

# Relationship between cohesion and tensile strength in wet sand at low normal stresses

## Le rapport entre la cohésion et la limite élastique à traction dans le sable mouillé aux tensions normales basses

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### ABSTRACT

Within the unsaturated zone, a steep excavated surface can be sustained for some unknown but finite time, and some slopes may remain stable for extended time periods due to capillary forces. These observations clearly indicate small but non-zero values for attraction strengths (cohesion and tensile strength) in unsaturated sands. The relationship between attraction strengths is often considered by Mohr-Coulomb and Chen & Drucker models. However, for small compressive stress ranges (less than tens of kPa) and particularly in the tensile stress regime, the Mohr-Coulomb criterion often can not be accurately used for representing the behavior of sand. To find out the relationship between cohesion (one of shear strength terms) and tensile strength, uniaxial tensile and direct shear tests were carried out in moist sands ( $D_r = 30\%$ ). Tensile strength, which is significantly different from zero, increases with increasing moisture content. Apparent cohesion strength is also identified in moist sands. A simple relationship between tensile strength and apparent cohesion is proposed using the obtained data.

### RÉSUMÉ

Dans la zone non saturée, une surface excavée raide peut être soutenue depuis quelque temps inconnu mais fini et quelques pentes peuvent rester fermes pendant les périodes de temps prolongées en raison des forces capillaires. Ces observations indiquent clairement des valeurs petites mais non-zéro pour les forces d'attraction (la cohésion et la limite élastique à traction) dans les sable non saturés. Le rapport entre les forces d'attraction est souvent considéré par le Mohr-coulomb et les modèles de Drucker et Chen. Pourtant, pour de petites gammes de tension de compresseur (moins des dizaines de kPa) et particulièrement dans le régime de tension extensible, le critère de Mohr-coulomb ne peut pas souvent être exactement utilisé pour représenter la conduite de sable. Découvrir le rapport entre la cohésion (un de termes de force de tondage) et de limite élastique à traction, uniaxial les épreuves de tondage extensibles et directes ont été réalisés dans les sables moites ( $D_r = 30\%$ ). La limite élastique à traction, qui se distingue de façon significative du zéro, les augmentations avec la teneur en humidité augmentante. La force de cohésion apparente est aussi identifiée dans les sables moites. Un rapport simple entre la limite élastique à traction et la cohésion apparente est proposé en utilisant les données obtenues.

Keywords : capillary forces, attraction strength, apparent cohesion, tensile strength, moist sands

## 1 INTRODUCTION

Unsaturated soils, especially soils having very low moisture contents appear near the surface far away from the water table. We frequently observe the existence of cohesion as well as tensile strength in unsaturated soils even at low moisture contents.

Let us consider the stability of excavated cuts in very fine sand. If the sand is in a dry state, the excavated surface will readily slip or adjust to its natural angle of repose. Excavation under water, such as below the ground water table or in an free water boundary setting, such as coastlines, submarine slopes, etc., will also result in collapse, since capillary action clearly does not exist in these conditions. However, above the ground water table and within the unsaturated zone, a stable excavated surface can be sustained due to capillary forces in the soil medium. Capillary forces also have a significant role in the stability of sand slopes. These slopes remain stable, if the sand is unsaturated, as capillary forces acting in the medium provide apparent cohesion. However, the slopes will fail with the loss of apparent cohesion upon saturation of the soil by infiltrating water.

Capillary forces induced by interstitial water can significantly influence the engineering behavior of soils. Even

at low moisture contents, small amounts of water form bridges at contact points, and as the water content increases these bridges become larger and more developed. This results in capillary bonding between particles, giving rise to both cohesion and tensile strength. Capillary bonding generally leads to two force components at low water content levels: 1) surface tension force acting along the water-particle contact line, and 2) force due to the difference in the pressures outside and inside the bridge acting on the cross-sectional area (Rumpf 1961, Schubert 1984, Pierrat & Caram 1997, Lu et al. 2007, Kim & Sture 2008). The surface tension tends to force the particles together, whereas the force due to the pressure difference can only contribute to particle adhesion if there is a net pressure deficiency within the bridge. Due to the presence of water bridges between the particles, these two forces act together as a bonding force.

