# Statistical models for reliability assessment of rock strength

## Modeles statistiques pour evaluer la resistance des roches

B. Look
*Connell Wagner Pty Ltd, Australia*

V. Wijeyakulasuriya
*Main Roads, Queensland, Australia*

ABSTRACT

The Gateway Upgrade Project in the State of Queensland, Australia, is the largest road and bridge infrastructure project in the state's history. A statistical analysis of intact rock strength properties of the sub-horizontally interbedded sandstone stratum underlying the main river span of the duplication bridge was carried out using pre-construction geotechnical data. These findings were then compared with an analysis based on additional borehole data captured during construction. However, this has been confined to the additional data obtained at the southern pier location of the main span only for this paper. Having a borehole at each socket location at this pier showed that signification variation of ground conditions could occur locally between piles, even for the same pier location.

Given the data variation, an appropriate statistical density function is required for statistical modeling to asses the reliability of the design. Using the probabilistic models identified, the study undertakes to rationalize the design rock strength input model adopted for socket design. The impact of rock strength anisotropy on the design UCS is also investigated. The implications of using the design UCS with various probability distribution models to satisfy limit state material characterization requirements are briefly discussed. Assumption of normality in the data distribution was shown to have the potential to significantly affect the design values.

RÉSUMÉ

Le projet "Gateway Upgrade" dans l'etat de Queensland en Australie est le plus grand projet de ponts et chaussees de l'histoire de cet etat. Une analyse statistique de la resistance des strates de gres sousjacentes la portee principale du pont de deviation de la riviere Brisbane a ete prepare en utilisant les resultats des investigation geotechniques preliminaires. Ces resultats furent ensuite compare avec les investigations geotechniques detailee, durant la construction. Nous notons que cet article ne considere que les resultats additionels obtenu a la pile Sud de la portee principale. Le sondage effectue a chaque pieu de foundation de la pile montre que pour une pile de pont donnee il peut existe des variations significative de condition de foundation entre les pieux.

A cause de ces variation de donnee, une distribution probabilistique approprie est necessaire pour l'analyse statistique de la capacite de l'ouvrage. En utilisant un ensemble de modeles probabilistiques identifie dans l'article, l'etude propose une resitance de calcul de la roche adoptee pour le calcul des sabots de pieu. L'impact de l'anisotropie de la resitance de la roche sur les conditions d'etude a aussi ete evalue. Les consequences survenant de l'utilisation de critere de calculs base sur un modele probabilistique pour satisfaire une condition de calcul en etat limite sont brievement discutees. L'utilisation d'une distribution normale semble affecter significativement les parametres de calculs.

Keywords : rock strength, rock sockets, point load index, statistical distribution models, characteristic values, anisotropy, Queensland

## 1 INTRODUCTION

The Gateway Upgrade Project in the State of Queensland, Australia involves a major road upgrade and construction of a second six- lane Gateway Bridge immediately downstream from the existing bridge. The bridge structure spans 1.6 km between abutments, with a central river span ~260 m. This main span of the bridge spanning piers 6 & 7 (with compressive ultimate limit state socket loads up to 36 MN), consists of 24 bored piles of 1.8 m diameter at each pier location. Permanent steel liners were installed to about RL -10 with socket lengths extending to about RL -30.

This site is located close to the mouth of the Brisbane river and has Holocene (young material) generally overlying the Pleistocene (older) Alluvium. The following major sequences were intercepted at piers 6 & 7:

- Weak estuarine soils overlying, alluvial sands and gravels,
- An upper low to very low strength cap rock and low grade coal overlying,
- Interbedded sandstone, siltstone and mudstones.

The rock within the zone of influence of the sockets belongs to the Aspley – Tingalpa formation of the Triassic period. This formation does not show significant folding, but is known to have faulting as a consequence of crustal tension in the Tertiary period (Willmott and Stevens (1992)).

The bored piles were socketed into the underlying sedimentary rock which is generally flat lying in terms of the geological structure with only shallow bedding dips. It is interbedded and generally brecciated with depth. Spread footings were feasible at the southern piers of the bridge where competent shallow rock conditions were established. Constant normal direct shear and pressuremeter testing formed part of the investigation suite. During construction, to confirm the design geotechnical model, a borehole was drilled at each rock socket location to depths generally deeper than the socket bases at these pier locations. To further corroborate the design parameters, two land based test piles with Osterberg cells (O-cells) were also carried out prior to construction.

