Predicting effective stress parameter of unsaturated soils in plane strain condition using neural networks

La prévision du paramètre de contrainte effectif pour les sols non saturés en utilisant les réseaux de neurones

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ABSTRACT
Accurate prediction of the shear strength of unsaturated soils is essential for a cost optimized design of these structures. Based on the effective stress equation proposed by Bishop (1959) for unsaturated soils, shear strength in these soils depends on the parameter χ which is a function of soil suction.

An adaptive learning neural network method is utilized to predict the effective stress parameter, χ, in plane strain condition. The proposed network is a multilayer perceptron network that consists of 6 neurons in the input layer representing air entry value, volumetric water content at residual and saturated conditions, slope of Soil Water Characteristic Curve, net vertical stress and suction. A database prepared from direct shear test results available in the literature are used to train and test the network. The results indicate suitability of the proposed approach for estimating the effective stress parameter of unsaturated soils.

RÉSUMÉ
La prévision précise de la résistance au cisaillement des sols non-saturés est essentielle pour une conception optimisée de coût des travaux géotechnique. Basé sur l'équation de contrainte effectif proposée par Bishop (1959) pour les sols non saturés, la résistance au cisaillement dans ces sols dépend du paramètre χ qui est une fonction de succion des sols.

Une méthode d'étude adaptative de réseau neural est utilisée pour prévoir le paramètre de contrainte effectif, χ. Le réseau proposé est un réseau multicouche de perceptron qui se compose de 6 neurones dans la couche d'entrée représentant la valeur d'entrée d'air, la teneur en eau volumétrique aux conditions résiduelles et saturées, la pente de la courbe caractéristique de l'eau et de sol, la contrainte vertical nette et la succion de sol. Une base de données préparée à partir des résultats d'essai de cisaillement direct disponibles dans la littérature sont employées pour former et examiner le réseau. Les résultats indiquent la convenance de l'approche proposée pour estimer le paramètre de contrainte effectif pour les sols non saturés.

Keywords: Unsaturated Soil, Shear Strength, Neural Network, Effective Stress Parameter

1 INTRODUCTION

Accurate prediction of the shear strength of unsaturated soils is essential for a cost optimized design of earth structures, foundations and natural slopes. Assuming Mohr-Coulomb failure criteria, shear strength of soils is express as:

$$\tau = \sigma' \tan(\phi') + c' \tag{1}$$

Where, τ is the shear strength, σ' is the effective normal stress on failure plane and c' and ϕ' are the effective shear strength parameters of the soil. Effective stress for unsaturated soils is written as follows (Bishop, 1959):

$$\sigma' = (\sigma - u_a) + \chi(u_a - u_w) \tag{2}$$

In which, σ is total stress, u_a is pore air pressure, u_w is pore water pressure and χ is an effective stress parameter, being 0 for completely dry soil and 1 for fully saturated soil. Some researchers declared that the assumption of $\chi = S_r$ is adequately precise to predict the shear strength of unsaturated soils (Oberg and salfors, 1997). But, this parameter strongly depends on the soil structure (Loret and Khalili, 2002).

Fredlund et.al. (1996) proposed another relationship to predict the shear strength of unsaturated soils based on two independent stress state variables, as follows:

$$\tau = c' + (\sigma - u_a)\tan\phi' + (u_a - u_w)\tan\phi'\left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right) \tag{3}$$

Where θ is the volumetric water content at critical state, θ_r is the residual water content and θ_s is the volumetric water content at saturated condition. Similar equation has been proposed by Vanapalli et al. (1996):

$$\tau = c' + (\sigma - u_a)\tan\phi' + (u_a - u_w)\tan\phi'(\bar{\theta})^{\kappa} \tag{4}$$

In which, κ is a fitting parameter and $\bar{\theta}$ is equal to $\frac{\theta}{\theta_s}$. Garven and Vanapalli (2006) found a relationship between κ and plasticity index of the soil.

Khalili and Khabbaz (1998) demonstrated that the effective stress parameter (χ) is unity at suctions equal to bubbling pressure and the relationship between the χ and logarithm of the matric suction is linear. Based on these observation, they proposed a new equation for χ, as:

$$\chi = \left(\frac{u_a - u_w}{(u_a - u_w)_b}\right)^{-0.55} \tag{5}$$

Where, $(u_a\text{-}u_w)_b$ is the air entry value in drying process and equals to air expulsion value in the wetting condition.

