

Measurement of soil variability for probabilistic slope stability analysis

La mensuration de la variabilité du terroir pour l'analyse de la probabilité de la stabilité des talus

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ABSTRACT

It has been known for some time that the factor of safety is of little physical significance. Attempting to predict future performance of geotechnical structures using the deterministic factor of safety is fraught with uncertainty and risk. Probabilistic analysis of slope stability can allow the quantification of the input parameter uncertainty and temporal forecasting to be more accurately achieved. Accounting for the spatial correlation structure of soil deposits is essential to proper estimation of probabilities of failure. The current study involves three-dimensional cluster analysis of Cone Penetration Testing with Pore-Pressure measurements (CPTu). The clustered data are to be used in an assessment of the autocorrelation statistics for lacustrine clay foundation soils of a water retention dyke. Dyke 17W is located at the McArthur Falls hydro-electric generating station owned and operated by Manitoba Hydro, located near Winnipeg, Manitoba, Canada. The results of the geostatistical analysis will be modeled as a random field in a probabilistic Monte-Carlo simulation.

RÉSUMÉ

Il est connu depuis quelque temps que le coefficient de sécurité à peu d'importance physique. Essayer de prédire la performance d'une structure géotechnique en utilisant le coefficient de sécurité déterministe est chargé de risque et incertitude. L'analyse de la probabilité de la stabilité des talus peut permettre l'incertitude de la quantification du paramètre d'entrée et de la prévision temporelle à être plus précis. La justification de la corrélation spatiale de la structure du terroir est essentielle pour bien estimer la probabilité d'éboulement. Cette étude comprend l'analyse de groupement à trois dimensions de la pénétration au cône avec la mesure de pression interstitielle (CPTu) et une évaluation des statistiques d'autocorrélation pour les sédiments lacustres qui forment la fondation d'une digue à la rétention de l'eau. Digue 17W est située aux chutes McArthur, une station générée par l'hydro-électricité. Elle est gérée par Manitoba Hydro, proche de Winnipeg, Manitoba, Canada. Les résultats de l'analyse géostatistique seront modélisés comme un champ aléatoire qui dans une analyse probabilistique à la méthode de Monte-Carlo.

Keywords : probabilistic slope stability, soil variability, cluster analysis, autocorrelation, stratigraphic delineation

1 INTRODUCTION

In deterministic slope stability analysis, the factor of safety (FS) is used as an indicator of stability. Traditionally, critical values for input parameters have been selected using judgment-based analysis of available data, thus resulting in a single FS estimate. The calculated FS value is thus uncertain since it is based on uncertain input parameters, yet it yields no information regarding the degrees and sources of uncertainty. A prescriptive FS allowance greater than unity has typically been used in design to provide a margin of safety against all such sources of uncertainty. It has been recognized that the factor of safety has little physical meaning and that the establishment of an acceptable value is uncertain and risky (Christian et al. 1994).

The use of probability and statistics in geotechnical engineering has been gaining popularity, since probabilistic methods promote a quantitative assessment of uncertainty and risk, whereas traditional deterministic methods rely upon experiential judgment for design that generally leads to the use of conservative design parameters. The objective of probabilistic slope stability analysis is to estimate a distribution for the factor of safety based on statistical descriptions of input parameters to the slope stability model and to estimate the probability of failure for the slope. Recently, El-Ramly (2001) developed a Monte-Carlo simulation technique that generated thousands of random combinations for input parameters from their respective distributions, resulting in a factor of safety distribution for a pre-defined slip surface. Previously, computational processing limitations restricted the use of

Monte-Carlo simulation techniques. The probabilistic slope stability methodology developed by El-Ramly (2001) was demonstrated on several cases, including Norway's Lodalén slide (El-Ramly et al. 2006), a failed cut slope in residual soil in Hong Kong (El-Ramly et al. 2005), a tailings dyke in the Alberta oilsands (El-Ramly et al. 2004), and the James Bay hydroelectric dykes (El-Ramly et al. 2002). A common and important conclusion from these studies was that the spatial correlation structure of soil deposits has a significant impact on the likelihood of specific landslide hazards.

Vanmarcke (1977) proposed the concept of a spatial correlation structure, whereby soil properties are correlated within a distance called the scale of fluctuation (δ), beyond which the degree of correlation decreases. A major consequence of this theory is that the variance in spatially averaged material properties decreases as the averaging distance increases beyond the scale of fluctuation. Within the scale of fluctuation, the variance of the spatial average is equal to the variance in point properties, which represents a maximum variance.

