# Dynamic impedance functions of machine foundations on sandy soils by physical model tests

Impedance dynamique des foundations de machine sur des soles sableuse avec des modèles

physiques

Fardin Jafarzadeh & Hossein Ghasemzade Mashhadi Sharif University of Technology, Tehran, Iran

#### ABSTRACT

In order to improve new methods of design of machine foundations by means of impedance function, implementation of experimental investigations to calibrate these methods is vital. The physical model tests were carried out in a steel container with dimension of  $1 \times 1 \times 0.9 \text{ m}$ . The Babolsar sand was employed in the model. To eliminate reflection of elastic waves, a sawdust layer was used in the lateral walls and bottom of the container. Also, in order to have homogeneous medium with unified density, air pluviation of sand method was employed. Response of the models to harmonic excitations was measured using various instruments deployed around the foundation. Results of the tests are shown in the impedance function form and effects of inertia and embedment of footing and dynamic loading level on the impedance function are discussed.

# RÉSUMÉ

Afin d'améliorer des nouvelles méthodes de design des fondation de la machine par fonction d'impédance résistance, l'instauration de vérifications expérimentales afin de calibrer ces méthodes est essentielle. Les tests de modèles physiques ont eu lieu dans un contenant de métal de dimension de 1x1x0.9 m. Le sable Babolsar a été utilisé dans le modèle Afin d'éliminer la réflexion d'ondes élastiques, une couche de poussière de bois a été utilisée sur les murs latéraux et le fond du contenant. De plus, afin d'avoir une consistance homogène avec une densité uniforme, la méthode de pluviationué de sable a été utilisée. La réponse des modèles aux occilation harmoniques a été mesurée en utilisant divers instruments déployés autour de la base. Les résultats de ces tests sont décrits dans la formule de la fonction d'impédance résistance et les effets de l'inertie et de la fixation enchassement de la base et les degrés de charge dynamique sur la fonction d'impédance résistance sont décrits.

Keywords : dynamic response, machine foundation, impedance function, Babolsar sand, physical model

## 1 INTRODUCTION

The design of a machine foundation should be resulted in a safe and economically foundation block. For satisfying existing criteria such as operational, structural and psychological criteria related to machinery, many investigators have studied this subject since the last several decades. Theoretical investigations in this subject have been carried out extensively since primitive classical works based on the elastic waves propagation theory such as classical work by Lamb 1904. By the time, new methods based on single degree of freedom system, lumped parameters and dynamic impedance function were developed. Theoretical impedance function for various condition such as foundation embedment, infinite or layered soil, homogeneous or nonhomogeneous media has calculated by analytical, semi analytical and numerical approaches (Gazetas 1983, Gazetas 1991, Novak 1987). In contrast, experimental data which is necessary for verification and reliable application of these theoretical studies in practice are rare. Particularly, determination of foundation-impedance function has a significant effect on the soil-structure interaction problems. Hence, experimental evaluation of impedance function by the physical models can help to the investigators to compare and calibrate impedance function come from theoretical studies. Also, effect of important parameters on the dynamic impedance such as embedment and inertia of footing and dynamic loading intensity have been considered very limited in previous experimentally studies.

On the other hand, experimental investigations in the vibration foundations problems have performed in various

levels; post construction measurement of the response of real foundations; (Crouse et al. 1990), small or relatively large scale field experiments; (Baidya & Rathi 2004) and small scale laboratory experiments, usually conducted on soil filled in a container; (Nii 1987, Asadi Nik 2006). Two factors in the latter experiments have important role in achievement of the accurate and applicable results. First, determination of insitu dynamic properties of soil profile in the model such as shear modulus and Poisson ratio, v. Second, boundary/box effect caused by reflection of elastic waves from finite boundary of soil model emanate from foundation. These spurious waves can profoundly affect the amount of radiation damping (Gazetas 1983).

In this paper the results of some small scale physical model tests are presented. The tests were conducted by various depths of embedment and different static weights. Results of the test are shown in the form of impedance function.

