Small strain stiffness of uniform granular materials based on dynamic and static measurements

Rigidité des petites déformations de matériaux granulaires uniformes par des mesures dynamiques et statiques

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ABSTRACT
A triaxial test apparatus combined with two independent wave measurement methods, i.e. Trigger Accelerometer (TA) and Bender Element (BE), was employed. Specimens of Toyoura sand, Hime gravel, crushed glass, and glass beads were prepared under dry and saturated conditions. Small strain cyclic loading and wave measurements were performed at several designated stress states of isotropic and anisotropic conditions.
Analyses were performed on the test results to obtain statically and dynamically measured moduli. The values of shear modulus resulted from static measurement, Trigger Accelerometer, and Bender Element methods were compared. With gravel material having particle size with D_{50} > 1 mm, it was found that values of shear modulus resulted from Bender Element method yielded lower values as compared to that from Trigger Accelerometer method and static measurement, which is due possibly to the effects of bedding error at the interface between the bender element and the specimen. A graph of the ratio of dynamic and static stiffness ($G_{dynamic}/G_{static}$) versus the ratio of half wave length in dynamic measurement and grain's mean diameter ($(\lambda/2)/D_{50}$) was used to observe behavior of the geomaterials. Stress induced anisotropy's formula was employed to compare values of the shear modulus under anisotropic condition.

RÉSUMÉ
Un dispositif triaxial muni de deux méthodes indépendantes de mesure d'ondes : « Trigger Accelerometer » (TA) et « Bender Element » (BE) est utilisé. Des échantillons de sable de Toyoura, de gravier de Hime, de verre concassé et de billes de verre ont été préparés dans des conditions sèches et saturées. Des chargements cycliques à petites déformations et des mesures d'ondes ont été réalisés à divers états de contraintes dans des conditions isotropes et anisotropes.
Les résultats des essais ont été analysés afin d'obtenir les modules mesurés de manière statique et dynamique. Les valeurs des mesures de cisaillement résultant des mesures statiques, des méthodes TA et BE sont comparées. Concernant les matériaux graveleux pour lesquels D_{50} > 1 mm, les modules de cisaillement résultant de la méthode BE se sont révélés être plus faibles que ceux obtenus par la méthode TA et les mesures statiques. Cela serait dû aux effets des erreurs d'interface entre le dispositif BE et l'échantillon. Le graphe du rapport des rigidités dynamique et statique ($G_{dynamic}/G_{static}$) en fonction du rapport de la demi longueur d'onde de la mesure dynamique et du diamètre moyen du grain ($(\lambda/2)/D_{50}$) est utilisé pour observer le comportement des géomateriaux. La formule de l'anisotropie due aux contraintes est employée pour comparer les valeurs des modules de cisaillement dans les conditions anisotropes.

Keywords : small strain, cyclic loading, trigger accelerometer, bender element (petites déformations, chargement cyclique, trigger accelerometer, bender element)

1 INTRODUCTION

Laboratory wave measurements have found their ways to contribute in the exploration of soil properties together with in-situ ones, such as cross-hole method. Along side with those achievements, term of "dynamic" and "static" properties has become one of many issues in geotechnical society. Some researchers have recognized that dynamic and static properties are no more different from each other except for the strain level. Precise static small strain measurements in the laboratory tests have bridged the gap of strain level between "dynamic" and "static" behavior (Tatsuoka and Shibuya, 1992). However, following the pioneer work by Tanaka et al. (2000), AnhDan and Koseki (2002) found that the difference on dynamic and static properties is not only caused by strain level, but also by some other factors like grain size and wave length.

In this study, in order to perform statically and dynamically stiffness measurements consecutively to a specimen, a triaxial apparatus combined with dynamic measurement tools (i.e. Trigger Accelerometer (TA) and Bender Element (BE)) were employed. The values of shear modulus resulted from static measurement, TA, and BE methods were compared at different stress levels. Furthermore, for the ratio of dynamic and static stiffness ($G_{dynamic}/G_{static}$), the effects of mean diameter of grain size relative to the wave length were discussed.

2 MATERIAL, EQUIPMENT, AND TEST PROCEDURES

2.1 *Specimen and Apparatuses*

Air-dried Toyoura sand (Gs=2.635, e_{max}=0.966, e_{min}=0.600, D_{50}=0.20 mm), Hime gravel (Gs=2.650, e_{max}=0.709, e_{min}=0.480, D_{50}=1.72 mm), crushed glass (D_{50}=3.08 mm), and glass bead having diameter of 1 mm and 4 mm were used as the test material. The material particles were pluviated through air to prepare cylindrical specimens with dimension of 50 mm in diameter and 100 mm in height. The specimens were initially consolidated with a confining stress (σ_c') of 25 kPa in the air-pressured triaxial cell. Eight tests from 8 specimens of those geomaterials having different dry densities (ρ) were performed as presented in Table 1.