The averaged shear strength over the length of a potential slip surface will tend towards the average strength of the soil deposit as the number of fluctuations about the mean value increases. That is, the smaller the scale of fluctuation, the more likely the average shear strength will be near the mean, and thus the smaller the variance in the spatial average. The likelihood of larger slope failure mechanisms increases as the scale of fluctuation increases.

The variability in soil parameters can be described using random field theory by the mean value (μ), the standard

deviation (σ) and the scale of fluctuation (δ) (Vanmarcke 1983, El-Ramly 2001). However, the use of a single scale of fluctuation (δ) in both vertical and horizontal directions relies on the assumption that spatial variability is isotropic, which rarely is the case. The horizontal scale of fluctuation (δ_h) can range between 40 to 60 times as large as the vertical scale of fluctuation (δ_v), indicating that soil properties vary over a much smaller scale in the vertical than in the horizontal (Phoon et al. 1999). Thus, estimating the isotropic scale of fluctuation based on vertical profiles may be unconservative.

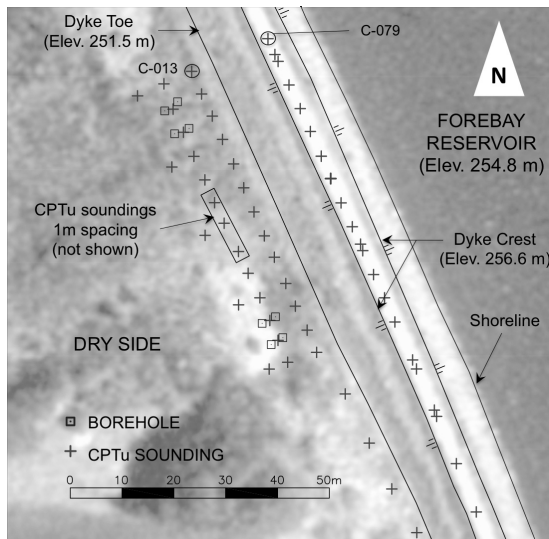


Figure 1. Plan view of CPTu test site at Dyke 17W

2 STUDY DESCRIPTION

The objective of the current study was to develop an objective site characterization and probabilistic modeling approach for the stability assessment of earthfill dykes for Manitoba Hydro. Changes to the Canadian Dam Association dam safety guidelines (CDA 2007) will require reassessment of design elevations for existing water retention dykes, as a result some of the current structures may require upgrading. The site characterization approach developed in this study included drilling, sampling and laboratory testing to complement an in-situ test program using the Cone Penetration Test with pore-pressure measurement (CPTu).

The site investigation approach was tested at the McArthur Falls Generating Station Dyke 17 West (17W) on the Winnipeg River near Lac du Bonnet, Manitoba. Dyke 17W is approximately 7km in length, has a maximum height of about 6m and side slopes of 1.5H:1V (dry side) and 2.2H:1V (wet side). The dyke was constructed in 1955 with compacted fill obtained from a clay-till borrow source and is founded on soft glacio-lacustrine Lake Agassiz clays. The fill is intermediate to high plastic, stiff to very stiff with some granitic gravel and cobble inclusions. The upper 2-4 m of the foundation, typically known as the Upper Complex zone, is often silty, firm and stratified with silt lensing and seams. The lower clays are typically unweathered, high plastic, massive and soft.

The perceived uncertainty in the computed factor of safety for Dyke 17W was relatively high, given the lack of site specific laboratory testing and site investigation data, resulting in highly conservative selections of input parameters. Deterministic slope stability analyses for dam safety review calculated a factor of safety lower than was deemed acceptable, despite acceptable performance and minor long-term settlements of the dyke since construction. It is anticipated that a probabilistic modeling approach that incorporates the spatial

correlation structure of the foundation soils can improve the accuracy of the estimated level of stability. The focus of this paper is on the characterization of the spatial correlation structure of the foundation soils.

A material sampling and laboratory-testing program was conducted to complement the in-situ testing program for the foundation soils and for the dyke fill. Undisturbed samples of the foundation clays were obtained using thin-walled Shelby tube cylinders from the borehole locations shown in Figure 1. Two test-pits were excavated in the freeboard section of the dyke to the south of the CPTu test site. Shelby tube sampling of the fill was attempted, but unsuccessful due to tube damage from cobbles encountered during pushing. The objective of the test-pit excavation was to obtain undisturbed samples of the fill if possible, otherwise to obtain a dry density profile for the fill materials in order to form recompacted specimens in the laboratory. A dynamic cone penetrometer (DCP) was fitted with a thin-walled tube to obtain undisturbed samples of the dyke fill at regular depth intervals in each test-pit.