Xu (2004) found a similar formula for the effective stress parameter using fractal theory. In this approach, surface fractal dimension of the soil could be estimated from the Soil Water Characteristic Curve.

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Russell and Khalili (2006) developed equation (5) to predict the effective stress parameter of sands as follows:

$$\chi = \begin{cases} 1 \quad for \quad \frac{(u_a - u_w)}{(u_a - u_w)_b} \le 1 \\ (\frac{(u_a - u_w)}{(u_a - u_w)_b})^{-0.55} for \quad 1 < \frac{(u_a - u_w)}{(u_a - u_w)_b} \le 25 \\ 25^{0.45} (\frac{(u_a - u_w)}{(u_a - u_w)_b})^{-1} for \quad \frac{(u_a - u_w)}{(u_a - u_w)_b} > 25 \end{cases} \tag{6}$$

Arvin et al. (2007) developed a numerical method to estimate the effective stress parameter based on the percolation theory. In their method, pore size distribution of the soil was determined from the soil water characteristic curve and a conceptual model was constructed employing the percolation theory. Then, the effective stress parameter is determined directly from the conceptual model. But this method needs high performance computer systems to extend the prediction domain to high suction range.

Recently some researchers utilized Artificial Neural Network to relate the shear strength of unsaturated soils with their physical properties (Lee, et al. 2003; Kayadelen, 2007). But, their modeling does not take into account the influence of sample preparation method and the stress state. It is notable that, processing the results of triaxial unsaturated shear tests shows that the effective stress parameter will change significantly when net mean stress changes and suction remains constant (Fig.1). It is worth noting that the results of the tests have demonstrated that the net pressure has the same influence on the soil water characteristic curve (Lee et al., 2005).

It is worth noting, although the algebraic representation of the effective stress parameter for volume change and shear strength of unsaturated soils, is different; but in the range of practical matric suction, this discrepancy is negligible.

In this paper, an Artificial Neural Network method is utilized to predict effective stress parameter (χ) required for proper estimation of the shear strength of unsaturated soils. The input variables for the network include the matric suction, net vertical stress and SWCC parameters of the soil. These parameters are explained in the following section.

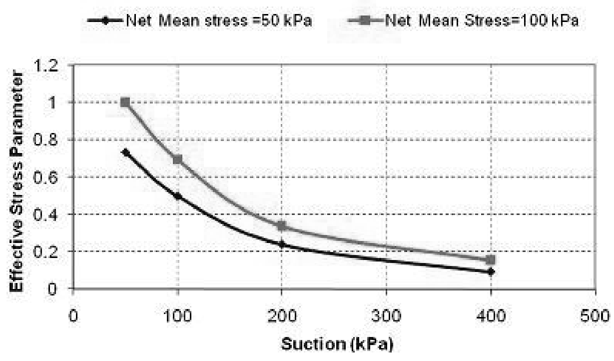

Fig. 1. Effect of net mean stress on the effective stress parameter of the Kurnell Sand (Raw Data from Russell and Khalili, 2004, 2006)

2 SOIL WATER CHARACTERISTIC CURVE (SWCC)

SWCC is usually utilized as a tool to estimate the permeability and shear strength of unsaturated soils. A few parameters can be defined from the SWCC: saturated volumetric water content, θ_s, bubbling pressure, $(u_a\text{-}u_w)_b$, and residual water content, θ_r (Vanapalli et al. 1999). Brooks and Corey (1964) equation for SWCC is written as follows:

$$\begin{cases} \frac{\theta - \theta_r}{\theta_s - \theta_r} = (\frac{(u_a - u_w)_b}{u_a - u_w})^{\lambda} & for\ (u_a - u_w) \ge (u_a - u_w)_b \\ \theta = \theta_s & for\ (u_a - u_w) \le (u_a - u_w)_b \end{cases} \tag{7}$$

In which, λ is a fitting parameter, θ_s and θ_r as defined before.

3 NEURAL NETWORK METHOD

A three-layer feed forward back propagation neural network was used in this study. To provide an intelligent weight initialization, Nguyen and Widrow (1989) method was utilized. The network was trained by a training function that updates weight and bias values according to Levenberg (1944) and Marquardt (1963) adaptive optimization method. The input layer consists of 6 neurons including bubbling pressure, volumetric water content at residual and saturated conditions, λ parameter, net vertical stress and suction.

