

Modulus of subgrade reaction for foundations on clay from unconfined compression tests

Nick Barounis

Geophysics Co., Accredited Soil and Rock Mechanics Laboratories, Kallithea, Athens, Greece

Trevor L.L. Orr

Dept. of Civil and Structural Engineering, Trinity College, Dublin University, Ireland

Paul H. McMahon

Built Environment and Engineering, University of Bolton

Aristides Barounis

Geophysics Co., Accredited Soil and Rock Mechanics Laboratories, Kallithea, Athens, Greece

ABSTRACT

The unconfined compression strength test for soils can be used to determine the modulus of subgrade reaction k_{unc} in the laboratory. It proves more economical and saves time compared with in situ plate loading tests. The modulus obtained from such tests is considerably lower than the modulus obtained from consolidation tests due to the absence of lateral confinement. Stress-subgrade reaction modulus graphs (σ - k) are presented which show the behavior of k during loading. Some typical k_{unc} values are given and also K_c values, the slope of the σ - k curve. Design values of foundations with dimensions B and L can be calculated by using proposed methodology.

RÉSUMÉ

L'essai de force illimité de compression pour des sols peut être employé pour déterminer le module du k_{unc} de réaction de sous-grade dans le laboratoire. Il prouve plus économique et épargne le temps comparé aux essais de chargement in situ de plat. Le module obtenu à partir de tels essais est considérablement inférieur au module obtenu à partir des essais de consolidation dus à l'absence de l'emprisonnement latéral. On présente des graphiques de module de réaction de soumettre à une contrainte-sous-grade (σ - k) qui montrent le comportement de k pendant le chargement. Quelques valeurs typiques de k_{unc} sont indiquées et également les valeurs de K_c , la pente de la courbe de σ - k . Des valeurs de conception des bases avec les dimensions B et L peuvent être calculées en employant la méthodologie proposée.

Keywords : unconfined compression strength test, clay, modulus of subgrade reaction k_{unc} and k_{cons} , mode of failure, K_c index, brittle failure, plastic yielding, partial shear

1 INTRODUCTION

The modulus of subgrade reaction k is a conceptual relationship, which is defined as the soil pressure σ exerted divided by the measured deflection, δ :

$$k = \sigma / \delta$$

The parameter k is widely used in the structural analysis of foundation members (shallow and deep foundations) and has units of KN/m^3 . The usual procedure for measuring k is to perform in situ plate load tests. The test is fully described in BS EN 1997-1:2004 (BSI, 2004), DDENV 1997-3:2000 (BSI, 2000b), ASTM D 1194-72, Tomlinson (1980), BS 1377:1990 (Part 9), Clayton et al. (1995). The plate load test for a number of reasons has limitations (Barounis et. al., 2007). Therefore is proposed the possibility for measuring k in the laboratory by means of the unconfined compression test. The usual application of such measurements is for shallow foundation design.

2 ALTERNATIVE METHOD FOR EVALUATING K FOR CLAYS

In the unconfined compression strength test a preferably undisturbed sample of clay is compressed to failure without lateral confinement ($\sigma_3=0$). The test is fully described in ASTM

D2166-06. Shortening of the sample is measured by means of a strain gage. The typical report of the test includes the stress-strain diagram, calculation of E and unconfined strength q_u . The proposed method is to obtain k directly from unconfined compression tests by dividing the stress applied on the sample by the amount of deflection measured by the strain gage. In each step during loading the readings of the dial gage are usually recorded for 1mm or 0,01cm of compression. Thus $k = \sigma / 0,01$ and when σ is in kg/cm^2 then k is in kg/cm^3 . The k values should be computed for each loading step of the test until failure occurs and beyond failure when readings are available. Then a diagram can be obtained with k values vs. applied stress or σ - k diagrams. From this graph further conclusions can be drawn for the behavior of clay under testing. It is proposed that the test report should include the σ - k diagram in addition to the standard data.

3 APPLICATION OF THE METHODOLOGY

The application of the methodology has been applied to an extensive site investigation scheme of nine building projects in the greater Larisa city area, Greece. Undisturbed samples from boreholes have been recovered and tested in UC. The classification data of the samples and measured k values are shown in Table 1.