Oedometer consolidation tests will be used to assess the over-consolidation ratio (OCR) profile for the foundation clays. Triaxial and direct shear tests will be used to assess the variance in shear strength parameters for both the foundation and fill layers for slope stability analysis. The specimens will also be used for a variety of index classification tests, including Atterberg plasticity tests, grain size distribution and specific gravity tests to classify the soil deposit profile.

Altogether, the laboratory testing on the foundation clays and dyke fill specimens will form the basis for a statistical description of shear strength, including estimation of the mean (μ) and standard deviation (σ) for each engineering parameter.

The primary objective of the CPTu investigation component was to estimate the spatial correlation statistics for the foundation clays. The majority of the CPTu soundings were located at the CPTu test site and arranged in a grid pattern adjacent to the dyke spaced from 1m to 10m apart, as shown in Figure 1. Soundings were also located along the centreline of the dyke spaced between 5m and 1000m. The current investigation was planned conservatively for the number and location of CPTu soundings, with the intent of gaining a better understanding of the number and spacing of soundings for future investigations at other similar structures. The cone penetration test program will be used to estimate the vertical and horizontal scales of fluctuation (δ_v and δ_h) as well as other related geostatistical descriptors of soil variability of the foundation soil layers and of the dyke fill, such as the autocorrelation distance (r_k) or the semi-variogram (γ).

The geostatistical analysis results will be input to a probabilistic slope stability analysis using a Monte-Carlo simulation process than simulates soil variability. The outcome will be a probability of failure estimate for the McArthur Falls Dyke 17W and a methodology for assessing spatial variability and its effect on the probability of failure of other Manitoba Hydro dam and dyke structures.

3 CONE PENETRATION TEST RESULTS

Figures 2 and 3 show the normalized CPTu profiles (Robertson 1991) located at the dyke centreline (C-079) and adjacent to the dyke (C-013). Some soundings were pre-punched 0.6 m (2 ft) using a larger cone tip in order to penetrate frozen soil at the ground surface. Soundings through the dyke centreline and adjacent to the dyke extended to a maximum depth of 15m and 10m below ground surface, respectively.

By visual inspection of C-013, the upper 3m are likely part of the Upper Complex zone, with characteristically higher tip resistance (q_c), sleeve friction (f_s) and lower pore-pressure

response (u_2). The tip resistance, sleeve friction and pore-pressure response in the lower 5 m of C-079 appear consistent with those in the lower portion of C-013 and are typical of results from this site. Visual inspection however, is an inefficient and subjective method towards stratigraphic delineation and site characterization. A more systematic and objective method for processing CPTu results is needed.

Many methods have been proposed in the literature for statistical analysis of CPTu profiles, including 2-dimensional and 3-dimensional clustering (Hegazy and Mayne, 1998; Liao and Mayne, 2007) and the intra-class correlation coefficient (Hegazy et al. 1996, Wickremesinghe and Campanella, 1991). Three-dimensional cluster analysis has been used in this study.

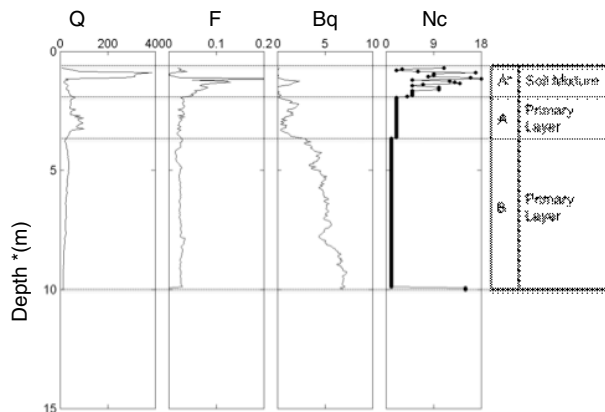


Figure 2. CPTu sounding C-013 adjacent to dyke, normalized tip resistance (Q, F, Bq) and stratigraphic delineation from three-dimensional cluster analysis.