Table 1. Soil properties of database for training ANN

References	h_b	θr	θs	λ
Zhan & Ng 2006	5	28.81	41	0.21
Zhan & Ng 2006	25	0	45	0.09
Vanapalli, et al., 1996	80	0	32	0.07
Vanapalli, et al., 1996	150	23.62	54.3	0.67
Escario& Juca, 1989	250	0	48.7	0.1
Escario& Juca, 1989	100	2.832	32.4	0.3

Bubbling pressure and water content at saturation condition were determined directly from the SWCCs and the other two parameters, λ and θ_r, were determined by fitting equation 7 to the data points. Data from direct shear test and pressure plate/filter paper test results available in the literature were used to train and test the network.

Table 2. Soil properties of database for testing ANN

References	$\mathbf{h_b}$	θr	θs	λ
Escario& Juca, 1989	30	0	23.8	0.19
Miller & Hamid, 2006	6	0	23.7	0

4 RESULTS AND DISCUSSION

The ANN was trained using the results from 58 CD direct shear tests (table 1) and tested using the results from 12 CD tests (table 2) that were not exposed to the network during the training part. Although, there was no need for early stopping during the training procedure, the number of epochs was limited to 80.

Figures 1, 2 compare the predicted effective stress parameter with the actual data for training and testing.

In order to assess the importance of input parameters on the performance of the network, each parameter was eliminated in turn from the input layer, to develop 5 other ANN models. Table 4 compares the performance of these 6 models. It is obvious that model 1 has a significantly superior performance. Therefore, it may be concluded that the effective stress parameter of unsaturated soils strongly depends on the whole soil water characteristic curve defined by four parameters, θ_s, θ_r, $(u_a-u_w)_b$ and λ. Besides, the net mean stress had a strong influence on the accuracy of the results. Connection weights of model 1 are presented in Table 5.

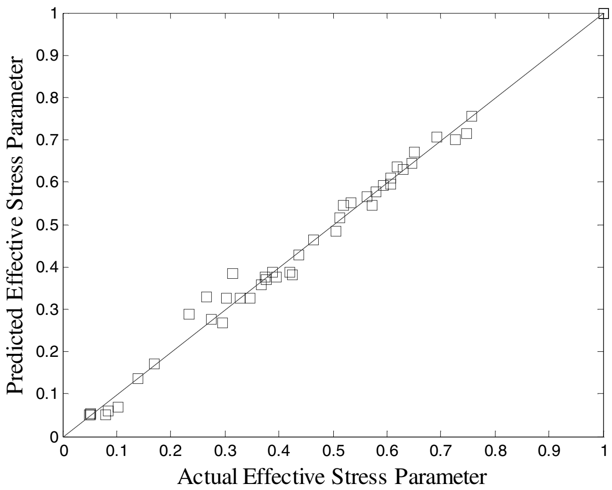

Fig. 2. Predicted versus actual χ values for training data (RMSE= 0.02)

Figure 4 demonstrates the variation of effective stress parameter with net vertical pressure while SWCC parameters remain unchanged. From this figure it may be concluded that there is a threshold value for the net vertical stress below which the χ parameter strongly depends on its variation. This behavior may be explained by the following arguments. The SWCC parameters change with the plastic volumetric strain and as the applied mean stress surpasses the preconsolidation pressure; significant plastic deformation occurs. Hence, variation of the net stress has considerable effect on the χ parameter. However, as the rate of plastic strains decreases with the applied net stress; the influence of the stress on the χ parameter will vanish.

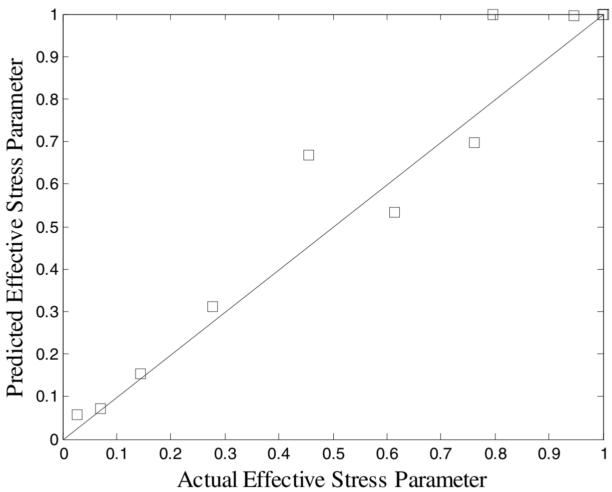

Fig. 3. Predicted versus actual χ values for testing data (RMSE= 0.093)

