The evolution of grain-size distribution of sands under 1-D compression

L'évolution de la granulométrie des sables en compression 1-D

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ABSTRACT

Results of a laboratory research programme on the crushing of dry sands compressed under one-dimensional conditions, at vertical effective stresses σ'_v up to 120 MPa, are reported in the paper. Tests have been carried out on calcareous bioclastic and on quartz sands, with initial coefficient of uniformity C_u ranging from 1.10 to 1.42. The crushing of bioclastic sand particles is considerable even at low stresses, whilst for quartz sands it becomes appreciable for stresses higher than 10 MPa. It is shown that the evolution of the grain-size distribution of a given sand can be effectively represented by the following relation between the absolute value of the decrement of the generic characteristic diameter ΔD_i and σ'_v: $\Delta D_i = h/(K(1+C\, e^{-h \lg \sigma'_v}))$, where C, h, K are parameters depending upon the nature and the initial grain-size distribution of the sand. This Verhulst type relation properly accounts for the existence of an upper limit to ΔD_i. The analysis of published data shows that this type of relation also applies for other, quite different, sands.

RÉSUMÉ

L'article présente les résultats d'une recherche de laboratoire sur l'écrasement des grains de sables secs comprimés en condition de déformation oedométrique à contraintes verticales σ'_v jusqu'à 120 MPa. Les essais ont été réalises sur des sables calcaires bioclastiques ainsi que sur des sables quartzeux, avec un coefficient d'uniformité C_u comprise entre 1.10 et 1.42. L'écrasement des grains des sables bioclastiques est important même sous faibles contraintes, alors que pour les sables quartzeux il ne devient remarquable que pour des contraintes supérieures à 10 MPa. L'évolution de la composition granulométrique d'un sable donné peut être efficacement représentée par la relation entre la valeur absolue du décrément du diamètre caractéristique générique ΔD_i et σ'_v: $\Delta D_i = h/(K(1+C\, e^{-h \lg \sigma'_v}))$, où C, h, K représentent des paramètres dépendant de la nature et de la composition granulométrique initiale du sable. Cette relation du type Verhulst montre qu'il existe une limite supérieure à ΔD_i. L'analyse des résultats publiés démontre que ce type de relation s'applique également à d'autres sables très différents.

Keywords: particle crushing; characteristic diameter; grain-size distribution; one-dimensional compression.

1 INTRODUCTION

The process of particles crushing is one of the fundamental mechanisms leading to deformation of particulate materials at medium and high stresses. It brings about profound changes in the fabric and the structure of the material (Bolton, 1999; Hardin, 1985; Valore, 1994; Valore & Ziccarelli, 2001). As crushing progresses in consequence of increasing applied stresses, the grain-size distribution undergoes modifications, causing changes in the porosity and in the grain-to-grain contacts and their density, and in the complex process of birth and death of particles.

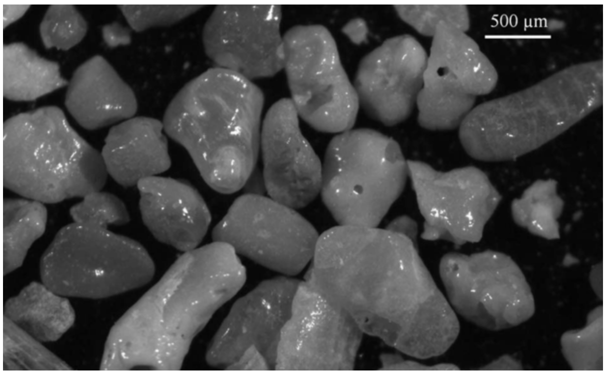

Figure 1. Uncrushed grains of SH sand C (0.30mm<d< 0.42mm).

The process is relevant for many geotechnical problems – e. g. for piles in sand and high rockfill dams – and for grasping the mechanical behaviour of the material as well. As a matter of fact the modification of the grain-size distribution plays a key role for the structural stability, the stress-strain relation, the dilatancy and the shear strength of sands and of particulate materials (Mitchell & Soga, 2005; Rahma, 1998).

In the present paper results of one-dimensional compression tests on two widely different sands are reported and trends in the evolution of characteristic diameters of the sands are identified that appear to be shared alike by other sands.

