented Point Relation Algebra* ($\mathcal{OPRA}_m$) [5] is the set of oriented points (points in the plane with an additional direction parameter). The calculus relates two oriented points with respect to their relative orientation towards each other. The exact set of base relations distinguished in $\mathcal{OPRA}_m$ depends on the granularity parameter $m \in \mathbb{N}$. For each of the two related oriented points, m lines are used to partition the plane into $2m$ planar and $2m$ linear regions. Fig. 1 shows the partitions for the cases $m = 2$ (a) and $m = 4$ (b). The regions are numbered from 0 to $4m - 1$, region 0 always coincides with the orientation of the point. An $\mathcal{OPRA}_m$ base relation consists of a pair (i, j) where i is the number of the region of $\vec{A}$ which contains $\vec{B}$, while j is the number of the region of $\vec{B}$ that contains $\vec{A}$ (written as $\vec{A}\ {}_m\angle^j_i\ \vec{B}$). In case point positions coincide (see Fig. 1(c)), the relation is determined by the number s of the region of $\vec{A}$ in which the orientation arrow of $\vec{B}$ is positioned We denote the set of base relations by $\mathcal{BR}_{\mathcal{OPRA}_m}$.



(a) $m = 2$: $\vec{A}\ {}_2\angle^1_7\ \vec{B}$ (b) $m = 4$: $\vec{A}\ {}_4\angle^3_{13}\ \vec{B}$ (c) case where $\vec{A}$ and $\vec{B}$ coincide: $\vec{A}\ {}_2\angle 1\ \vec{B}$

Figure 1. Two oriented points related at different granularities.

The domain of the *Alignment Calculus* ($\mathcal{AC}$) is the set of oriented lines. Given two oriented points (o-points) $\vec{A}$ and $\vec{B}$, two oriented lines are induced by their reference directions, which we call *o-lines*. Four different base relations ($\mathcal{BR}_{\mathcal{AC}}$) can be distinguished regarding the angle between two o-lines: (1) parallel (*P*) (2) opposite-parallel (*O*) (3) positive alignment (+), and (4) negative alignment (-).

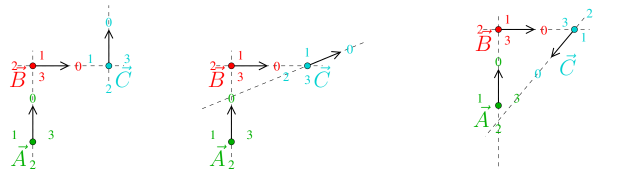
For further details on the calculi we refer to [2].

3 COMPOSITION OF $\mathcal{OPRA}^*_m$ RELATIONS

The base relations of the combined calculus $\mathcal{OPRA}^*_m$ consist of the $\mathcal{OPRA}_m$ part (${}^m_m\angle^j_i$ or ${}_m\angle s$) and a valid $\mathcal{AC}$ orientation [2] (o) resulting in ${}^o_m\angle^j_i$, ${}^o_m\angle s$ respectively. Given two $\mathcal{OPRA}^*_m$ relations ${}^x_m\angle^j_i$ and ${}^y_m\angle^l_k$ a composition result can be derived

[2] Not all orientations are possible for each $\mathcal{OPRA}_m$ relation.

on the basis of the separate composition tables of $\mathcal{OPRA}_m$ and $\mathcal{AC}$, i.e., ${}_{m}^{a}\angle_{s}^{t} \in ({}_{m}^{x}\angle_{i}^{j} \circ {}_{m}^{y}\angle_{k}^{l})$, where $a \in (x \circ y)$ and ${}_{m}\angle_{s}^{t} \in ({}_{m}\angle_{i}^{j} \circ {}_{m}\angle_{k}^{l})$. As example, Fig. 2 shows that bipath composition provides incorrect results: given the base configuration $\vec{A}\ {}_{1}^{-}\angle_{0}^{3}\ \vec{B}$ and $\vec{B}\ {}_{1}^{+}\angle_{0}^{1}\ \vec{C}$ (Fig. 2(a)) one of the resulting relations is $\vec{A}\ {}_{1}^{+}\angle_{3}^{3}\ \vec{C}$ (Fig. 2(c)) which is an invalid configuration considering the base configuration.



(a) base configuration (b) consistent configuration: $\vec{A}\ {}_{1}^{-}\angle_{3}^{3}\ \vec{C}$ (c) inconsistent configuration: $\vec{A}\ {}_{1}^{+}\angle_{3}^{3}\ \vec{C}$

Figure 2. Examples of bipath composition results for $\mathcal{OPRA}_m^*$ relations $\vec{A}\ {}_{1}^{-}\angle_{0}^{3}\ \vec{B}$ and $\vec{B}\ {}_{1}^{+}\angle_{0}^{1}\ \vec{C}$.

