A 2D mathematical model for the CBR of a road sub base in Ilorin, Nigeria

Un modèle mathématique bidimensionnelle pour la CBR d'un matériel de sous-bas en Ilorin Nigeria

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ABSTRACT.
This paper reports the development of a two dimensional mathematical model for the California Bearing Ratio (CBR) of a typical sub base material in terms of the density and moisture to replace the hitherto graphical model of an iso-CBR plot. Conventional laboratory compaction and CBR tests were simultaneously conducted on samples of soils obtained from the highway borrow pits around Ilorin township according to relevant portions of the British standard testing procedure; BS 1377: 1990. The outcomes of the test were used for the formulation of the relationships of the CBR with the resultant of the vectorially combined normalized dry density and moisture content at low compactive and medium compactive energy levels (LCE and MCE). The most efficient forms of the models for the samples from 0 -1m and 1-2 m depths below the ground surface are the quadratic and third order equations, with the R^2 values of more than 90 %. Indeed the differentials of the quadratic model proved that the maximum CBR occurs at Л value of √2 which is the actual indication of the optimum condition when the two normalized parameters are each equal to unity, their maximum values.

Keyword: CBR, normalized density and moisture intelligent compaction control.

RÉSUMÉ
Ce document rapporte le développement d'un modèle mathématique bidimensionnel pour le rapport de roulement de la Californie (CBR) d'un matériel typique de sous-bas en termes de densité et humidité pour remplacer le modèle jusqu'ici graphique d'une parcelle de terrain OIN-CBR. Le tassement conventionnel de laboratoire et les essais de CBR ont été simultanément effectués sur des échantillons de sols obtenus à partir des emprunts de route autour de la banlieue noire d'Ilorin selon les parties appropriées de la procédure d'essais de norme britannique ; 1377:1990 des BS. Les résultats de l'essai ont été employés pour la formulation des rapports du CBR avec la résultante de la densité sèche et de la teneur en eau normales vectorially combinées aux forces de compactage de compactage et moyennes basses (LCE et ECM). Les formes les plus efficaces des modèles pour les échantillons provenant des profondeurs de 0 -1m et de 1-2 m au-dessous de la surface au sol sont les équations quadratiques et de troisième ordre, avec les valeurs R2 de plus de 90 %. En effet les différentiels du modèle quadratique ont montré que le maximum CBR se produit à la valeur de Л de √2 ce qui est l'indication réelle de l'état optimum quand les deux paramètres normalisés sont chacun égale à l'unité, leurs valeurs maximum.

Mot-clé : CBR, densité normale et commande intelligente de tassement d'humidité.

1 INTRODUCTION

Conventionally a soil to be employed for the construction of pavement for highways and airports should be subjected to compaction and strength tests, whose outcomes have important applications including workmanship (relative compaction), strength (CBR) based quality control of earth works and the selection of appropriate compaction plants and economic number of roller passes for desirable specifications/ compactive energy when complimented with outcome of compaction trial sections. Basically, soil compaction at constant compactive effort and different moisture contents generally produces a dry density – moisture relationship with a well defined maximum dry density and corresponding optimum moisture content. Many attempts have produced, though separately; moisture – density, density – strength and moisture – strength (CBR) or moisture – density - strength predictive models either reflecting the soil index or some other properties. Examples abound in the form of (i) a fourth order polynomial for plasticity, silt clay fractions and or normalized moisture content (defined as prevailing moisture – optimum moisture)[1] (ii) the ISO CBR conceptualized as Soil Strength Signature[2] (iii) graphical plots [3, 4 5] and specifically (iv) in terms of a multivariate linear regression model for a Lateritic soil [6, 7]. Actually the usefulness of CBR – Moisture – Dry Density relationship needs not much emphasis again, however the need to express in mathematical forms reflecting the interwoven factors of prevailing moisture (w), density (γ) and the energy (E) at constant weather or climatic conditions or other forms of description of the factors for a more efficient analysis and or quick results is desirable, typified by equations 1 and 2 below.

$$CBR = f(w, \gamma, E) \quad (1)$$

$$CBR = f(w, \gamma)\, E \quad (2)$$

The development of an alternative mathematical and a completely statistically correlated model of CBR with the two independent variables of moisture content and dry density for a typical road soil is desirable for analytical and effective strength based intelligent compaction control in earth construction. Indeed the development of best fit multivariate models in natural or transformed forms of the regressors reflects on improved time savings and surer ways of ascertaining that the values of the density, moisture and the CBR are obtained on site at the same time, which shall constitute the inputs for the envisaged intelligent compaction quality control. The main objective of this paper therefore is to develop a two dimensional mathematical model with the CBR as the dependent variable and the moisture content and dry density as the two joint independent variables.

