Prediction of the onset of strain localization in non-coaxial plasticity

Prévision de l'apparition de la localisation de déformation dans la plasticité non-coaxiale

M.S. Huang, X.L. Lu, J.G. Qian
Department of Geotechnical Engineering, Tongji University, Shanghai, China
W.D. Wang
East China Architectural Design & Research Institute, Shanghai, China

ABSTRACT

A 3D non-coaxial elasto-plasticity model with a novel yield function is proposed and employed to study the onset of strain localization in plane strain and true triaxial tests. Bifurcation analysis under plane strain conditions shows that the proposed model is capable of giving an accurate prediction of the onset of shear band and producing a right trend of the band angle varying with the initial confining stress. Simulations of true triaxial tests show a good prediction of the influence of the intermediate stress ratio to the pre-bifurcation stress-strain relationships and the onsets of shear bands. Comparison with the predictions by the coaxial model indicates the introduction of the non-coaxial flow rule is critical to the prediction of strain localization.

RÉSUMÉ

Un 3D modèle non-coaxial de l'élastoplasticité avec une nouvelle fonction de la surface de charge est proposé et utilisé pour étudier l'apparition de la localisation de déformation dans les essais en déformations planes et triaxiaux vrais . L'analyse de bifurcation en état déformations planes prouve que le modèle proposé est capable de donner une prévision précise de l'apparition de la bande de cisaillement et de produire une bonne tendance de l'angle de bande variant avec le contrainte confiné initial. Les simulations des essais triaxiaux vrais montrent une bonne prévision de l'influence du rapport de contrainte intermédiaire en la relation entre contrainte-déformation de pré-bifurcation et l'apparition des bandes de cisaillement. La comparaison avec les prévisions par le modèle coaxial indique que l'introduction de la règle non-coaxiale d'écoulement est critique à la prévision de la localisation de déformation.

Keywords : strain localization; bifurcation; non-coaxial; plane strain; true triaxial

1 INTRODUCTION

Strain localization or shear band, which often occurs in the loading process of homogeneous soils and leads the loss of bearing capacity, is now a hot topic in geotechnical engineering. A special workshop, which is aiming to gather researchers all over the world together to discuss 'Bifurcations and Instabilities in Geomechanics', has been set and already held for eight times since 1988.

In the literature, various kinds of experimental, analytical and numerical approaches have been adopted to understand the physical property of strain localization. Theoretically, the formation of strain localization can be regarded as the bifurcation from the homogeneous deformations; meanwhile the predictions of the onset of shear band are critically dependent of the constitutive descriptions. Numerous researches have shown the deficiency of conventional plasticity models which are based on the assumption that the plastic strain-rate is coincident with the principal stress (i.e. coaxiality) (Rudnicki and Rice, 1975; Molenkamp, 1985; Vardoulakis and Graf, 1985; Papamichos and Vardoulakis, 1995). In order to improve the accuracy of theoretical predictions, non-coaxial behavior, which has long been observed in experiments (Ishilhara and Towhata, 1983; Gutierrez et al., 1991) and validated by theoretical analysis (Yang and Yu, 2006; Qian et al., 2008), should be considered. The non-coaxial term has been added into various plasticity model, such as Drucker-Prager model (Rudnicki and Rice, 1975), Mohr-Coulomb model (Bardet, 1991), sub-loading surface model (Hashiguchi and Tsutsumi, 2001) and deformation theory of plasticity (Vardoulakis and Graf, 1985), by use of which, the prediction of the onset of shear band was significantly improved.

The aforementioned non-coaxial models were built on 2D stress space and were validated only in biaxial tests, which induced the limitations of their applications. Qian et al. (2008) presented a non-coaxial plasticity modeling platform in 3D stress space by introducing the third stress invariant. However, the proposed 3D model was not suitable for true triaxial test since it has not been validated by true triaxial tests. The main objective of this paper is to propose a 3D non-coaxial elasto-plasticity model which is capable of simulating the stress-strain relationship and predicting the onset of strain localization of soil in true triaxial tests. The influence of intermediate stress ratio in formation of shear band and strength of soils is also studied.

2 NON-COAXIAL ELASTO-PLASTICITY MODEL

2.1 *General description*

According to the fundamental assumptions of the non-coaxial flow theory of plasticity, the strain increment $\dot{\varepsilon}_{ij}$ is the sum of elastic strain $\dot{\varepsilon}_{ij}^{e}$, coaxial plastic strain $\dot{\varepsilon}_{ij}^{cp}$ and noncoaxial plastic strain $\dot{\varepsilon}_{ij}^{np}$, i.e.