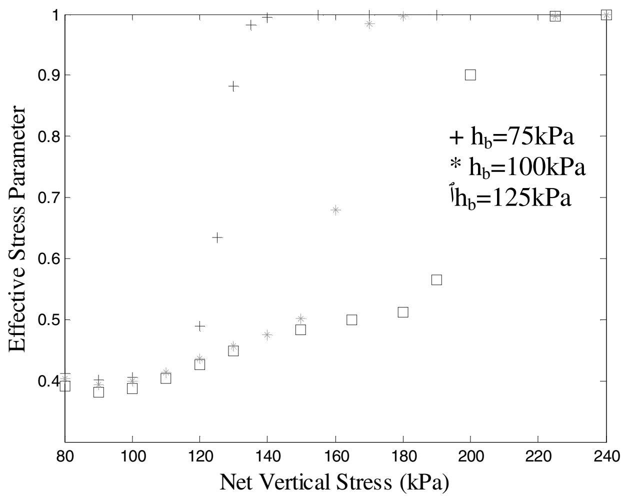

Fig.4. Estimated variation of effective stress parameter with respect to net vertical stress, Bubbling pressure=*25kPa,* θ_s *=45,* θ_r *=0,* λ*=0.1,*

5 CONCLUSION

A feed forward back propagation algorithm was used to investigate the ability of artificial neural network method for determination of the effective stress parameter, χ, in unsaturated soils. ANN model indicates a good ability to predict the target values for the datasets.

Network analysis indicated that the χ-parameter is strongly dependent on the net mean stress. This dependency is more pronounced below a threshold value which is different for different soil types.

It is notable that, the range of the bubbling pressure of soils considered in this research were limited to 250 kPa and therefore, the validity of the ANN should be tested for finer soils with higher bubbling pressure.

Since, data base employed in this study is from a collection of Consolidated Drained direct shear tests, performance of the network to predict the results of the Constant Water tests can not be evaluated in this stage.

It is worth noting, although the algebraic representation of the effective stress parameter for volume change and shear strength of unsaturated soils, is different; but in the range of practical matric suction, this discrepancy is negligible.

Table 3. Performance of ANN Models

Model	Inputs	Architecture	Transfer Function	RMSE (Training)	RMSE (Testing)
1	$\sigma_{3,net}$, suction, h_b, θr, θs, λ	6-6-1	logsig-logsig	0.02	0.093
2	$\sigma_{3,net}$, suction, θr, θs, λ	5-5-1	logsig-logsig	0.07	0.14
3	$\sigma_{3,net}$, suction, h_b, θs, λ	5-5-1	logsig -logsig	0.045	0.16
4	$\sigma_{3,net}$, suction, h_b, θr, λ	5-5-1	logsig-logsig	0.06	0.23
5	$\sigma_{3,net}$, suction, h_b, θr, θs	5-5-1	logsig-logsig	0.1	0.13
6	suction, h_b, θr, θs, λ	5-5-1	logsig-logsig	0.07	0.33

Table 4. Connection weights for ANN model 1.

Hidden layer	Net Vertical Stress(kPa)	Suction(kPa)	h_b	θ_r	θ_s	λ	Input Bias	Output Neuron
1	-0.0236	0.0413	-0.2718	-0.563	0.5383	0.0557	5.4646	4.4079
2	0.2114	-0.0041	0.3812	0.2893	-1.0463	0.0006	-1.422	1.805
3	0.0937	0.0533	-0.0421	0.3572	0.8095	-0.0585	6.1918	6.0201
4	0.5082	-2.2377	-0.9952	-2.263	-0.3359	-0.0837	-1.105	-1.1761
5	-0.3683	-0.0777	0.0434	-0.8547	-0.3269	0.1015	-7.652	5.3304
6	5.6063	1.6526	1.9873	5.4595	-0.7143	0.7079	6.8575	-2.1714
Output Bias	-	-	-	-	-	-	-	2.063

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