# 2 THEORETICAL BACKGROUND

Hsieh in 1962 discovered that dynamic behavior of vertically loaded massive foundation can be shown by a mass-springdashpot system. Equation of motion of a one degree of freedom system can be described by:

$$Q = mU + CU + KU \tag{1}$$

in which U,  $\dot{U}$  and  $\ddot{U}$  are displacement, velocity and acceleration; respectively, of the oscillating mass. Q is the external dynamic force related to operation of the machines. The lumped parameters are the equivalent mass, m, effective damping, C, and effective stiffness K. The lumped parameters were presented to approximate the response of one-DOF system in vertical mode of vibration in the low and medium frequency ranges as (Lysmer 1966):

$$K = \frac{4Gr_0}{1 - v} \quad ; \quad C = \frac{3.4Gr_0^2}{1 - v}\sqrt{G\rho} \tag{2}$$

where  $r_0$ , G and  $\rho$  are equivalent radius, shear modulus and density of soil; respectively. Based on Equation 2, the stiffness and dashpot parameters are frequency-independent. In reality and significantly in the high frequency ranges, these coefficients are frequency-dependent. To determine these lumped frequency-dependent parameters in current method of dynamic analysis, impedance function, *I*; of a rigid but massless foundation are employed. Impedance function mathematically is defined by:

$$I = \frac{R(\omega)}{U(\omega)} \tag{3}$$

in which  $R(\omega)$  and  $U(\omega)$  are Fourier transform of R(t) and U(t); respectively. U(t) and R(t) are displacement of the footing and the dynamic loading exerted on the interface between footing and soil; respectively; and  $\omega$  is circular frequency of the dynamic loading. It is interesting to note that displacement and dynamic force are out of phase and Equation 3 can then be shown in the complex form as:

$$I = I_1(\omega) + iI_2(\omega) \tag{4}$$

Also, Frequency Response Function, *FRF*, can exhibit the dynamic characteristic of a system by considering inertial force induced from footing. Mathematically, FRF is defined by (in the subject of dynamic response of foundation):

$$FRF = \frac{U(\omega)}{Q(\omega)} \tag{5}$$

An analogy between one-DOF system and three dimensional massless foundation-soil systems based on Equations 1 and 3 can be carried out (Gazetas 1983). This analogy results in:

$$I = \frac{R}{U} = K - m\omega^2 + iC\omega \tag{6}$$

This result confirms Equation 4. Real part of Equation 6 denotes the stiffness and inertia effects of supporting soil and imaginary part shows radiation and material damping of the system. Also, another result which can be drawn is dependence of inertia and damping on frequency of vibration. To separate the static stiffness, K, Equation 6 can be rewrite as:

$$I = K(k + ic_s \omega) \tag{7}$$

where k and  $c_s$  are named stiffness and damping coefficients; respectively. This equation implies that the dynamic impedance of a single degree of freedom may be expressed as a product of K times a complex number which contains inertia and damping of system. The results of the tests are presented in following section using these parameters. Furthermore, R and Q can be related as:

$$P(t) + mU(t) = Q(t)$$
(8)

where *m* in this study is mass below the load cell (mass of footing and its attachments). In this study, *FRF* directly is calculated by analyzer apparatus. Referring to Equation 6, if  $m\omega^2$  is added to the reverse of real part of *FRF* function, the real part of impedance function is achieved. Imaginary part of impedance function equals with the reverse of imaginary part of *FRF* function.

In this scaled simulation tests for application of its results in practice by massive machine foundation, results in the case of frequency of vibration are present in form of dimensionless frequency. This dimensionless number defines as:

$$a_0 = \frac{\omega r_0}{V_s} \tag{9}$$

where  $\omega$  and  $V_s$  are circular frequency of vibration and shear wave velocity in the soil medium; respectively.

## **3 PREPARATION OF THE MODEL**

The laboratory experiments were carried out in a steel container which its dimensions are 1, 1, 0.9 *m* in width, length and depth; respectively. Figure 1 shows the schematic experimental set up. A square footing by area and equivalent radius equal to  $177 \text{ cm}^2$  and 7.5 *cm*; respectively, was employed. Original weight of this footing is 3.5 *Kgf*. One of the parameters which may affect the dynamic response of foundation is inertia of footing. To study this factor, static weights were used. These weights were attached symmetrically to footings in several stages and in each stage impedance function is calculated. The static weights (each about 0.4 *kgf*) attached to the square footing are shown in Figure 2a.