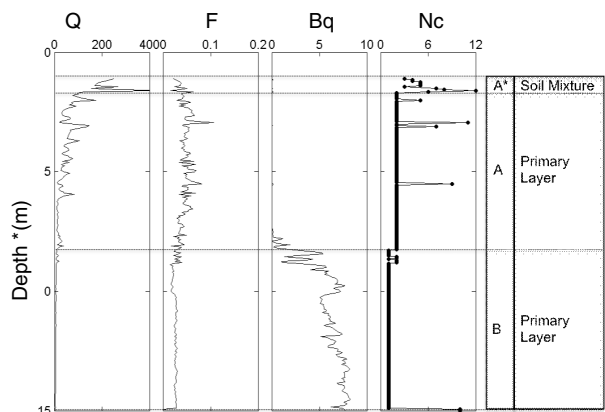


Figure 3. CPTu sounding C-079 through dyke crest, normalized tip resistance (Q, F, Bq) and stratigraphic delineation from three-dimensional cluster analysis.

Table 1. Comparison of CPTu parameter statistics for primary soil layers in C-013 and C-079

	C-013 (Foundation Soils)			C-079 (Dyke Fill and Foundation)		
	Q	F	Bq	Q	F	Bq
A - UPPER COMPLEX						
μ	57.38	0.026	1.131	41.36	0.046	0.067
σ	20.48	0.005	0.584	28.82	0.011	0.532
cv	0.36	0.203	0.516	0.70	0.229	7.926
B - FOUNDATION CLAY						
μ	23.63	0.024	5.189	6.44	0.025	6.567
σ	7.39	0.004	1.124	1.71	0.004	1.154
cv	0.31	0.152	0.217	0.27	0.136	0.176

[μ = mean, σ = standard deviation, $cv = \sigma/\mu$ = coefficient of variation.]

3.1 Stratigraphic delineation using clustering of CPTu data

Clustering is a technique used in a variety of fields to categorize data based on similar characteristics. For the analysis of CPTu measurements, cluster analysis groups point measurements based on similarity in a three-dimensional space of the measured response parameters. Clustering has been used previously to differentiate between zones of distinct soil behaviour types (SBT) based on cone tip resistance and sleeve friction (Hegazy and Mayne, 1998) and also including pore-pressure response (Liao and Mayne, 2007). While clustering alone has no direct statistical formulation, it is valuable for a cursory classification of soil zones and outliers in a CPTu profile. Clustering of CPTu profiles can identify subtle changes in a response profile and can differentiate between major and minor layers, soil mixtures, transition zones and outliers, if used effectively.

Dyke 17W CPTu data was used in a three-dimensional cluster analysis of normalized (Q, Bq, F) cone parameters (Robertson 1991) using nearest-neighbor clustering based on minimum Euclidean distance (Hegazy and Mayne 2002). The optimum number of cluster groups was selected as the minimum number of cluster groups (N_c) to produce the maximum number of primary layers.

A layer thickness of at least $t \geq 0.5$ m was required for secondary soil layers, while $t \geq 1.0$ m was selected as a for primary soil layers, consistent with previous studies (Hegazy and Mayne 2002, Liao and Mayne 2007). The layer classification algorithm proposed by Hegazy and Mayne (2002) was used for this study. A primary layer (A) is a continuous zone of an individual cluster with $t \geq 1.0$ m. A secondary layer (a) is a continuous zone of an individual cluster with $0.5 \text{ m} \leq t < 1.0$ m. A soil mixture is a zone with $t \geq 0.5$ m (a^*) or $t \geq 1.0$ m (A^*) where no continuous zone of an individual cluster has $t \geq 0.5$ m. Transitions are zones with 3 or more increasing or decreasing clusters numbers, or between two primary or secondary layers. Anomalies such as rocks or lenses within an otherwise homogeneous soil are identified as a set of clusters with $t < 0.5$ m surrounded by a primary or secondary layer of different cluster number.

The optimum number of clusters for C-013 and C-079 was 18 and 12, respectively, which resulted in two primary layers in each profile as shown in Figures 2 and 3. In both cases, a soil mixture zone was identified near the surface (A^*), followed by two primary soil layers (A and B). Primary layer A in C-013 can be cross-referenced to index testing as part of the Upper Complex zone, while layer A in C-079 is the dyke fill materials.

The statistics for the normalized CPTu parameters from each layer are shown in Table 1. The mean and standard deviation of the lower layer B is similar for both soundings, however the variability in the CPTu parameters in the Upper Complex and dyke fill zones are noticeably different. Most notably, the coefficients of variation (COV) for the CPTu parameters are quite similar in both foundation soil zones. The mean, standard deviation and COV for the normalized sleeve friction are nearly identical for both foundation soil layers.