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{e} + \dot{\varepsilon}_{ij}^{p} = \dot{\varepsilon}_{ij}^{e} + \dot{\varepsilon}_{ij}^{cp} + \dot{\varepsilon}_{ij}^{np} \tag{1}$$

where the coaxial strain $\dot{\varepsilon}_{ij}^{cp}$ is determined from the conventional plasticity theory. By the composition of deviatoric

Proceedings of the 17th International Conference on Soil Mechanics and Geotechnical Engineering
M. Hamza et al. (Eds.)

doi:10.3233/978-1-60750-031-5-574

plastic strain which is shown in Fig. 1, the noncoaxial strain $\dot{\varepsilon}_{ij}^{np}$ is

$$\dot{\varepsilon}_{ij}^{np} = \frac{1}{H_t}\dot{s}_{ij}^{n} \tag{2}$$

where $s_{ij} = \sigma_{ij} - \delta_{ij}\,\sigma_{ii}/3$; H_t is the non-coaxial hardening modulus; $\dot{s}_{ij}^{n}$ denotes the non-coaxial stress rate, which is

$$\dot{s}_{ij}^{n} = \dot{s}_{ij} - \frac{\dot{s}_{kl}s_{kl}}{s_{mn}s_{mn}}s_{ij} - \frac{\dot{s}_{kl}S_{kl}}{S_{mn}S_{mn}}S_{ij} \tag{3}$$

where $S_{ij} = s_{ik}s_{kj} - \frac{2}{3}J_2\delta_{ij} - \frac{3}{2}\frac{J_3}{J_2}s_{ij}$; $J_2 = \frac{s_{ij}s_{ij}}{2}$; $s_{ij} = \sigma_{ij} - \delta_{ij}p$; $J_3 = s_{ij}s_{jk}s_{kl}/3$.

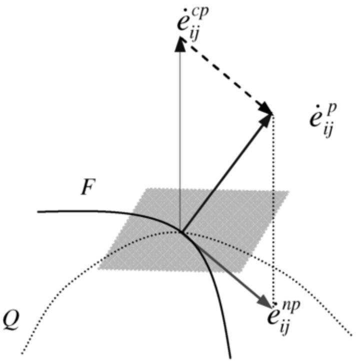


Figure 1. Non-coaxial strain rate

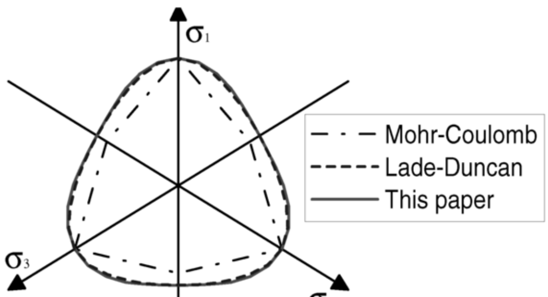


Figure 2. Yield surfaces in the deviatoric plane

2.2 *Non-coaxial Mohr-Coulomb model*

The yield function and plastic potential function of Mohr-Coulomb model are

$$\begin{cases} F = q - M\cdot g(\theta_\sigma)\cdot p = 0 \\ Q = q - M_c\cdot g(\theta_\sigma)\cdot p\ln(\dfrac{p}{p_0}) = 0 \end{cases} \tag{4}$$

where $p = \sigma_{kk}/3$, $q = \sqrt{3J_2}$, $\theta_\sigma = \sin^{-1}(3\sqrt{3}J_3/2J_2^3)/3$, $J_3 = \sqrt{s_{ij}s_{jk}s_{ki}/3}$; p_0 is the confining pressure; $g(\theta_\sigma)$ is a function of θ_σ, which describe the shape of yield surface in the deviatoric plane. Here, we adopt the function which was first proposed by William and Warnkle (1975),

$$g(\theta_\sigma) = \frac{1}{4(1-\beta^2)\cos^2(\frac{\pi}{6}-\theta_\sigma)+(2\beta-1)^2}\Big[2(1-\beta^2)\cdot\cos(\frac{\pi}{6}-\theta_\sigma) + \sqrt{4(1-\beta^2)\cos^2(\frac{\pi}{6}-\theta_\sigma)+\beta(5\beta-4)}\cdot(2\beta-1)\Big] \tag{5}$$

With the parameter β being defined as $\beta = M_{TE}^{f}/M_{TC}^{f}$, where M_{TE}^{f} and M_{TC}^{f} can be obtained form triaxial tension tests and triaxial compression tests. The applicability of Eq.(5) in modeling true triaxial tests has been validated (Lu, 2009).