In the following we present a method for computing compositions of tight combination of $\mathcal{OPRA}_m$ and $\mathcal{AC}$. We assume three o-points $\vec{A}$, $\vec{B}$, and $\vec{C}$ that constitute a positively oriented proper triangle, i.e., the vertices are positioned in counter clockwise order and they are not collinear. We do not consider cases with at least two coinciding o-points explicitly, as they can be processed in a similar manner. We denote the three corresponding $\mathcal{OPRA}_m^*$ relations with $\vec{A}\ {}_{m}^{x}\angle_{i}^{j}\ \vec{B}$, $\vec{B}\ {}_{m}^{y}\angle_{k}^{l}\ \vec{C}$, and $\vec{A}\ {}_{m}^{z}\angle_{t}^{s}\ \vec{C}$. Let the line that connects the points A and C be denoted l_{AC}. Then we call the angle between l_{AC} and the linear region from $\vec{A}$ positioned clockwise next to it α_t. Likewise we define the angles α_i, β_j, β_k, γ_l, and γ_s (cf. Fig. 3). Using these notations we are able to find equivalent expressions for the relations $+$, $-$, P, and O added in $\mathcal{OPRA}_m^*$, e.g.,

$$\begin{aligned} &\vec{A}\ {}_{m}^{O}\angle_{t}^{s}\ \vec{C} \iff \vec{A}\ {}_{m}\angle_{t}^{s}\ \vec{C} \text{ and} \\ &\left(\alpha_t + \lfloor t/2 \rfloor \cdot \tfrac{\pi}{m} - \left(\gamma_s + \lfloor s/2 \rfloor \cdot \tfrac{\pi}{m}\right)\right) \bmod 2\pi = 0, \end{aligned} \tag{1}$$

where $\lfloor x \rfloor$ denotes the integral part of x. The interpretation of (1) is as follows. The term $\alpha_t + \lfloor t/2 \rfloor \cdot \frac{\pi}{m}$ stands for the angle between the o-point's reference direction (0-th linear sector) of $\vec{A}$ and the line l_{AC}. If this is equal to $\gamma_s + \lfloor s/2 \rfloor \cdot \frac{\pi}{m}$ which stands for the angle between the reference direction of $\vec{C}$ and l_{AC}, then they make up alternate interior angles arisen from the line l_{AC} and the two reference directions. In the same way we can reformulate remaining $\mathcal{OPRA}_m^*$ relations to conjunctions of $\mathcal{OPRA}_m$ relations and linear equations/inequalities. Using this information in addition to the constraints from the underlying triangle construction we can prune incorrect results from the bipath composition (see Algorithm 1).

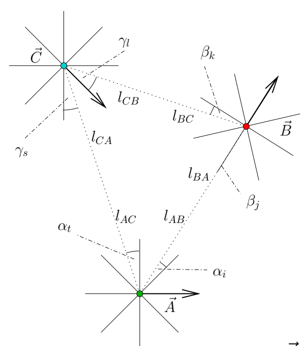


Figure 3. An exemplary triangle configuration for $\vec{A}\ {}_{m}^{x}\angle_{i}^{j}\ \vec{B}$, $\vec{B}\ {}_{m}^{y}\angle_{k}^{l}\ \vec{C}$, and $\vec{A}\ {}_{m}^{z}\angle_{t}^{s}\ \vec{C}$.

Algorithm 1: An algorithm for deriving $\mathcal{OPRA}_m^*$ composition

function: comp(r_1, r_2) with $r_1 = {}_{m}^{x}\angle_{i}^{j}$ and $r_2 = {}_{m}^{y}\angle_{k}^{l}$

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C = ∅
S = m∠_i^j ∘ m∠_k^l                                   {OPRA_m composition}
while S ≠ φ do
    Choose m∠_t^s ∈ S
    S = S\{ m∠_t^s }
    M = {a | a is a AC base relation conform to m∠_t^s }
    if |M| = 1 then
        C = C ∪ { z/m∠_t^s } with z ∈ M
    else
        while M ≠ φ do
            Choose z ∈ M
            M = M\{z}
            Generate systems of equations and inequalities w.r.t. z/m∠_t^s and
            apply Fourier-Motzkin elimination                        {cf. [7]}
            if one of the systems has a solution then
                C = C ∪ { z/m∠_t^s }
            end if
        end while
    end if
end while
return C                          {the composition result of r1 and r2}
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4 CONCLUSION

In this paper, we sketched an approach for tight combination of two calculi based on their algebraic specifications. The work presented is a step in the direction for a general approach to deal with relation dependencies in combined calculi. Based on the result we have been able to identify the minimal set of base relations. For further details we refer to [2].

As long as dependencies can be expressed by linear equations or inequalities the interdependency function can be built in the manner presented in this paper. If dependencies are of higher order, polynomial systems solving methods (e.g., cylindrical algebraic decomposition) are to be used.

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