This paper presents a statistical analysis of intact rock strength properties at Piers 6 & 7 using pre-construction geotechnical data (10 boreholes). These findings are then compared with an analysis based on additional borehole data captured during construction. However, this has been confined to the additional Pier 6 data (24 boreholes) only. Analysis is extended with this additional Pier 6 Point load index data to

*Proceedings of the 17th International Conference on Soil Mechanics and Geotechnical Engineering*
*M. Hamza et al. (Eds.)*
*© 2009 IOS Press.*
*doi:10.3233/978-1-60750-031-5-60*

identify suitable probability density functions for statistical and reliability modeling. Using the probabilistic models identified, the study undertakes to rationalize the design rock strength input model (10 MPa design UCS) adopted for socket design against the background of the Rowe & Armitage (1987) methodology which was the stipulated primary design method in the project brief. The implications of using the design UCS with the probability models identified to satisfy limit state material characterization requirements are briefly discussed. The impact of rock strength anisotropy on the design UCS is also investigated.

## 2 STATISTICAL ANALYSIS OF DATA

### 2.1 *UCS – Point Load Index Relationship*

Broch and Franklin (1972) established a ratio of 24 between the point load strength index tests [$I_s$ (50)] and the UCS value. However, variations of the UCS / $I_s$ (50) ratio of 5 to 17 has been found for much of the "soft" meta - sedimentary rocks in south east Queensland (Look and Griffiths, 2001, 2004). The lower ratios were for the bedded and oldest bedrock of the Brisbane area – the Neranleigh-Fernvale Formation and Brisbane Phyllites of approximately Devonian to Carboniferous age.

Table 1 shows the results of regression (with forced intercept of zero) between the Point Load Index test and UCS (on vertical core specimens) for the interbedded sandstone – siltstone layer, with outliers eliminated using judgment to enhance confidence in the $R^2$ statistic (Figures 1 and 2). Using the automatically generated regression lines from spreadsheets can result in lines with a questionable relationship.

Table 1. Correlation between UCS and / $I_s$(50) for the interbedded sedimentary layer

| Test | Pre-construction data – Piers 6 & 7 | | | During construction data – Pier 6 | | |
|---|---|---|---|---|---|---|
| | Speci-men | $R^2$ | UCS / $I_s$ (50) | Speci-men | $R^2$ | UCS / $I_s$ (50) |
| Axial | 68 | 0.52 | 24 | 17 | 0.74 | 28 |
| Diamet. | 62 | 0.53 | 40 | 17 | 0.56 | 42 |

A strong anisotropy is seen in the results due to the sub-horizontally bedded nature of the rock. The UCS / $I_s$(50) ratio for the axial to diametral directions is ~ 0.5 which is called the anisotropy factor (AF) in this study.

The UCS / $I_s$ (50) ratios computed in this study are therefore much higher than those previously quoted by Look and Griffiths (2004). The lower ratios previously determined by these authors were for shallow meta–sedimentary rocks of South-East Queensland, whereas the majority of the sedimentary rocks tested for this site are deeper, and of a different geological period. A ratio of UCS / $I_s$ (50) = 40 was adopted for the detailed study of diametral point load data reported in what follows. Table 1 also confirms the validity of the pre-construction UCS / $I_s$ (50) model as it is in general agreement with the findings from the construction phase testing.

Due to difficulties in physical inspection of the deep socket bases, sockets were designed to carry the serviceability loads predominantly in shaft friction. Therefore the UCS of horizontal core specimens which are likely to be much weaker, especially given the strong anisotropy due to sub-horizontal bedding, would be relevant given the radial nature of normal stresses from socket loadings. As such UCS data is very expensive to obtain and almost operationally infeasible for deep borings, to investigate the impact of anisotropy, the UCS / Is (50) diametral value of 40 was factored by AF, the anisotropy factor, defined earlier. Therefore the impact of anisotropy was thus investigated conservatively via a reduced UCS / $I_s$ (50) diametral = 20.