From these observations, it is apparent that capillary forces exist in unsaturated sandy soils and lead to apparent cohesion or adherence between the grains, which allow the soil to remain stable. At this point, contrary to the common Mohr-Coulomb strength assumption, we may assume that the tensile strength (and cohesion) resulting from small amounts of water in sands exists (Fig. 1).

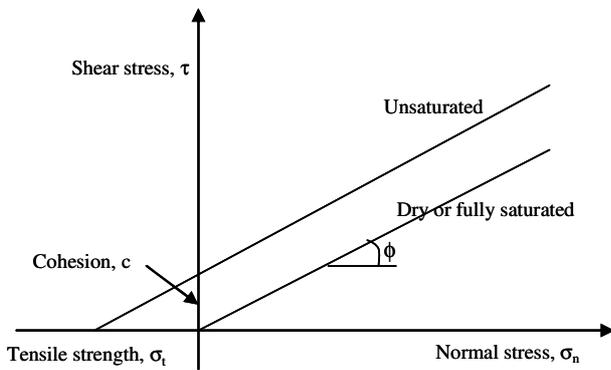


Fig. 1 Typical shear strength envelope of sandy soils

For stress in the large compressive ranges (from tens to hundreds of kPa), the Mohr-Coulomb criterion is generally an adequate representation for the shear strength behavior of sand. However, experimental evidence has shown that for small compressive stress ranges, say less than tens of kPa, and particularly in the tensile stress regime, the Mohr-Coulomb criterion often can not be accurately used for representing the behavior of sand.

One should also consider the possibility that cohesion (one of shear strength terms) and tensile strength are not necessarily related. For instance, while the shear strength of two glass plates containing a small water film between the plates is very low and allows the plates to slide relative to one another under tangential action, the tensile strength is substantial, when one attempts to pull the plates apart under normal action only. Another radically different example is the combination of two brushes pushed against one another: although the two intertwined bristles show significant resistance to shearing, the tensile strength is very low, and they are easily separated. These two examples show how different cohesion and tensile strength phenomena can be, and in some situations they have little to do with one another. Figure 2 illustrates how these two strength criteria may appear together. The cohesion and the tensile strength may not necessarily be related in sandy soils. For instance, while the interlocking between the particles has insignificant influence on tension, the effect of interlocking becomes significant during shear at low normal stress and low relative density levels, and thus provides an interlocking cohesion

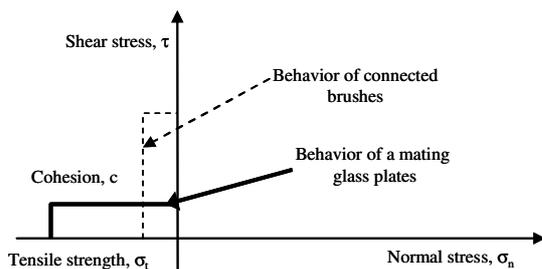


Fig. 2 Cohesion and tensile strength behavior for different materials or composite systems

Based on these observations, it is necessary to investigate the relationship between the tensile strength and the apparent cohesion in moist sand with low relative density at low normal stress levels. Within the scope of this work where purely mechanical water-induced strength issues are investigated, tensile strength and cohesion due to physico-chemical interactions in the solid-fluid electrolyte system and chemical cementation between particles will not be addressed.

## 2 EXPERIMENTS

Direct tension and direct shear tests were conducted on specimens having water contents focused on the range of less than 1% to study the relationship between tensile strength and apparent cohesion. F-75 Ottawa silica sand, which is a fine-grained uniform sand with a mean particle size of 0.22 mm is used for these tests. Two categories of F-75 sand mixtures were prepared; F-75-C (no containing particles smaller than 0.075mm and F-75-F (having 2% of fines by weight). The experiments were only conducted on loose specimens,  $D_r = 30\%$ , because it is believed that the effect of tensile strength, or cohesion, on the overall strength would be most pronounced at low friction state.

### 2.1 Direct tension experiments

The purpose of these experiments is to obtain the tensile strength in moist sand, and examine the variation in tensile strength as a function of water content. A direct tension apparatus was developed (Kim & Sture 2008). Because granular materials with weak bonding stress, a special specimen container ( $178 \times 178 \times 178$  mm) which can split in two equal halves was adopted. Although it is unlikely that uniaxial conditions are actually maintained for specimens initially compacted into confining molds, it was assumed that the tensile strength measured via this apparatus is the uniaxial tensile strength.