We use a hyperbolic relationship between the stress ratio and the plastic shear strain to define the hardening function M , i.e.

$$M = \frac{\varepsilon_s^p}{A+\varepsilon_s^p}M_{TC}^{f} \tag{6}$$

where *A* is a model parameter.

3 NUMERICAL SIMULATIONS

3.1 *Plane strain tests*

The proposed model is used to simulate the stress-strain relationships of a series of plane strain tests performed by Han and Drescher (1993). All model parameters are summarized in Table 1.

Table 1. Parameters for plane strain tests

Elastic parameters	Plastic parameters
	M_f=1.70
E=175Mpa	M_c=0.965
ν=0.1667	β =0.705
	A=0.001

The predicted stress-strain relationship and bifurcation points are plotted in Fig. 3, which shows the non-coaxial parameter H_t is critical to the determination of the bifurcation point. The bifurcation point is the earliest in coaxial plasticity (i.e. $H_t = \infty$) and is delayed with the decreasing of H_t, as H_t approaching to 0.04*G*, the predicted result almost coincides with experiment.

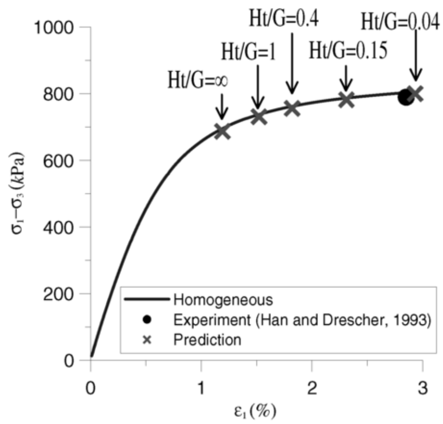
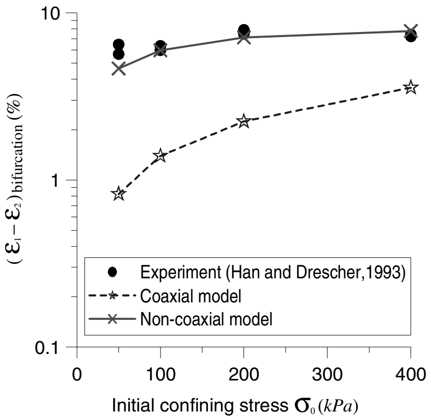
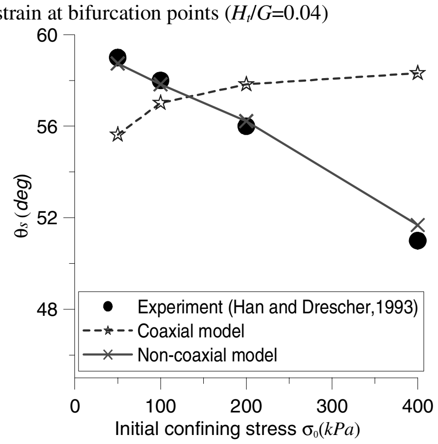


Figure 3. Influences of H_t on bifurcation points (experimental data after Han and Drescher, 1993)



(a) Shear strain at bifurcation points (H_t/G=0.04)



(b) Shear band inclination (H_t/G=0.04)

Figure 4. The influence of confining stress (experimental data after Han and Drescher, 1993)

As shown in Fig. 4, the non-coaxial model provides an accurate onset of shear band and reflects the right trend of the influence of initial confining stress, these results compare well with the experiments.

3.2 *True triaxial tests*

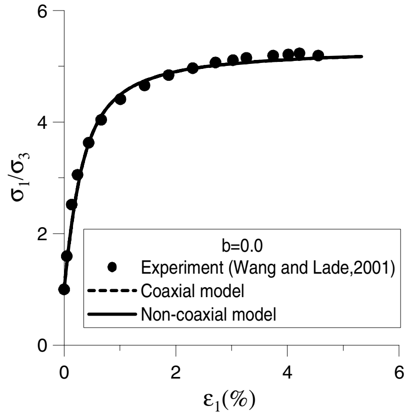
The proposed non-coaxial model is also employed to simulate the true triaxial tests (Wang and Lade, 2001) in which the initial confining stress is 49kPa, all model parameters are summarized in Table 2.