2 MATERIALS AND TESTING PROCEDURE

2.1 *Properties of materials*

The tests have been carried out on two sands. The first one – shortly termed "shell sand" or SH sand – has been recovered from a beach deposit located in the vicinity of the city of Palermo. The sand was immersed in flowing tap water for at least 48 hours to wash away salts from the surface of grains. It is composed almost entirely of small fragments of calcareous marine shells. Particles are from subangular to rounded as shown in figure 1. The initial grading of tested SH sands is shown in figure 2. Sands from B to G have been obtained from the natural sand A by sieving. The uniformity coefficient C_u of tested sands ranges from 1.18 to 1.42; the specific weight is 27 kN/m^3. The initial void ratio e_0 ranges from 0.56 to 1.08.

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The second sand is commercially available; it is entirely composed of angular quartz grains obtained in a crusher plant. This sand is termed QZ. The granulometric composition of the tested QZ sands is shown in figure 2. C_u ranges from 1.10 to 1.18; the specific weight is 26 kN/m^3; e_0 varies from 0.60 to 0.65.

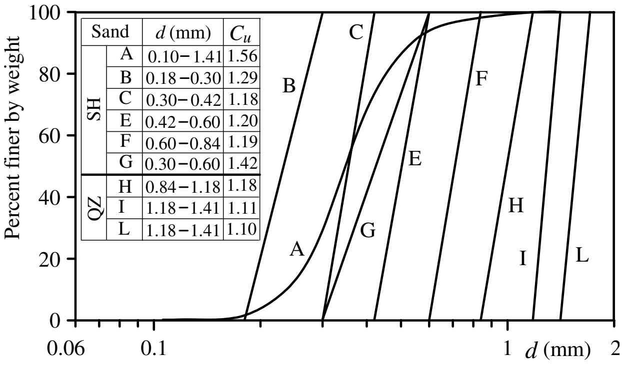

Figure 2. Initial grading of calcareous (SH) and quartz (QZ) sands. A: natural sand, not tested at high stresses.

2.2 *Testing procedure*

One-dimensional compression tests have been carried out in a specially built oedometer capable of withstanding high stresses (Ziccarelli, 1999). The diameter and the height of the specimens were 73mm and 20 mm respectively. The sand has been placed dry in the oedometer and then gently tamped.

The maximum vertical stress σ'_v reached 120 MPa. The vertical load was applied by means of a hydraulic press at almost constant rate of axial deformation of 0.5 mm/min, high enough that creep effects can be deemed negligible.

Conventional incremental loading tests were also carried out, up to maximum vertical stresses σ'_v of 19.5 MPa, on specimens with 56mm diameter and 20mm height.

A thin film of polyethylene terephtalate was put over the inside surface of the oedometer in order to reduce friction at the steel-sand interface.

The vertical settlement of the sand was measured by conventional millesimal micrometers. After testing, the specimens were removed from the oedometer and sieved. All tests results are reported elsewhere (Ziccarelli, 1999).

3 EXPERIMENTAL RESULTS

83 tests have been carried out on calcareous SH sands, see table 1. Typical modifications in granulometric composition experienced by the sands in function of the applied stress σ'_v are shown in figures 3 and 4. These data show that grain crushing of SH sands occurs even at low stresses; for the coarser sand F it is appreciable for stresses as low as 0.2 MPa. It has been observed that a few "large" grains do survive compression at stresses so high as 100 MPa.

26 tests have been carried out on quartz QZ sand, as indicated in table 2. Typical results are plotted in figure 5. It can be noted that particle crushing becomes significant for applied stress higher than about 10 MPa.

The amount of crushing can be expressed quantitatively by the relative breakage index B_r (Hardin, 1985), defined as the ratio between the total breakage B_t (i.e. the area on the semilogarithmic plot bounded by the initial grading curve, the grain-size curve pertaining to the given applied stress and the vertical line drawn at the abscissa d=0.074mm) and the breakage potential B_p (i.e. the area bounded by the initial grading curve, the horizontal line through the 100% point of the initial grading curve, and the vertical line at d=0.074mm). Typical trends of B_r with applied stress σ'_v are shown in figure 6 for both SH and QZ sands.

Table 1. Tests on SH sands.