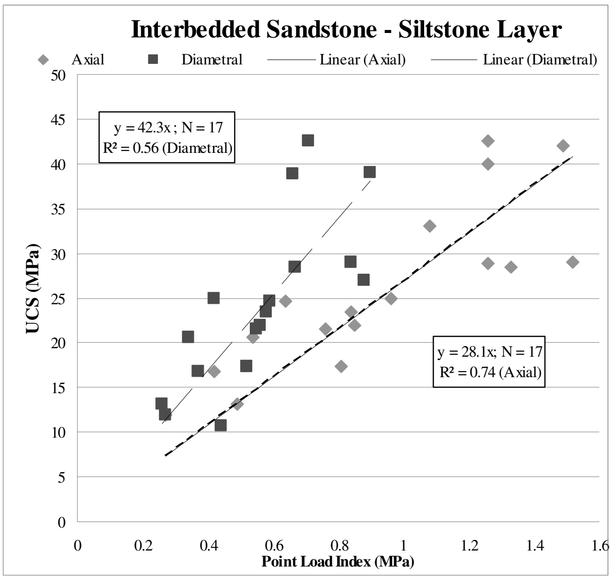


Figure 1. UCS / $I_s$ (50) ratios without outliers

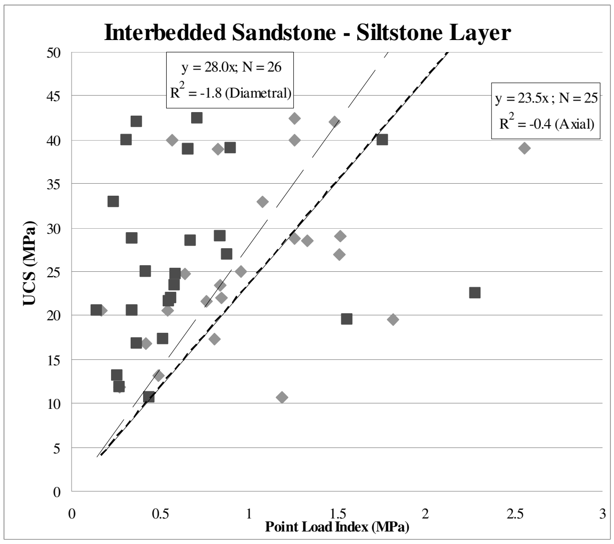


Figure 2. UCS / $I_s$ (50) ratios with outliers

### 2.2 *Statistical Distribution Model*

The Normal distribution generally adopted for material modeling suffers from allowing negative values and the resulting error is exacerbated with higher coefficient of variation (COV) values (Fenton & Griffiths, (2008)). Thus the appropriateness of alternative distribution models needs to be assessed and goodness-of-fit tests are commonly used for this purpose to discern differences between a hypothesized and the observed distribution. Three widely used tests are the Chi-Square, Kolmogorov-Smirnov (K-S) and the Anderson-Darling (A-D); the A-D test is very similar to the K-S test, but unlike both former tests, places more emphasis on tail values owing to the logarithmic nature of its test statistic and has better discerning ability. Hence the A-D test was used.

The statistical analysis of Pier 6 construction phase $I_s$ (50) (diametral) data was carried out using @Risk software. This identified the best fitting distribution in each case from a set of 35 candidate distributions. However, the best fit distributions identified depend on the significance level assumed and the type of fitness test used, and hence may not be unique. Table 2 shows the best fit distributions and the ranking of Normal and the logNormal at selected piles as well as for the combined diametral data (330 Nos.), at Pier 6.

Table 2 and Fig. 3 show that the loglogistic distribution is overall the best. In addition, the lognormal, by far, has outperformed the Normal distribution statistical model. Further, some of the difficulties with the Normal distribution are evident in Table 3 when the 10% characteristic values are examined, when it yields negative $I_s$ (50) values. Given the greater familiarity of the logNormal over the loglogistic among engineers, and the difficulties associated with the Normal, the logNormal distribution is proposed for use despite it not being the best ranking candidate.