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Table 1. Information of specimen

Test	Material	Condition	Density, ρ (gr/cm^3)
T1	Toyoura sand	Dry	1.602
T2	Toyoura sand	Saturated	1.602
T3	Toyoura sand	Dry	1.588
T4	Hime gravel	Dry	1.735
T5	Hime gravel	Saturated	1.568
T6	Glass bead ∅ 1 mm	Dry	1.593
T7	Glass bead ∅ 4 mm	Dry	1.606
T8	Crushed glass	Dry	1.515

To evaluate dynamic measurement based on elastic wave propagation, two independent wave measurement methods were employed, i.e. Trigger Accelerometer and Bender Element. A digital oscilloscope (8855 Memory Hicorder of Hioki Co.) with sampling interval of 10^{-6} sec and a resolution of 16-bit was employed to display and to record the transmitted signals.

2.2 *Trigger Accelerometer (TA)*

Trigger Accelerometer (TA) method, as shown in Figure 1, is a set of dynamic measurement tools attached to the triaxial apparatus that consists of a combination of triggers and accelerometers. Both P and S waves are possible to be excited through the specimen under the assumption of wave propagation in an elastic rod.

Multi-layered piezoelectric actuator made of ceramic, called trigger, were used to generate the dynamic wave. Its commercial name is AE1010D16 of Tokin Company, having dimension of 10 mm x 10 mm x 20 mm, mass of 35 grams, and natural frequency of 69 kHz.

Accelerometer is a sensing transducer that produces an electrical output signal proportional to the acceleration according to piezoelectric phenomenon. In this study, accelerometer 352A24 and built-in signal conditioner 482A22 of PCB Piezotronics, Inc was employed.

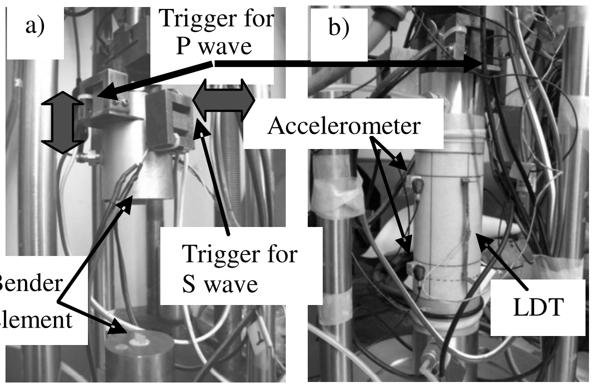

Figure 1. Specimen, LDT, Trigger Accelerometer, and Bender Element

2.3 *Bender Element (BE)*

Bender Element (BE) is a two-layer piezoelectric transducer that consists of two conductive outer electrodes made of deposited nickel or silver and two piezoceramic sheets. As a piezoceramic material, BE has capability of converting mechanical energy into electrical energy and vice versa. Photo of BE that was fixed on the top cap and pedestal is shown in Figure 1a.

The BE used in this study was made of lead titanate material (PZT = Pb(Zr.Ti)O3). Its commercial name was C-6 of Fuji Ceramics Corporation having piezoelectric charge constant of d_{31} = -210x10^{-12} m/V. The transmitter was fixed to the top cap; on the other hand, the receiver was fixed to the pedestal.

2.4 *Test Procedures*

Except for test T3, all specimens were subjected to isotropic stress condition. Following to the initial consolidation stage, the isotropic stress levels were increased to 50 kPa, 100 kPa, 200 kPa, and 400 kPa. At each stress level, after a 10 minute stage with constant stress state, 11 cycles of vertical loading with a single axial strain amplitude of about 0.002% measured by Local Deformation Transducer, LDT (Figure 1b), were carried out as a static measurement. During the cyclic loading stage, the vertical stress was unloaded first then reloaded to the original stress level under drained condition. After completing the cyclic loading, another stage with constant stress state was maintained while conducting the dynamic measurement using TA and BE methods.

Test T3 was consolidated under anisotropic condition with Ko = 0.5 subsequent to the initial condition. The confining stresses were set to σ_c' = 25, 50, 100, and 200 kPa, consecutively. At each stress level, 3 stages including 10 minute constant stress state, cyclic loading, and another constant stress state were carried out using procedures that were similar to those of the isotropic condition.