Sand	Range of e_0	Applied stresses MPa	Number of tests
B	0.56	10-120	12
C	0.56-0.76	5-120	18 [(1)]
E	0.71-0.95	2-120	21
F	0.77-1.08	0.2-120	20 [(2)]
G	0.71	5-120	12

[(1)] The test at σ'_v=120 MPa has been repeated 4 times.
[(2)] Some tests have been repeated.

Table 2. Tests on QZ sands.

Sand	Range of e_0	Applied stresses MPa	Number of tests
H	0.65	10-100	6
I	0.60-0.62	5-100	7
L	0.62	5-100	13

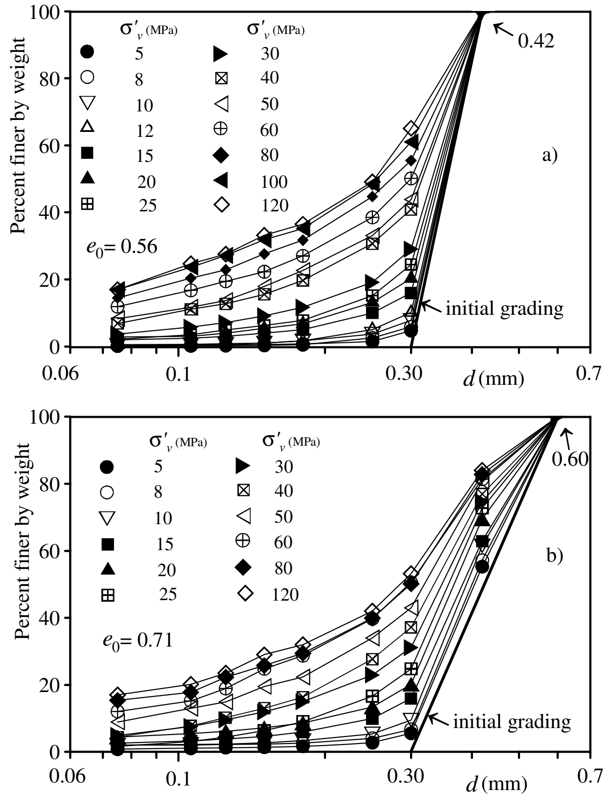

Figure 3. Modifications of the grain-size distribution as a function of the maximum applied stress σ'_v for SH sands. a) sand C; b) sand G.

4 EVOLUTION OF THE GRAIN-SIZE DISTRIBUTION

The changes in the grading induced by the applied stress σ'_v depend on many and interdependent factors such as: the void ratio e_0; the initial form, dimensions, angularity and the strength of particles; the defects and flaws of the grains; the granulometric sorting and the related coefficient of uniformity; the distribution of grain-to-grain contacts and its possible non-uniformity within the sand sample; the variability within the sand mass of the coordination number of the grains; the loading rate. It does not surprise, therefore, that the quantitative description of the crushing process in deterministic as well as in probabilistic micromechanical terms is extremely difficult.

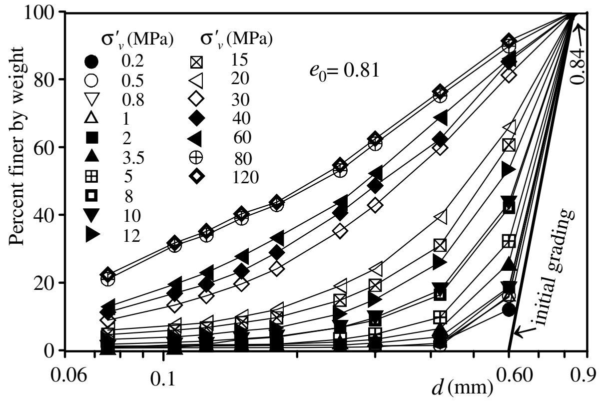

Figure 4. Modifications of the grain-size distribution as a function of the maximum applied stress σ'_v for SH sand F.