Figure 1. Schematic experimental set up of physical models.

In process of preparation of the model, two factors were vital: boundaries and existence of homogeneous medium. In order to absorb elastic waves emanate from foundation vibration and prevent rigid boundary wave reflection, the sand was surrounded by a 10 *cm* sawdust layer along the rigid walls. The sawdust was selected because it can be expected, in view of its properties to dissipate elastic waves receiving to this region. Also, a 15 *cm* sawdust layer was placed at the bottom of container for simulating extension of medium in vertical direction in reality, as an infinite half space. Also, In order to have homogeneous medium with unified density, air pluviation of sand method was employed. Figure 2b shows the box and pluviation of the sand used in these tests.



Figure 2. a) Footing, static weights and attachment of static weights on the footing; b) air pluviation of sand method.

The box was filled with uniform sand classified as SP in the Unified Classification, called Babolsar sand provided from Caspian Sea shores. The density calibration tests were implemented by means of several sieves (opening diameters of 2, 4, 6 and 8 mm) and different heights of pluviation (20, 30 and 70 cm). Finally, relative density was calibrated based on the height and sieve number. Based on calibration tests to create a homogeneous medium by relative density equal to 50%, the sieve with 6 mm opening diameter and 64 cm height of pluviation were selected.

## 4 EXCITATION AND INSTRUMENTATION PROCESS

In this investigation, model footing was subjected to vertical mode of harmonic vibration by means of an apparatus which generated harmonic dynamic loading. This harmonic force used to model the operation of reciprocating machines. These harmonic signals generated by an analyzer were transmitted into an electromagnetic shaker through a B&K amplifier causing oscillation of the rod connected to the footing. The shaker, rod and footing are shown in Figure 1. In these tests, frequency span of the dynamic loading, based on the customary operation of the harmonic machines and condition of simulation based on dimensionless frequency ( $a_0$ ), was selected 5-300 Hz. The dynamic force amplitude induced to the footing was constant. Therefore, the dynamic loading was force controlled.

Two piezoelectric accelerometers and load cell were placed at the top and center of footing and were then linked to the analyzer to measure the vibration amplitude and force; respectively. The load cell was connected to the rod and footing to measure exciting harmonic forces (Q).

In order to monitor stress and displacement resulted from propagation of elastic waves inside of the soil produced by the foundation vibration, six strain gage type sensors were placed at the various depths of the soil in the container (Figure 1). These sensors include three accelerometers and three pressure cells. Two LVDTs were also placed on the footing to measure displacement amplitude of vibrations.

# **5 COMPLEMENTARY TESTS**

The most important lack in other similar experimental investigations in literature, is experimental assessment of shear modulus, G, of medium. To achieve this parameter and Poisson's ratio, v, ; accelerometers placed inside the soil were used. Several tests by means of impact of the hammer on the surface of the soil were performed. Applying the accelerometer embedded in soil, these pulses in various depth of the medium were sensed and the velocity of shear and compressive waves were calculated. Based on the results tests, shear modulus, G, in the soil medium is rather constant and equal to 19 *MPa*. Poisson's ratio was then calculated 0.35 in the medium. In other tests the static loads by the analyzer exerted on the footings and static stiffness of system was calculated. Based on these tests, the static stiffness was 2.5 MN/m.

Maximum and minimum density of the Babolsar sand was measured according to the ASTM-D4253 standard and was reported 1.915 and 1.646  $grf/cm^3$ ; respectively. The total unit weight of sawdust used at model boundaries was 6.5  $kN/m^3$ .

#### 6 RESULTS AND DISCUSSIONS

By considering Equation 2 and shear modulus, Poisson ratio and equivalent radius, static stiffness was calculated 8.8 MN/m. Reason of discrepancy between static stiffness calculated by Equation 2 and test results reported in previous section (2.7 MN/m) is the assumption of infinite elastic medium in Equation 2. This comparison shows amount of faults in modeling of infinite half space by use of sawdust layer as an absorber of elastic waves. In following section results of the tests in the form of dynamic impedance are compared by typical theoretical impedance function for infinite half space or other status of layering.