#### 2.1.1 Specimen preparation and experimental procedure

The container consisting of front and rear compartments was first tightly taped to prevent the movement of the container assembly during compaction. Sand and water were then thoroughly mixed to provide a homogeneous specimen. Distilled water was used to avoid introducing other physico-chemical factors during mixing. Individual specimens were prepared in four lifts within the container box by compaction with a drop hammer furnished with an angular foot, which facilitates compaction in corner regions. To achieve uniform specimen density, the number of blows, the weight of the hammer and the drop height were controlled. During and after specimen preparation, thin plastic wraps were used to cover the top surface of the assembly to minimize evaporation. However, complete uniformity in low moisture content is not possible. This was of particular concern in this case, since small variations in moisture content led to large variations in measured tensile strength. Thus, large specimens were chosen, because the larger the specimen, the lower the measurement and instrument errors

The tensile load on the sample was slowly and steadily applied by introducing water into the front loading container at a rate of about 170 g/min or about 0.03 N/sec until failure occurred. Careful measurements of the empty apparatus found that about 115 g of this observed load was necessary to overcome the friction in the apparatus itself, and this was subtracted from the applied tensile load after testing to correct for this. Variations in apparatus friction were small, and given the magnitude of the total measured failure load, potential variations in friction within the device itself could be expected to be quite small.

This somewhat simple loading procedure provided excellent results, and proved to be highly repeatable. The error in load-mass measurement was  $\pm 0.01$  g. The tensile strength was calculated by dividing the failure load by the separated area. Immediately after completion of each test, the density and moisture content of the entire specimen were measured. Brittle failure occurred at very small displacements ( $< 0.1$  mm) in all experiments.

2.2 Direct shear experiments

The purpose of these experiments was to obtain the shear strength parameters for moist sand at low normal stress levels. The direct shear apparatus used which has also a relatively large failure surface, with inside dimensions of 178 × 178 mm along the plane of shear and thickness of 15 mm (Kim & Sture 2008). The large shear surface relative to the thickness facilitates development of significant shear resistance under low normal stress levels and achieves high degree of uniformity in the stress state by decreasing boundary effects (Perkins, 1991). The same support table and the same loading system (consisting of pulleys, wires, and containers) used for the direct tension experiments were adopted for the direct shear experiments. The horizontal displacement of the lower shear compartment was measured by a dial gauge with accuracy of ± 0.025 mm, and the top shear compartment remained stationary.

2.2.1 Specimen preparation and experimental procedure

The shear compartments were positioned using set screws so that approximately 1.0 mm of separation existed between the two halves of the box prior to testing. The thickness of the specimen (15 mm) was selected considering both the ASTM D 3080 standard method and the estimated shear band thickness of the sand, which was in the range of 3.19 to 3.29 mm (Alshibli & Sture, 1999). The specimens were prepared by compaction with the same drop hammer technique used to prepare the direct tension specimens. The total weight of the prepared specimen was measured to calculate the in-place density of the specimen. After weighing, a thin acrylic plate was placed on the top surface of the specimen to prevent moisture from evaporating and also to transmit normal force to the specimen.

After specimen preparation, a surcharge load was applied to the top of the specimen to achieve the desired normal stress level, taking into account the self-weight on the upper half of the specimen. Teflon sheets with thickness 1 mm were inserted in the gap between the upper and lower compartments to reduce friction in case contact occurred during shearing. The upper shear compartment was locked in place, and the loading containers (front and rear) were connected with the wires, which were tied to a bolt attached at the lower shear compartment. The balance of the loading system was checked prior to testing. The clamps used to prevent the relative movement of the compartments during specimen preparation were then carefully removed. Loading was then slowly and steadily applied by introducing water into the front loading container at a rate of about 170 (g/min) similar to the direct tension tests. As indicated in direct tension experiments, to avoid a possible measurement error during testing the initial friction ( $F_{int} = 2.45$  N) required to mobilize the empty shear box was carefully measured and subtracted from the applied shear load after testing.