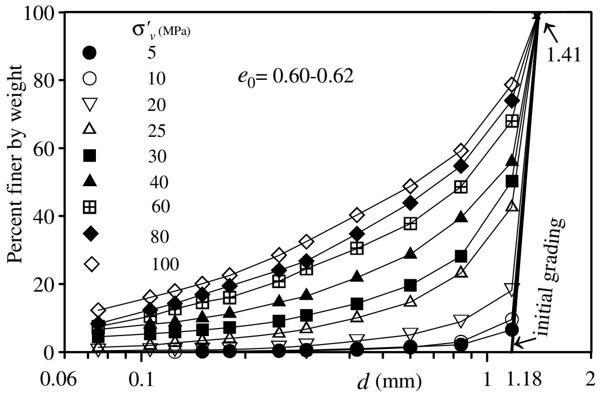

Figure 5. Modifications of the grain-size distribution as a function of the maximum applied stress σ'_v for QZ sand I.

Nevertheless, the experimental results of the present research permit to identify some general macroscopic trends in the evolution of the granulometric composition. In figures 7 and 8 the variation of the absolute values of the decrement ΔD_i of the characteristic diameter D_i (such as D_{10}, D_{15}, D_{25}, D_{50}, D_{75}) are plotted in function of the applied stress σ'_v. D_i is the diameter corresponding to the percentage i finer by weight.

The characteristic diameters always decrease with increasing σ'_v. However, experimental results show that there is an upper limit, $\Delta D_{i,max}$, to ΔD_i. $\Delta D_{i,max}$ is lower than the initial value of D_i. The rate of increase of ΔD_i with $\lg\sigma'_v$ is lower the higher D_i. ΔD_i can be related to σ'_v by the following function:

$$\Delta D_i = \frac{1}{K}\frac{h}{(1+Ce^{-h\lg\sigma'_v})} \qquad (1)$$

where K, h, C are positive parameters depending on the initial properties and the nature of the sand, and can be found by curve fitting methods. Other functions may be used to fit experimental data, too. Function (1) plots as a logistic or Verhulst curve on the plane ΔD_i - lg σ'_v; it has two horizontal asymptotes (ΔD_i=0 and $\Delta D_{i,max}$=h/K), and a point of inflection at abscissa:

$$\lg\sigma'^{*}_v = \frac{1}{h}\ln C \qquad (2)$$

The slope p_f at the inflection point is:

$$p_f = \frac{h^2}{4K} \qquad (3)$$

The existence of an upper bound $\Delta D_{i,max} < D_i$ implies that at high stresses the grains of the sand become so tiny or so mutually coordinated that further crushing becomes negligible (Kendall, 1978).

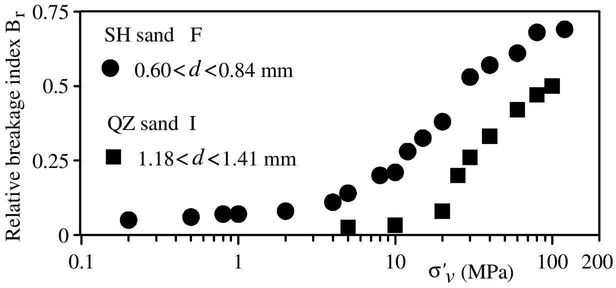

Figure 6. Typical trends of the relative breakage index B_r for SH and QZ sands.

The proposed relation (1) fits satisfactorily experimental data (figures 7 and 8), for both calcareous (SH) and quartz (QZ) sands. Some values of parameters h, K and C for SH and QZ sands are reported in table 3, in which $\Delta D_{i,max}$, the stress σ'^{*}_v and the slope p_f (both referred to the inflection point) are also given. It may be observed that σ'^{*}_v, h and K decrease, while the slope p_f and $\Delta D_{i,max}$ increase, as the initial dimensions of grains increase.

The parameter h varies within a relatively narrow range; its inverse seems related to the propensity to breakage of sand grains. The parameter K accounts for the resistance to changes of D_i that is higher the higher the number of particles for unit volume and, consequently, the smaller the dimensions of particles. The parameter C varies within a very wide range, and its physical meaning is not as yet clear.

The proposed relation between ΔD_i and σ'_v fits quite well other experimental data on such diverse materials as Ottawa sand, figure 9, angular glass, figure 10, (Hagerty et al., 1993), subjected to 1-D compression; it seems to portray satisfactorily also the granulometric evolution under triaxial compression, as shown in figure 11.