3 PROCEDURES FOR ANALYZING TEST RESULTS

Four different types of stiffness resulted from 3 different testing methods were analyzed in particular procedures as described in Table 2. The values of stiffness modulus were evaluated from each obtained data, followed by a converting calculation to result apparent compared stiffness moduli.

Tabel 2. Scheme of evaluation procedures

Method	Obtained Data	Evaluated stiffness	Compared stiffness
Static	Stress-strain curve	E_{sta}	G_{sta}
TA	$V_{P,TA}$	$E_{D,TA-P}$	$G_{D,TA-P}$
	$V_{S,TA}$	$G_{D,TA-S}$	$G_{D,TA-S}$
BE	$V_{S,BE}$	$G_{D,BE}$	$G_{D,BE}$

To evaluate the static measurement, the stress-strain relationship during the cyclic loading was fitted by a straight line and the quasi elastic vertical Young's modulus (E_s) was evaluated from the slope of the line. Furthermore, to obtain the representing value of the Young's modulus at each stress level, the average of two values of the Young's modulus resulted from the fifth and the tenth cycles of 11 cyclic loading was used.

In the analysis of data obtained by the dynamic measurement, a wave velocity (V) is evaluated. In addition, by knowing density of the specimen (ρ), the soil stiffness can be obtained. Since Primary (P) and Secondary (S) waves are employed, both values of dynamic Young's modulus (E_D) and dynamic shear modulus (G_D) can be evaluated using Equations 1 as follows:

$$E_D = \rho V_P^2 \qquad G_D = \rho V_S^2 \tag{1}$$

where V_P and V_S are wave velocities corresponding to P and S waves, respectively.

In addition, to compare all stiffness moduli obtained from each test, the values of the dynamic Young's modulus (E_D) and the static Young's modulus (E_s) were converted to those of the dynamic shear modulus (G_D) and the statically measured shear modulus (G_{sta}), respectively. Equations 2 based on an anisotropic elasticity modeling (Tatsuoka et al., 1999) were employed for the conversion as follows:

$$G_{sta} = \frac{E_s}{2(1+\nu)} \frac{2(1-\nu)}{1 + aR^n - 2\sqrt{a} \cdot R^{n/2} \cdot \nu}$$

$$G_D = \frac{E_D}{2(1+\nu)} \frac{2(1-\nu)}{1 + aR^n - 2\sqrt{a} \cdot R^{n/2} \cdot \nu} \tag{2}$$

where ν is Poisson's ratio. The values of Poisson's ratio were set as 0.17 for Toyoura sand and 0.20 for Hime gravel, crushed glass, and glass bead. Coefficient a is that of the degree of inherent anisotropy (set as 1.0, by neglecting the possible effect of inherent anisotropy), R is stress ratio (= σ_v/σ_h), and n is coefficient of the degree of stress-level dependency.

Furthermore, in case of stiffness evaluation under isotropic condition (R=1.0), Equations 2 can be rewritten as Equations 3 as follows:

$$G_{sta} = \frac{E_s}{2(1+\nu)} \qquad G_D = \frac{E_D}{2(1+\nu)} \tag{3}$$

In wave velocity evaluation, the determination of travel-time plays an important role, considering the fact that it often depends on subjective interpretation of each researcher. In this study, to avoid the uncertainties and the unreliable wave forms while computing wave velocity, the travel-time values evaluated with sinusoidal excitation and rising-to-rising technique employing both the TA and the BE methods, were used as shown in Figure 3.

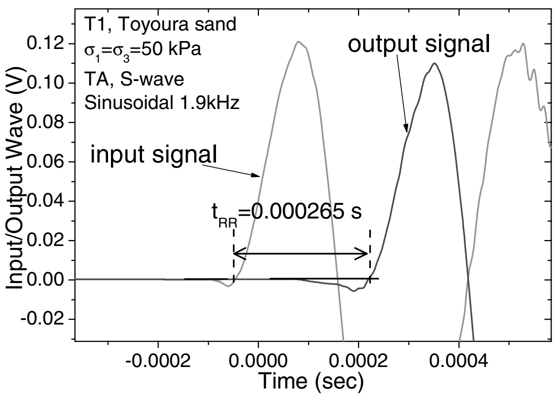

Figure 3. Definition of Rising to Rising travel time

4 TEST RESULTS AND DISCUSSIONS

Figures 4a and 4b show graphs of comparison between static and dynamic moduli of Toyoura sand and Hime gravel at different isotropic stress states under dry condition obtained from tests T1 and T4, respectively.