Table 2. Best Fit Distribution models at selected pile locations and with all 24 pile data combined

| Pier 6 | A – D Test ( 5% significance level ) | | |
|---|---|---|---|
| Pile # | Rank 1 | Normal - Rank | Log Normal - Rank |
| P6-5 | Expon | 6 | 13 (n/a) |
| P6-6 | Log Logistic | 7 | 3 |
| P6-7 | Weibull | 7 | 5 |
| P6-8 | InvGauss | 8 | 7 |
| P6-21 | Inv Gauss | 5 | 3 |
| P6-22 | LogLogistic | 8 | 4 |
| P6-23 | Normal | 1 | 5 |
| P6-24 | LogLogistic | 8 | 4 |
| P6-ALL | LogLogistic | 9 | 3 |

Table 3. Statistical analysis of $I_s$ (50) at selected pile locations and with all 24 pile data combined

| Pier 6 Pile # | Diametral $I_s$ (50) Statistics | | | 10% Characteristic (MPa) | |
|---|---|---|---|---|---|
| | Mean (MPa) | COV | No. of points | Normal | Log Normal |
| P6-5 | 0.85 | 39 % | 10 | 0.46 | n/a |
| P6-6 | 1.01 | 151% | 10 | 0.26 | 0.43 |
| P6-7 | 0.57 | 56% | 15 | 0.15 | 0.19 |
| P6-8 | 0.74 | 68% | 15 | 0.12 | 0.30 |
| P6-21 | 0.94 | 37% | 16 | 0.48 | 0.51 |
| P6-22 | 0.81 | 113% | 17 | (-0.13) | 0.20 |
| P6-23 | 0.81 | 40% | 13 | 0.40 | 0.45 |
| P6-24 | 0.61 | 87% | 18 | (-0.12) | 0.12 |
| P6-ALL | 0.82 | 91% | 330 | 0.03 | 0.24 |

The COV for the Diametral $I_s$ (50) varied from 31% to 156% for the 24 boreholes. Special attention needed to be given to those piles socketed in areas of low chraceteristic values and / or high COV.

## 2.3 *Characterization of the rock strength*

Two test piles (1.5 m dia.) with O-cells, conducted one on each bank, provided data on the load – settlement response of the sockets in the interbedded sedimentary layer. Based on confirmation from the load tests, a 10 MPa design UCS along with a value of 20 for UCS / $I_s$ (50) for the load carrying interbedded layer was adopted for rock sockets by the designer. The Rowe & Armitage method used stipulates the adoption of an *average* UCS for design.

In the reliability estimates shown in Table 4, the percentiles corresponding to the 10 MPa UCS design strength is lower than the percentiles for mean strengths (50% - 64%) for the probability models shown. This implies that the mean rock strengths under these distributions for the interbedded layer are higher than the 10 MPa design UCS. Therefore the use of a lower strength as the mean strength while being conservative satisfies the design requirement under Rowe & Armitage.

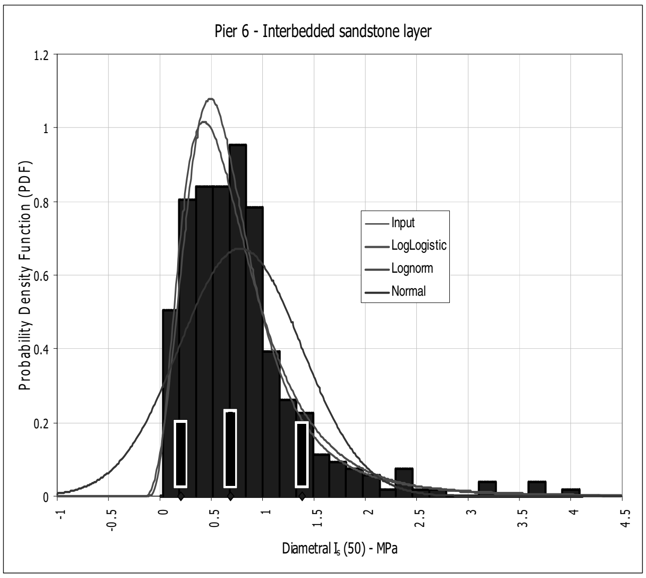


Figure 3. Comparison of distributions for the Is (50) Diametral values

Table 4. Characteristic Percentiles Corresponding to 10 MPa UCS Design Strength for various distribution models and UCS / $I_s$ (50) conversions

| LogLogistic Rank 1 | | Log Normal Rank 4 | | Normal Rank 9 | |
|---|---|---|---|---|---|
| UCS / $I_s$ (50) | | | | | |
| 40 | 20* | 40 | 20* | 40 | 20* |
| 10% | 34% | 11% | 35% | 18% | 31% |
| Mean Strength = 64% | | Mean Strength = 61% | | Mean Strength = 50% | |