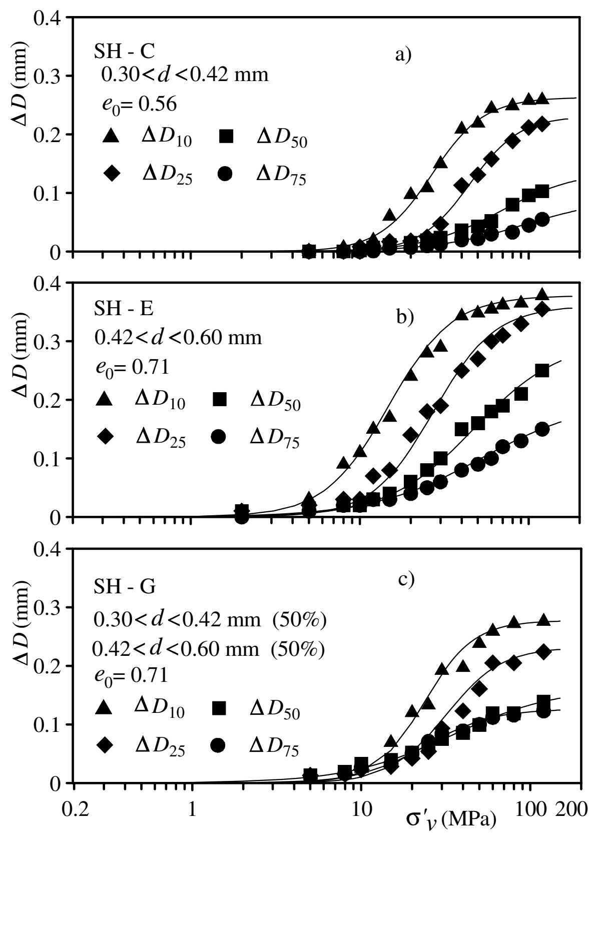

Figure 7 (a, b, c). Progress of the changes ΔD of characteristic diameters as a function of the applied stress σ′$_v$ for SH sands C, E and G. ΔD_i: reduction of characteristic diameter D_i.

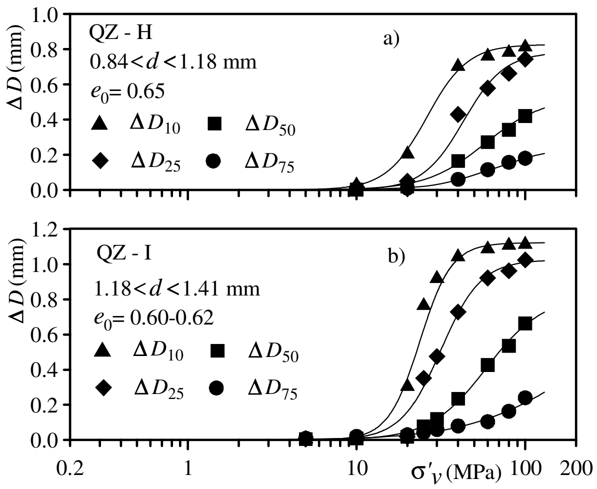

Figure 8 (a, b). Progress of the changes ΔD of characteristic diameters as a function of the applied stress σ′$_v$ for QZ sands H and I.

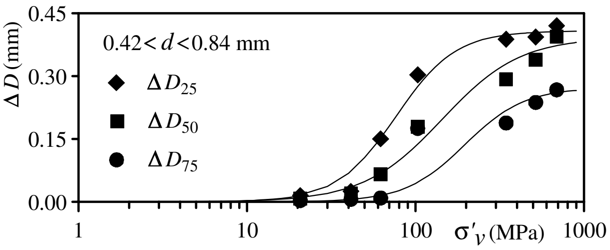

Figure 9. Progress of the changes ΔD of characteristic diameters as a function of the applied stress σ′$_v$ for Ottawa sand (1-D tests). Data from Hagerty et al., 1993.