As shown in Figure 4a, the values of dynamic shear modulus with the BE ($G_{D,BE}$) method were at largest 10% larger than those of the statically measured shear modulus (G_{sta}), but were smaller than those of the dynamic shear modulus with TA method using S wave ($G_{D,TA-S}$). Similar tendency was observed between the specimens under dry and saturated conditions.

Under dry condition, the values of $G_{D,TA-S}$ were 10% - 25% larger than those of G_{sta}. Meanwhile, the values of dynamic shear modulus with TA method using P wave ($G_{D,TA-P}$) were 20% - 40% larger than those of G_{sta}. In addition, the difference in the values between $G_{D,TA-S}$ and $G_{D,TA-P}$ decreased with the increase in the stress levels.

Under saturated condition, the values of $G_{D,TA-S}$ were 20% larger than those of G_{sta}, showing the tendency that was similar to those observed under dry condition. On the other hand, the values of $G_{D,TA-P}$ were significantly larger than those of G_{sta}

Comparisons between static and dynamic moduli of Hime gravel at different isotropic stress states under dry condition resulted from test T4 are shown in Figure 4b. In general comparisons: under dry condition, at largest 10% smaller values of $G_{D,BE}$ as compared to those of G_{sta} were observed; while under saturated condition, the $G_{D,BE}$ values were close to G_{sta} values; and the values of $G_{D,TA-P}$ were significantly larger than those of G_{sta}. Typical results obtained from glass bead and crushed glass (i.e. tests T6, T7, and T8) had a tendency that was similar to the results of Hime gravel (test T4).

In case of dry condition with Toyoura sand that having mean diameter of 0.2 mm, the shear modulus values resulted from dynamic measurements were higher than those from static one. Similar results of TA method using P wave and that using S wave were observed, while results of BE method were smaller than those of TA method.

In case of dry condition with uniform granular materials having mean diameter in the range of 1 mm – 4 mm, the results of TA methods, which were similar between using P and S waves, were higher than those of static one. On the other hand, the results of BE method were smaller than those of static one.

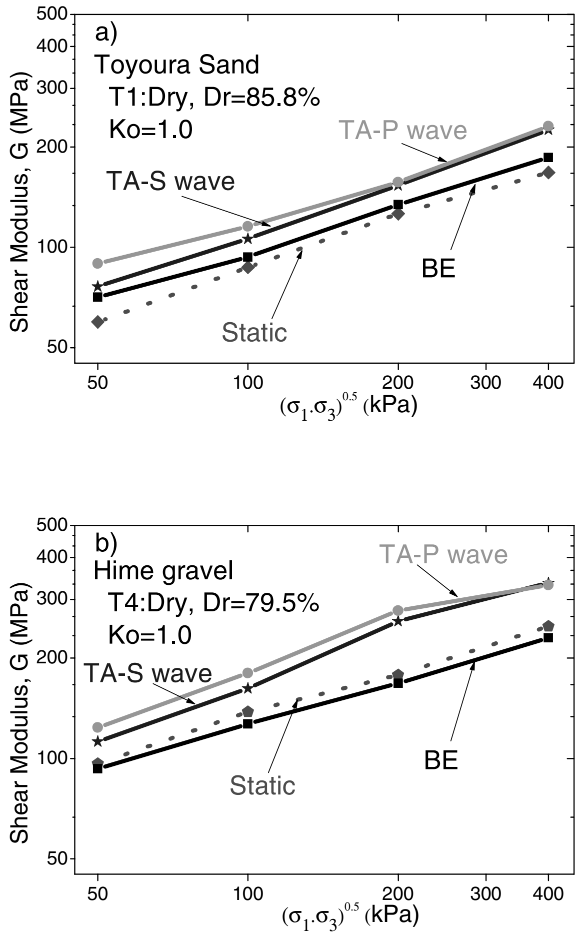

Figure 4. Typical results on Toyoura sand and Hime gravel under dry condition

The difference between statically and dynamically measured stiffness moduli is due possibly to the effects of heterogeneity of the specimen. In the static measurement, the stiffness modulus reflects the overall cross-sectional property of the specimen. On the other hand, in the dynamic measurement the wave travels through the shortest path made by interlocking of bigger particles that resulting into larger stiffness modulus as compared to those by the static measurement (Maqbool, 2005) and, as well, possibly to the effects of estimated strain levels during dynamic measurements. With the TA method, they were in the range of in the order of 10^{-4}% and 10^{-5}%, and with the BE method, they were in the range of in the order of 10^{-3}% and 10^{-6} %. Meanwhile, the static measurement was conducted at strain levels of about 0.002%. (Wicaksono, 2007a).