Table 2. Parameters for true triaxial tests

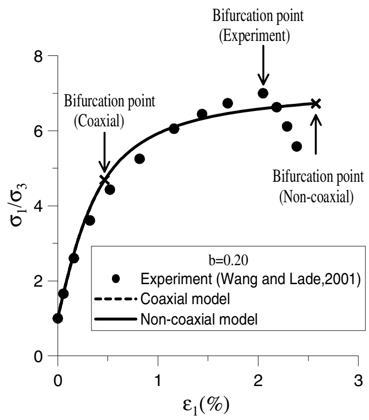
Elastic parameter	Plastic parameter
	M_f=1.75
E=175Mpa	M_c=0.965
v=0.1667	β=0.705
	A=0.001
	H_t=290kPa

The simulations of stress-strain relationships and bifurcation points in a series of intermediate principal stress ratios b are shown in Fig. 5. Coaxial model produces bifurcation point when 0.18<b<0.83, while non-coaxial model produces bifurcation point when 0<b$\leq$1. As shown in Fig. 6, the bifurcated strains ε_1^b predicted by the coaxial plasticity in different b values deviate largely from experiments, while these deviations can be largely reduced by adopting the non-coaxial model.

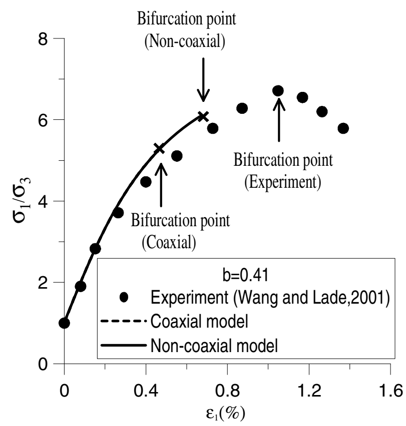
The peak stress ratio, which is shown in Fig. 7, indicates a reduction for the formation of strain localization. It is underestimated by the coaxial model between 0.2<b<0.8 and overestimated when 0<b<0.1 and 0.8<b<1.0. For the non-coaxial model, the peak stress ratio shows a significant increase from b=0 until about 0.20, after a fluctuation when b arrives about 0.5, it remains almost constant in range from 0.5 to unity, the predicted results compare well with the experiments.