2 METHODS AND MATERIALS

A total of six (6) different samples were obtained at two depths (0 – 1metre and 1 – 2metre) below the ground surface) and from three (A, B and C) of the Kwara State Ministry of Works borrow pits for road base and sub base around the Ilorin urban area, central Nigeria. The samples

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were subsequently subjected to preliminary, compaction and strength tests [5, 8]. Simultaneous compaction and CBR tests were further performed at the Lower and Medium compactive energy levels (LCE and MCE), culminating into twelve sample/data points for the statistical modeling. The two energy levels were used because they are easily achievable on site and commonly applied in Nigeria. Table 1 presents the preliminary, compaction and CBR properties of the studied soil samples. The data for joint compaction / CBR tests were further analysed for transformation of the two independent variables to their normalized forms (moisture/optimum moisture and dry density/maximum dry density) in order to ensure that the three variables are of the same dimension. It was further assumed that normalized forms of the two independent variables are mutually perpendicular and hence the resultant vector of their addition serves as a single independent variable. The computation was done with equation 3 and designated as Л, the independent variable or regressor for the CBR. Table 2 presents the outcome of this computation which was then tried for different statistical empirical models. The respective R^2 values of these models were applied for selection of the most attractive models (highest R^2 values).

$$Л^2 = (w/w_{opt})^2 + (\gamma_d/\gamma_{dmax})^2 \quad (3)$$

3 DISCUSSION OF RESULTS

Table 3 is the R2 values for the different fitted models for the twelve tested samples. The respective values for the linear, power, log and exponential models are just 0.66 – 0.84. The values that range from 90% and above correspond to the polynomials of 2nd and higher order. Indeed, the 4th order polynomial is 1.00 which indicates the completeness in the explanation of the model with the test data. This inference incidentally agrees with the reported CBR model in terms of the soil index properties by the US Army of Engineers.

Table1. Geotechnical Properties of soils.[9]

Property	A		B		C	
	0-1	1-2	0-1	1-2	0-1	1-2
D10, mm	0.2	0.16	0.22	0.75	0.14	0.15
D60, mm	1.10	0.60	1.18	4.75	0.90	2.16
LL, %	35	39	36	43	38	48
PI, %	19	22	18	23	17	28
N, %	1	1	0	1	2	2
AASH-TO CLASS	A26	A26	A26	A27	A76	A27
MDD g/cc	1.88	1.90	1.97	1.84	1.97	1.85
Omc, %	15	13.8	12.0	19.0	16.8	14.1

Fig. 2 and 3 are the sample displays of the fitted models for the CBR in terms of the Pi regressor respectively for the first and last of the tested sample points. The outcome for all the others follows the same pattern for the 2D relationship of the CBR with the normalized moisture content and dry density. An implication of the models is that their relationship can at least be described as a maximum value quadratic functions (equation 4) with coefficients of a, b and c; which together with the maximum dry density (mdd), optimum moisture content (omc) form the significant input parameters into the analogue for application of the model for the intelligent compaction control in practice. The global variation of these parameters and the estimates of the model coefficients for the 2° polynomial model for the tested samples within the Ilorin area are summarized in table 4. On testing these values for variation effects of the location, depth and compactive level for optimum moisture, maximum density and the model coefficients with the statistical computer package SPSS 13.0 software, it reveals that only the difference in mdd at 5% level is significant. This implies that their respective mean values can be taken as representative of the corresponding properties of the Ilorin laterite soil.

$$CBR = aЛ^2 + bЛ + c \quad (4)$$

Table3: R2 values of fitted models.

Model	A at 0-1m LCE 1	C at 1-2m MCE 2
Linear	0.74	0.86
Power	0.65	0.75
Log	0.72	0.83
Exponential	0.68	0.79
Polynomial 2nd	0.88	0.96
Polynomial 3rd	0.91	0.98
Polynomial 4th	1.00	1.00

1 Sample pit A 0-1m at Low energy.
2 Sample pit C 1-2m at Medium energy.

Table 4: Global variation for parameters.

a) Maximum Dry Density (g/cc).[9]

	LOCATION A		LOCATION B		LOCATION C	
	0-1m	1-2m	0-1m	1-2m	0-.1m	1-2m
LCE	1.86	1.83	1.94	1.90	1.90	1.86
MCE	2.10	2.08	2.08	2.18	2.10	2.28

b) Optimum Moisture Content (%).[9]

	LOCATION A		LOCATION B		LOCATION C	
	0-1m	1-2m	0-1m	1-2m	0-1m	1-2m
LCE	12.0	11.0	12.0	12.0	11.0	12.0
MCE	11.5	11.0	8.0	8.0	11.0	11.5

c) "a" Coefficient for 2° Polynomial.