A large difference on results of TA method using P wave excitation between under dry and saturated conditions was observed, which is due possibly to the fact that under saturated condition, the velocity of P wave reflects that on pore water, while it is not the case with S wave velocity.

The difference between two kinds of dynamic shear modulus measured with the TA and the BE methods is due possibly to the effects of bedding error at the interface between the bender element and the specimen at both the top-end and the bottom-end. When the BE is bent, larger size of the locally loose zone with gravelly material as compared to that with sandy soil seems to result in longer travel time (i.e. smaller dynamic shear modulus) (Wicaksono, 2007b).

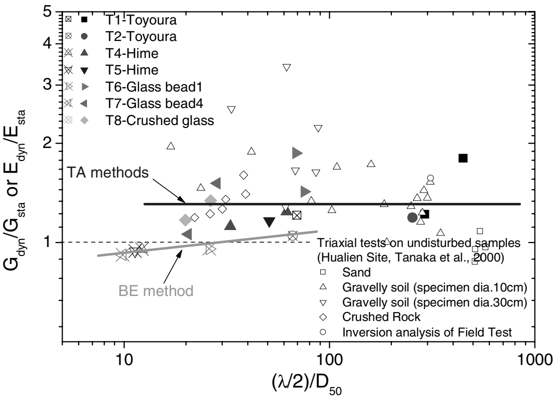

Figure 5. Effects of particle size and wave length

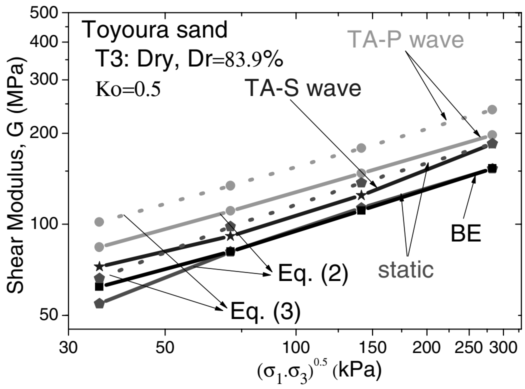

Figure 6. Employment of stress induced anisotropy formula

In this study, following to the previous researchers (Tanaka, 2000), to study the effects of particle size and wave length, the test results measured at confining stress of 50 kPa during isotropic stage were employed. A graph plotting $G_{dynamic}/G_{static}$ versus $(\lambda/2)/D_{50}$ is used as shown in Figure 5. It was found that the values of $G_{dynamic}/G_{static}$ obtained from TA methods seemed to be constant at 1.2 - 1.3. Meanwhile, those from BE method decreased with the decrease in the $(\lambda/2)/D_{50}$ values, which was observed that the values were lower than unity with particle size having mean diameter (D_{50}) of larger than 1 mm. Such evidence showed that the dynamic measurement using BE method were found to be possibly affected with bedding error.

Figure 6 shows the values of statically and dynamically obtained shear moduli from test T3. The values with dot line show those of $G_{D,TA\text{-}P}$ and G_{sta} that were converted using Equation 3 from those of respective Young's moduli ($E_{D,TA\text{-}P}$ and E_{sta}). The values of G_{sta} had a tendency that was similar to those of $G_{D,TA\text{-}}$S, while $G_{D,TA\text{-}P}$ were significantly larger than those of other shear moduli. In addition, when employing Equation 2 to consider stress induced anisotropy, the values of G_{sta} were similar to those of G_{BE} and the values of $G_{D,TA\text{-}P}$ approached to $G_{D,TA\text{-}}$S. In simpler terms, proper anisotropy's assumption was needed in the case of test under anisotropic condition.

5 CONCLUSIONS

a. For the uniform granular material having mean diameter of 0.2 mm, the values of shear modulus obtained from dynamic measurements (i.e. using TA and BE methods) tend to give larger than those of static measurement
b. Dynamic measurement using BE method yielded lower values of dynamic shear modulus as compared to that using TA method, which is due possibly to the effects of bedding error at the interface between the BE and the specimen with gravelly materials that having particle size with mean diameter of larger than 1 mm.
c. By employing TA method for uniform granular material in this study, the $(\lambda/2)/D_{50}$ values were in the range of 10 - 500, while the ratio of $G_{dynamic}$ and G_{static} values was 1.2 - 1.3.
d. For the evaluation of shear modulus obtained from triaxial test under anisotropic condition, proper anisotropy's assumption was needed.

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