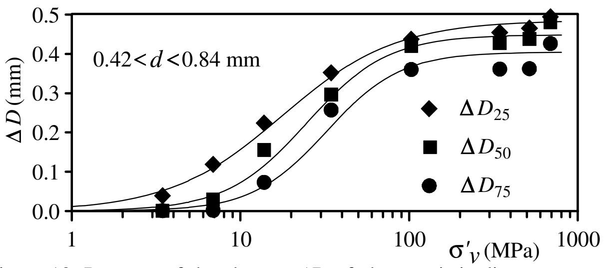

Figure 10. Progress of the changes ΔD of characteristic diameters as a function of the applied stress σ′$_v$ for angular glass (1-D tests). Data from Hagerty et al., 1993.

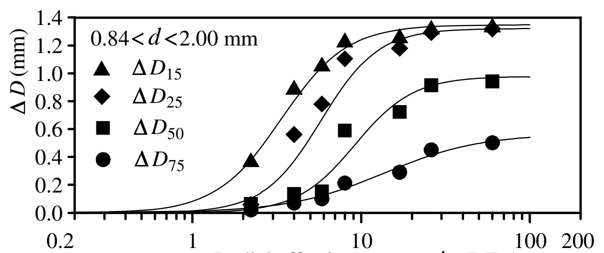

Figure 11. Progress of the changes ΔD of characteristic diameters as a function of effective radial stress σ′$_r$ for Cambria sand (triaxial tests). Data from Yamamuro, Bopp & Lade, 1996.

5 CONCLUSIONS

One-dimensional compression tests have been carried out on five calcareous sands (SH) and on three quartz sand (QZ) with different initial grading. The calcareous sands have been sieved out from a single sample of a natural beach sand composed of fragments of marine shells with a degree of roundness from subangular to rounded. The void ratio e_0 of the uncrushed material ranges from 0.56 to 1.08. Quartz sands have been produced in a crusher plant; their grains are equidimensional and angular; e_0 varies from 0.60 to 0.65.

Table 3. Value of parameters h, K, C in function (1).
σ'^{*}_v, p_f: stress and slope at inflection point; $p_f = d(\Delta Di)/d(lg\sigma'_v)$

Sand	i (%)	$\Delta D_{i,max}$ (mm)	σ'^{*}_v (MPa)	p_f	h	K	C
SH – B 0.18-0.30 mm	10	0.152	36	0.305	8.03	52.82	267054
	25	0.135	63	0.235	6.99	51.76	288860
SH – C 0.30-0.42mm	10	0.268	27	0.442	6.60	24.64	4496
	25	0.24	45	0.345	5.76	24.01	36954
SH – E 0.42-0.60 mm	10	0.382	15.3	0.406	4.25	11.12	153
	25	0.382	29	0.383	4.01	10.51	353.8
SH – G 0.60-0.84 mm	15	0.59	10.5	0.531	3.60	6.10	39.4
	25	0.584	14	0.514	3.51	6.01	55.1
QZ – H 0.84-1.18 mm	10	0.839	26	1.886	8.99	10.71	334384
	25	0.79	43	1.552	7.86	9.95	376569
QZ – I 1.18-1.41 mm	10	1.13	23	2.444	8.65	7.65	130451
	25	1.05	32	2.096	7.98	7.60	165663
QZ – L 1.41-1.71 mm	10	1.38	21	2.458	7.13	5.16	12357
	25	1.30	31	2.111	6.49	5.00	16088
Ottawa sand (*) 0.42-0.84 mm	25	0.405	78	0.55	6.83	16.87	410537
Angular glass (*) 0.42-0.84 mm	25	0.491	18	0.361	2.95	6.01	40.4

(*) Data from Hagerty et al., 1993.

The maximum applied stress reached 120 MPa. Results show that the grain-size distribution begins to undergo appreciable changes at stress σ′$_v$ as low as 0.2 MPa in the case of calcareous sands, and at stresses of about 10 MPa in the case of quartz sands. Common trends in the evolution of the grain-size distribution have been recognised for the two sands, despite the nature and properties of SH sands widely differ from those of QZ sands. It is shown that the decrement ΔD_i of the generic characteristic diameter D_i increases with the applied stress σ′$_v$ according to the relation $\Delta D_i = h/(K(1+C\, e^{-hlg\sigma'_v}))$, where C, h, K are parameters depending on the initial properties of the given sand. The above logistic or Verhulst function properly accounts for the existence of an upper bound to ΔD_i. The same type of function appears to apply for other sands according to data retrieved from geotechnical literature.

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