2.3 Results

The data clearly shows that the tensile strength is not zero even a small amount of water and tends to increase as the water content increases (Fig. 3). This can be explained by considering the capillary bonding forces between the particles. At low moisture levels, water bridges form at the particle-particle contact points. This results in capillary bonding forces between the soil particles, which lead not only to cohesion, but also to a certain amount of tensile strength in the soil. As the moisture level increases, the water bridges become more developed in the contact geometries, and the tensile strength increases.

The results for the F-75 sands are presented in Fig. 4. The data indicates a nonlinear relationship between the shear strength and normal stress. With a decreasing normal stress, the nonlinear behavior is clearly observed and the failure envelopes are convex-upward for both dry and moist specimens. This is

related to the decrease in dilatancy in the shear zones. For low normal stress levels, the individual grains in a shear zone can more easily move over one another, leading to increased dilatation in the shear zone. For high normal stress levels, this effect is suppressed, and the individual grains have to slide through irregularities. These effects of low confining stresses on strength have also been observed by others (Batiste 1998, and Sture et al. 1998). A non-linear relationship between shear strength and normal stress is often the case if the applied normal stress is less than several kPa. The ratio of shear stress to normal stress typically increases significantly as the normal stress decreases.

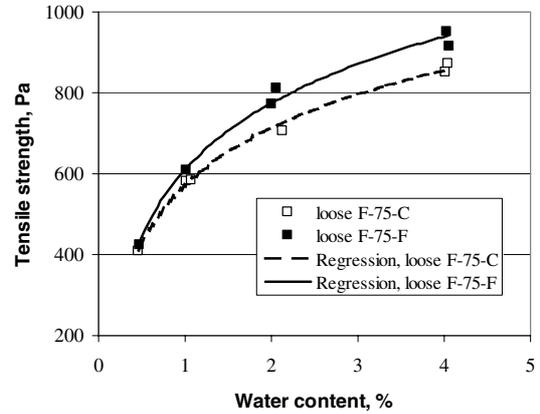


Fig. 3 Results of direct tension tests: F-75 sands ( $D_r = 30\%$ )

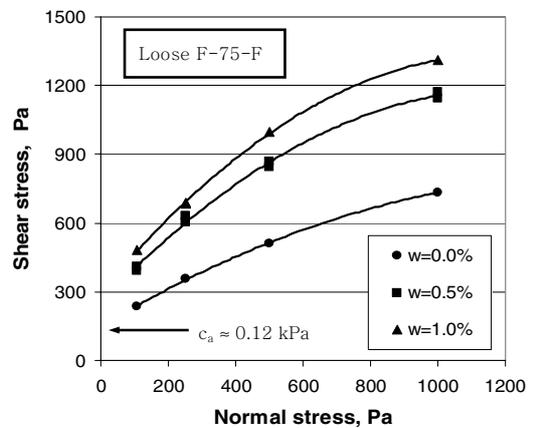
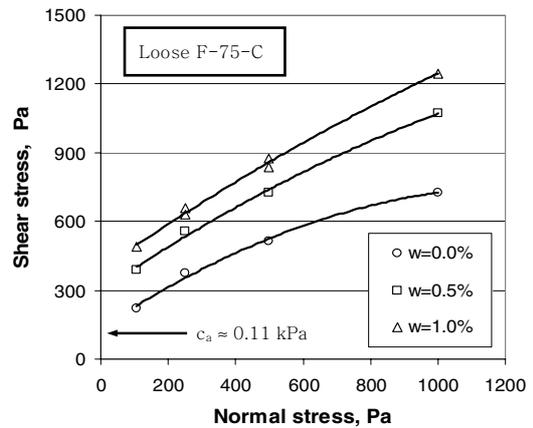


Fig. 4 Results of direct shear tests: F-75 sands ( $D_r = 30\%$ )

### 3 RELATIONSHIP BETWEEN TENSILE STRENGTH AND COHESION

In this section, considering the experimental data for the direct shear and the direct tension tests on loose specimens, a relationship between the tensile strength,  $\sigma_t$  and the apparent cohesion,  $c_a$  was developed for low water contents and low stress levels. To do this, first, tensile strength and apparent cohesion were calculated (see Tables 1 and 2).