*anisotropy conversion

On the other hand, if a limit state code requires a 10% (say) characteristic strength, then the variation in the interpretation of the 10 MPa design UCS strength with UCS / $I_s$ (50) (diametral) = 40, for the different distributions shown in the Table 4. In this regard, the advantage of using the logNormal which compares favorably with the best ranked distribution is evident.

Table 4 also shows how the situation is exacerbated if a strength reduction due to anisotropy were to be considered (i.e. the use of UCS / $I_s$ (50) (diametral) = 20). Despite these restrictions, the 10 MPa design UCS (with characteristic values of 31% - 35%) would still qualify under the Rowe & Armitage method as the mean percentiles are still higher (50% - 64%). Conversely, the adoption of the 10 MPa design UCS would be clearly unsafe under a 10% characteristic value, limit state code requirement for material strength.

At the quartile value the normal distribution is approximately comparable to the more highly ranked distribution models. Table 5 compares the Characteristic strengths based on these lower characteristic values.

Table 5. Characteristic UCS (MPa) using different distribution models with goodness of fit rank shown

| Characteristic Value | LogLogistic - 1 | Log Normal - 4 | Normal - 9 |
|---|---|---|---|
| 10% | 5.1(0.253 * 20) | 4.8(0.238 * 20) | 0.7(0.035 * 20) |
| 25% | 8.4 | 8.0 | 7.9 |
| Mean | 16.3(0.816 * 20) | 15.9(0.793 * 20) | 15.9(0.795 * 20) |

## 3 CONCLUSIONS

A statistical review of the intact rock strength data pertaining to the load bearing sub-horizontally interbedded sedimentary layer underlying the river piers of the Gateway Bridge duplication was undertaken. The high UCS / $I_s$ (50) ~ 40 computed for the diametral test is well supported by the investigation data while the UCS / $I_s$ (50) axial ~ 25 is almost the conventional ratio, i.e. 24. This has highlighted the need to account for strength anisotropy for socket design in view of the radial normal stresses on the socket wall.

UCS / $I_s$ (50) diametral distributions are not well modelled by the Normal distribution and while the lognormal is not the best fitting distribution, the use of the lognormal has been shown to have clear advantage in defining characteristic values for material strengths. The UCS strength of 10 MPa adopted for the design using Rowe and Armitage method clearly satisfies design requirements even under considerations of strength reduction due to anisotropy, i.e. with a use of UCS / $I_s$ (50) diametral ~ 20.

The use of the lognormal distribution for the rock strengths investigated has been shown to produce realistic strengths under limit state code requirements which generally stipulate characteristic strengths at low percentile values. Comparable predictions based on the Normal distribution could become unrealistically low, even bordering on negative values.

## ACKNOWLEDGEMENT

Chris Clark of Connell Wagner, Brisbane contributed to this discussion during the project delivery.

## REFERENCES

Broch, E. M. and Franklin, J. A. 1972. The point load strength test. Int. J. Rock Mech. Min. Sci. 9, pp 669-697.

Fenton, G. A. & Griffiths, D. V. 2008. Risk Assessment in Geotechnical Engineering, Wiley Publishers.

Look B. G. and Griffiths S. 2001. An engineering assessment of the strength and deformation properties of Brisbane Rocks. Australian Geomechanics Journal, Vol. 36, No. 3 pp 17 – 30.

Look B.G, and Griffiths S. 2004. Characterization of rock strengths in South East Queensland. Proc. 9th Aust. NZ Geomech Conf. Auckland, Vol. 1, pp 187 - 194.

Palisade Corporation. 2008. @ Risk – Risk analysis and simulation Add – In for Microsoft Excel

Rowe, R. K. & Armitage, H. H. 1987. A design method for drilled piers in soft rock. Can. Geotech. J., 24, pp 126-142.

Willmott W. and Stevens N. 1992. Rocks and Landscapes of Brisbane and Ipswich. Geological Society of Australia, Queensland Division.