3.2 Classification of Layers using Multivariate Statistics

Although a primary soil layer identified by cluster analysis can appear similar to a primary layer in an adjacent sounding, cluster analysis does not provide an objective means of defining a relationship between adjacent soundings. Primary and secondary layers from adjacent profiles need to be classified based on their distribution of CPTu response measurements.

A multivariate normal probability distribution can be defined by Eq. [1]. The mean (μ) and covariance (Σ) matrices can be estimated from a set of measurements from a given population (Eq. [2] and [3]). The sample variance $V[X]$ is the expected squared deviation of a random variable X from its mean. The covariance $Cov[X, Y]$ between two random variables X and Y describes the expected product of their respective deviations from their mean values.

$$f_i(x) = \frac{1}{(\sqrt{2\pi})^p} \exp\left\{-\frac{1}{2}(x - \mu_i)' \Sigma_i^{-1}(x - \mu_i)\right\} \quad [1]$$

$$\mu_i = \begin{bmatrix} \mu_Q \\ \mu_F \\ \mu_{Bq} \end{bmatrix} = \frac{1}{n} \begin{bmatrix} \sum Q \\ \sum F \\ \sum Bq \end{bmatrix} \quad [2]$$

$$\Sigma = \begin{bmatrix} V[Q] & Cov[Q, F] & Cov[Q, Bq] \\ Cov[F, Q] & V[F] & Cov[F, Bq] \\ Cov[Bq, Q] & Cov[Bq, F] & V[Bq] \end{bmatrix} \quad [3]$$

For an observation of 'p' variables, the probability of classification as class 'i' can be calculated as follows using Bayes' theorem (And and Tang 2007):

$$P(C_i | x) = \frac{f_i(x)}{\sum_i f_i(x)} \quad [4]$$

where x is an observation of 'p' random variables; $P(C_i | x)$ is the probability of classification as the i^{th} class; $f_i(x)$ is the multivariate probability density of the i^{th} class for x .

Similarly, for a set of n observations of p variables identified as the same cluster layer, the probability of classifying the clustered layer as the i^{th} class (out of m possible classes) is a function of classification probabilities of each individual measurement:

$$P(C_i | x_{j=1:n}) = \frac{\prod_{j=1}^n P(C_i | x_j)}{\sum_{i=1}^m \prod_{j=1}^n P(C_i | x_j)} \quad [5]$$

Given this theoretical basis, a "training set" of soundings from a specific site can be used in cluster analysis and cross-referenced to traditional site investigation data to establish a geological profile, and can also be used to estimate multivariate statistical distributions for primary layers. Subsequent soundings can then be clustered using the same approach and classified according to Eqs. [4] and [5].

3.3 Geostatistical analysis of primary layers

The data from each sounding can be used to estimate the vertical autocorrelation distance based on any or all of the CPTu response parameters. For a set of measurements $\{x_1 \dots x_n\}$ equally spaced by Δx , which may be in either the vertical or horizontal directions, the autocorrelation coefficient $C_k(x)$ can be estimated using the method of moments for a lag of k as follows in Eq. [6] (Baecher and Christian 2003):

$$C_k(x) = \frac{1}{n-k} \sum_{j=1}^{n-k} (x_j - \mu_x)(x_{j+k} - \mu_x) \quad [6]$$

where $k\Delta x$ is the separation distance; C_k is the autocorrelation coefficient at a lag of k measurements; n is the total number of measurements equally spaced by Δx ; μ_x is the expected value of X .

4 CONCLUSIONS

Clustering of CPTu data can provide a cursory classification of zones of similar measurements and can differentiate between primary and secondary layers and small-scale discontinuities, such as lenses and cobbles within an otherwise homogeneous material. Three-dimensional clustering was used to delineate primary layers within the soil profile at Dyke 17W, in order to classify CPTu measurements using multivariate statistics. Once classified, the autocorrelation distance in both the vertical and horizontal directions can be estimated for each layer. In order to accurately estimate the probability of failure of slopes using Monte-Carlo simulation, the spatial variability structure must be accounted for using geostatistics, such as the autocorrelation function. The proposed methodology for stratigraphic delineation, classification and geostatistical analysis will provide the parameters necessary for the generation of random fields of material properties for probabilistic slope stability analysis.

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