Table 1. Values of tensile strength in direct tension tests ( $D_r = 30\%$ )

Material type	Water content, $w$ (%)	Tensile strength, $\sigma_t$ (Pa)
F-75-C	0.5	410
	1.0	583
F-75-F	0.5	426
	1.0	609

Table 2. Values of apparent cohesion for different normal stress range in direct shear tests ( $D_r = 30\%$ )

Material type	Normal stress range, $\sigma_n$ (kPa)	Water content, $w$ (%)	Apparent cohesion, $c_a$ (Pa)
F-75-C	0.10 – 0.24	0.0	113
		0.5	264
		1.0	377
	0.25 – 0.50	0.0	238
		0.5	390
		1.0	431
	0.50 – 1.00	0.0	299
		0.5	382
		1.0	463
	0.10 – 1.00*	0.0	209
		0.5	343
		1.0	427
F-75-F	0.10 – 0.24	0.0	127
		0.5	240
		1.0	328
	0.25 – 0.50	0.0	208
		0.5	377
		1.0	376
	0.50 – 1.00	0.0	286
		0.5	554
		1.0	682
	0.10 – 1.00*	0.0	207
		0.5	380
		1.0	452

Note: \* Entire normal stress range

A simple power law for the nonlinear equation was devised as

$$y = Sx^2 + T \tag{4}$$

where  $S$  and  $T$  are parameters:  $S$  controls the slope of the curve and  $T$  represents the value of the tensile strength. The parameters ( $S$  and  $T$ ) in Eq. 4 were determined directly by using the mean value of the normalized tensile strength and apparent cohesion. Substituting the normalized apparent cohesion and tensile strength data in Eq. 4 instead of the variables  $x$  and  $y$ , the tensile strength-apparent cohesion relationship for moist F-75 sand can be expressed as

$$\frac{\sigma_t}{\sigma_{ta}} = 1.6 \left( \frac{c_a}{\sigma_{ta}} \right)^2 - 1 \tag{5}$$

where  $\sigma_{ta}$  represents the average tensile strength. The resulting relationship is shown in Fig. 7. This relationship describes the behavior of moist sandy soils with low water contents and low stress levels in tension as well as in shear. It should be noted that the parabolic relationship was chosen for the sake of simplicity and due to theoretical and analytical considerations.

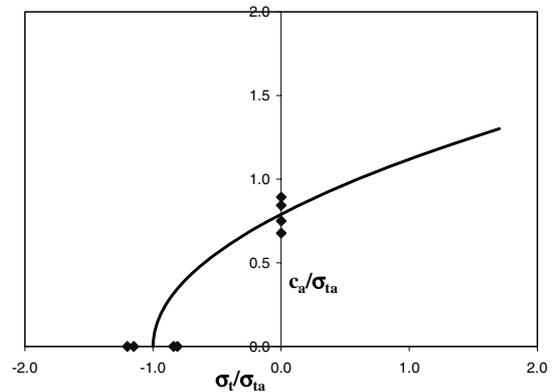


Fig. 5 Results of direct shear tests: F-75 sands ( $D_r = 30\%$ )

### 4 CONCLUSIONS

This study was carried out to assess the relationship between the tensile strength and the apparent cohesion in moist sand with low relative density at low normal stress levels. The following conclusions can be drawn:

- 1) For direct tensile tests for water contents in the range of  $0.5 < w < 1.0\%$ , the tensile strength of moist sand is not zero. For direct shear experiments for 0.0, 0.5 and 1.0% water contents, an apparent cohesion 0.11kPa due to interlocking between the soil particles was observed in dry sand. An additional apparent cohesion component due to capillary forces was observed.
- 2) A non-linear relationship between shear strength and normal stress is observed. Based on these experiment results, a simple relationship between tensile strength and apparent cohesion at low moisture and stress levels was proposed using a power law equation.
- 3) This relationship is important to describe the behavior of moist sandy soils in tension as well as in shear. For sand behavior under small compression regimes, such as under low or zero gravity, or under undergoing tensile failure, such as tensile zones in the crest area of hillslopes or behind retaining walls, it is important to consider the non-linear behavior. There are also additional applications involving wet granular media under tensile stress conditions, such as material processing, manufacturing, and transporting, and tensile failure in sandy soil under shallow environment.

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