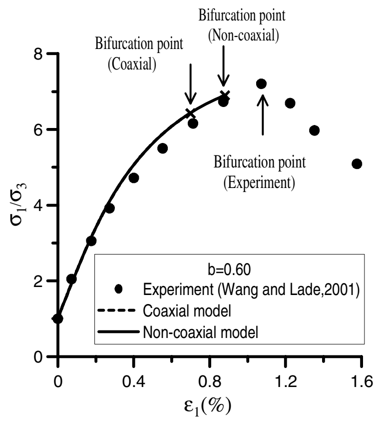
(a) b=0



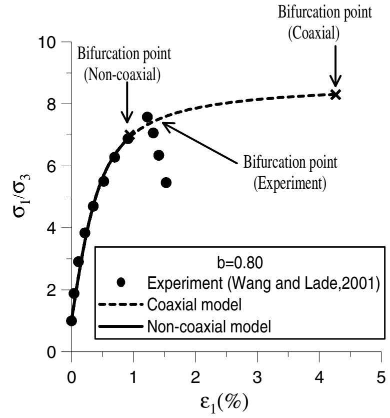
(b) b=0.2



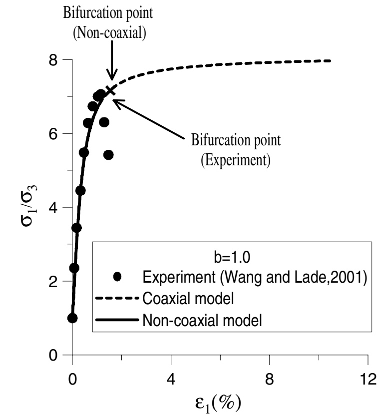
(c) b=0.41



(d) b=0.6



(e) b=0.8



(f) b=1.0

Figure 5. Major principal strain vs. Stress ratio, and bifurcation points

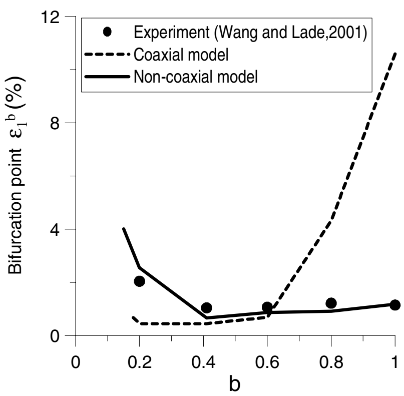


Figure 6. Major principal strain at the bifurcation point

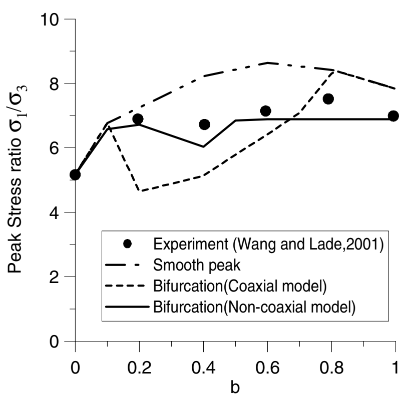


Figure 7. Peak stress ratio vs. intermediate principal stress ratio *b*

4 CONCLUSIONS

A novel 3D non-coaxial Mohr-Coulomb model is proposed and adopted to simulate the stress strain relationship in plane strain and true triaxial conditions. Bifurcation analysis in plane strain tests show the proposed model can predict the onset of shear band accurately and reflect the influence of initial confining stresses on band angles rightly. The simulations of true triaxial tests showed the proposed model can accurately predict the inception of shear band in various intermediate principle stress ratios.

ACKNOWLEDGEMENT

This work was financially supported by the National Natural Science Foundation of China under research Grant No. 50825803 and No. 50679090. These supports are gratefully acknowledged.

REFERENCES

Bardet, J. P. 1991. Orientation of shear bands in frictional soils. Journal of Engineering Mechanics, ASCE, Vol.117, No.7, pp. 1466- 1484.

Gutierrez, M., K. Ishilhara, and Towhata, I. 1991. Flow theory for sand during rotation of principle stress direction, Soils and Foundations, Vol. 31,No. 4, pp. 121-132.

Han, C. H. and Drescher, A. 1993. Shear bands in biaxial tests on dry coarse sand. Soils and Foundations, Vol. 33, No. 1, pp. 118- 132.

Hashiguchi, K. and Tsutsumi, S. 2001. Elastoplastic constitutive equation with tangential stress rate effect, International Journal of Plasticity, Vol. 17, No. 1, pp. 117-145.

Ishilhara, K. and Towhata I. 1983. Sand response to cyclic rotation of principal stress directions as induced by wave loads, Soils and Foundations, Vol. 23, No. 4, pp. 11-26.

Lu, X. L. 2009. Theoretical prediction and numerical modeling of strain localization in geomaterials, PhD thesis, Tongji University, Shanghai, China.

Molenkamp, F. 1985. Comparison of frictional material models with respect to shear band initiation, Géotechnique, Vol. 35, No. 2, pp. 127-143.

Papamichos, E. and Vardoulakis, I. 1995. Shear band formation in sand according to non-coaxial plasticity model, Géotechnique, Vol. 45, No. 4, pp. 649-661.

Qian, J. G., Yang, J. and Huang, M. S. 2008. Three-dimensional noncoaxial plasticity modeling of shear band formation in geomaterials, Journal of Engineering Mechanics, ASCE, Vol. 134, No. 4, pp. 322-329.

Rudnicki, J. W. and Rice, J. R. 1975. Conditions for the localization of the deformation in pressure sensitive dilatant materials, Journal of the Mechanics and Physics of Solids, Vol. 23, No. 6, pp. 371-394.

Vardoulakis, I. and Graf, B. 1985. Calibration of constitutive models for granular materials using data from biaxial experiments, Géotechnique, Vol. 35, No. 3, pp. 299-317.

Wang, Q. and Lade, P. V. 2001. Shear banding in true triaxial tests and its effect on failure in sand, Journal of Engineering Mechanics, ASCE, Vol. 127, No. 8, pp. 754-761.

William, K. J. and Warnke, E. P. 1975. Constitutive model for the triaxial behavior of concrete, International Association for Bridge and Structure Engineering Proceedings, Bergamo, Italy. 19.

Yang, Y. M. and Yu, H. S. 2006. A non-coaxial critical state soil model and its application to simple shear simulations, International Journal for Numerical and Analytical Methods in Geomechanics, Vol. 30, No. 13, pp. 1369-1390.