	LOCATION A		LOCATION B		LOCATION C	
	0-1m	1-2m	0-1m	1-2m	0-1m	1-2m
LCE	-201.7	-138.5	-228.9	-80.7	-143	-186.4
MCE	-178.3	-134.6	-64.8	-81.2	-173.4	-187.6

d) "b" Coefficient for 2° Polynomial.

	LOCATION A		LOCATION B		LOCATION C	
	0-1m	1-2m	0-1m	1-2m	0-1m	1-2m
LCE	630.3	431.3	691.6	531.1	459.4	555.0
MCE	528.6	394.1	242.8	311.9	531.9	539

e) "c" Coefficient for 2° Polynomial..

	LOCATION A		LOCATION B		LOCATION C	
	0-1m	1-2m	0-1m	1-2m	0-1m	1-2m
LCE	-449.7	-290.3	-474.3	-344	-326.4	-366.8
MCE	-339.3	-237.3	-179.9	-245.2	-354.8	-334.1

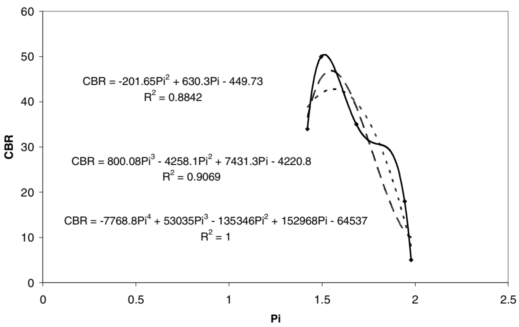

Fig 1 CBR Vs Pi for A @ 0-1 m (LCE) best fit

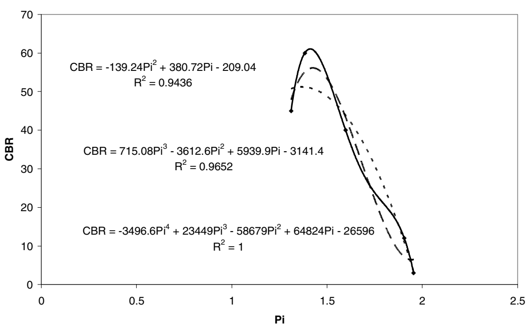

Fig 2 CBR Vs Pi for C @ 1-2 m (MCE) best fi

4 CONCLUSIONS AND RECOMMENDATION

The following conclusions can be made from the result of this study:-

a) The relationship between the CBR (on one hand) and dry density and moisture content (on the other) for a laterite soil can be expressed with a statistical model in two dimensions. The polynomial of the second to the fourth degrees were found to be most effective for the Ilorin sub base and base materials at R^2 values of 90 – 100% respectively.

b) The compactive energy level has significant effect at 5% on the maximum dry density values of a road soil, but negligible on the optimum moisture and the model coefficients. There is no effect of the position and depth of sample collected within 2 metres.

c) The respective means for the various parameters determined for the Ilorin laterites are mdd of 1.968g/cc and 2.009g/cc for low (BS) and medium (AASHTO) compaction respectively. The optimum moisture for the two energies is 10.2 – 11.7% while the statistical model parameters are -158.258, 487.25 and -328.125 respectively for a, b and c for the two degree polynomial. These values are recommended as inputs for analytical and mathematical prediction of CBR of Ilorin laterite in intelligent quality control works.

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Table 2: CBR – Л Data for the Studied Samples.

		LCE					MCE				
		1	2	3	4	5	1	2	3	4	5
A@0-1m	CBR.	34	50	35	18	5	45	60	45	17	6
	Л	1.421	1.496	1.684	1.943	1.979	1.392	1.435	1.644	1.940	1.986
A@ 1-2m	CBR	40	53	35	8	2	45	60	40	12	3
	Л	1.492	1.571	1.782	2.077	2.120	1.401	1.477	1.697	2.012	2.061
B@ 0-1m	CBR	40	55	40	10	3	40	55	40	12	3
	Л	1.389	1.462	1.654	1.919	1.957	1.703	1.800	2.130	2.617	2.699
B@1-2m	CBR	40	55	35	8	2	45	60	40	12	3
	Л	1.405	1.479	1.669	1.930	1.968	1.679	1.854	2.108	2.601	2.684
C@ 0-1m	CBR	35	50	35	18	5	45	60	45	17	4
	Л	1.463	1.542	1.755	2.056	2.101	1.394	1.470	1.691	2.008	2.057
C@ 1-2m	CBR	40	55	35	8	2	45	60	40	12	3
	Л	1.421	1.496	1.684	1.943	1.979	1.311	1.383	1.597	1.905	1.954