#### 6.1 Impedance function

Impedance function for the tests is calculated from Equations 5 through 7. In Figure 3a,b (without the static weights) the real and imaginary part of the original surface foundation are shown. The stiffness and dashpot coefficient is calculated in the form of Equation 7. Comparing these results by theoretical results for infinite half space (Gazteas, 1983) shows discrepancy between these results. These differences can be emanated from reflection of elastic waves from artificial boundaries. Also, these figures can be used in design process of machine foundation by calculating dynamic and dashpot coefficient of system in any frequencies.

## 6.2 Inertial effects

Real and imaginary parts of the impedance function for the 4, 12 and 20 static weights was calculated. Total weight by these static loads attached to the footings is 5.0, 8.3 and 11.1 kgf; respectively. These functions are shown in Figure 6a,b. Also, for comparison, this function for footing without static weights (the original weight of the footing is 3.5 kgf) is shown. Dynamic load level in all of these tests was selected 40N. According to Figure 3a,b, the dynamic stiffness and dashpot coefficient less than 25 Hz are independent from footing inertia, but in frequencies more than 25 Hz this factor increase by increasing in the inertia of the footing. These results show that inertia of footing has effect in moderate and high frequencies in dynamic response of foundation and it can affect real and imaginary part of dynamic impedance function. This conclusion in the case of dynamic stiffness coefficient can be described by effects of variation of inertia of footing on a part of soil mass which vibrate by oscillation of footing and affect the real part of impedance function.

#### 6.3 Dynamic loading level effects

Current theoretical methods in assessment of dynamic response of foundation assume elastodynamic behavior for 1dof system. Therefore, evaluation of dynamic impedance is independent from intensity of exciting force. To assess this parameter, the tests were performed in the various dynamic force levels. For example, results of several tests conducted by several dynamic loading amplitudes as 8, 20 and 35 N are shown in Figures 4a and b. Referring to these figures, damping of system in various dynamic loading is relatively constant and therefore is independent from dynamic loading level. Although dynamic stiffness is constant in dimensionless frequency less than up to 0.25, in other dimensionless frequency range, it decreases by increasing intensity of loading. This results show amount of nonlinearity in behavior of supporting soil in dynamic loading. As a result, application of the linear elastic model is not reliable and accurate particularly in the moderate or high frequency of loading.



Figure 3a, b. Effect of inertia of foundation on the dynamic response of foundations.

## 6.4 Embedment of footing effects

To study the effects of embedment on the dynamic response of foundations, the footings were embedded in two status; full and half embedment. In Figure 5a, b results of these tests on the square footing in dynamic loading level 20N are reported. Several points were observed in these results. The stiffness of system decreases by increasing in depth of embedment. This variation can be related to effect of inertia of supporting soil in dynamic stiffness; Footing by full embedment causes oscillation of higher amount of surrounding soil than half embedment footing. Variation of damping coefficient in this status is relatively constant and embedment of footing has dispensable effect on damping of system.

## 7 CONCLUSION

In current methods of design of machine foundation, impedance function has important role. Evaluation of effect of main parameters on the impedance function can be helped to improvement of existing theoretical approaches in this subject. In this investigation, effect of several factors on the dynamic response of foundations is investigated experimentally. Based on the results, various outcomes were achieved. The test results show amount of nonlinearity in dynamic behavior of the soil. This nonlinearity can be related to effect of loading level on dynamic stiffness of system while damping is independent from intensity of loading. Also, applying the impedance function in the machine foundations problems by variation in inertia must be considered particularly in moderate and high frequency of vibration. Effect of inertia on dynamic stiffness and damping in high frequency ranges are indispensable. In the subject of embedment of footing, the test results show decreasing of dynamic stiffness by increasing depth of embedment. In this status, dashpot coefficient is relatively constant in different depth of embedment.



Figure 4a, b. Effect of dynamic loading level on the dynamic response of foundations.



Figure 5a, b. Effect of footing embedment on the dynamic response.

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