Table 1: Classification data and measured k values of clay samples

# project	Sample Depth (m)	LL	PL	W(%)	USCS	γ ton/m ³ (bulk)	K at failure KN/m ³	q_u (KPa)
1	3.00-3.30	34	24	20,05	ML	1,897	54678	214,5
2	4.35-4.60	49	22	23,42	CL	1,914	32684	288,6
3	3.05-3.40	50	16	16,98	CH	1,934	10687	115,6
4	4.40-4.70	31	18	15,73	CL	2,109	50899	499,3
5	3.95-4.40	49	19	16,12	CL	1,921	48462	427,9
5	5.25-5.45	43	18	14,74	CL	2,015	12792	263,5
6	3.30-3.60	41	14	25,13	CL	1,893	4479	48,3
6	5.20-5.40	48	12	22,58	CL	1,988	68769	539,7
7	8.85-9.10	54	16	28,31	CH	1,929	13341	170,1
7	11.20-11.50	50	15	26,50	CH	1,944	11804	196,8
8	2.50-3.00	29	12	16,78	CL	2,036	35958	634,9
9	2.70-3.00	34	14	23,49	CL	2,073	4216	53,7

The advantages of measuring k in the laboratory were:

- Testing performed on undisturbed samples with low disturbance during sample preparation
- Testing in depths of interest which otherwise need excavation prior to plate loading testing thus increase of time and cost
- UCS and k measured can be correlated
- Low cost compared to plate load testing (7% cost relative to the cost of load plate test in Greece)
- Soil is taken to failure and the response of k towards failure is recorded. This is difficult to be achieved in situ with plate loading test and sometimes unnecessary.

Limitations compared to plate load and consolidation tests were:

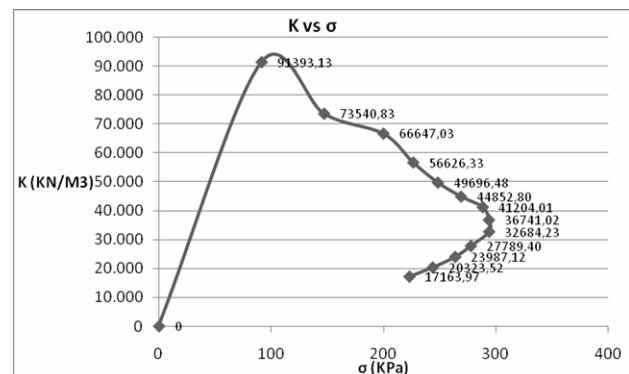
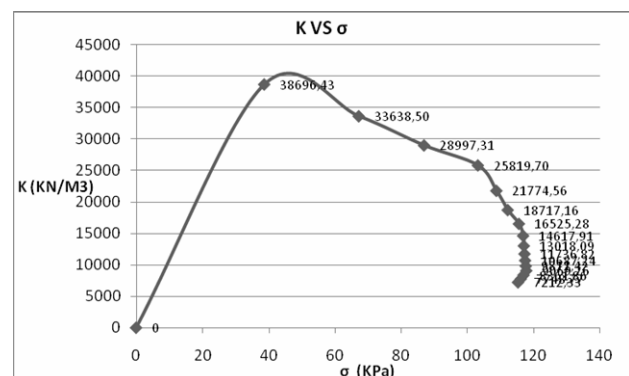
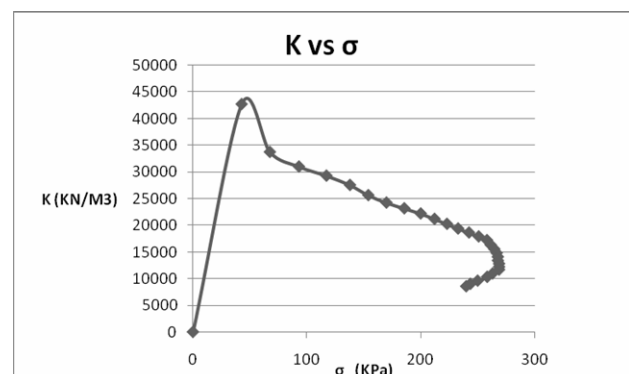
- The loading conditions do not reflect the in situ loading from foundations or plate loading tests. The main reason for that is the absence of lateral confinement of the sample ($\sigma_3=0$)
- Deeper strata cannot be taken into account
- Macrostructure cannot be taken into account or can be taken to a limited extent depending on sampling
- Compared to k obtained from consolidation tests, k from UCS tests is considerably lower due to absence of lateral confinement

4 MODE OF FAILURE AND MEASURED K

The failure of clays in compression can be of three modes: brittle slip failure, partial shear failure and plastic yielding (Whitlow, 1990). The mode of failure of the samples is shown in Table 2. The mode was recorded during the execution of test by optical inspection of the condition of the tested sample at the end of the test. It is feasible from the data of a UCS test and from the σ -k diagram to classify the clay in each of the failure modes. Three typical σ -k diagrams for each mode of failure are shown in Figures 1 to 3.

Table 2: Recorded mode failure of samples

Project #	Sample depth (m)	Mode of failure
1	3.00-3.30	BRITTLE
2	4.35-4.60	BRITTLE
3	3.05-3.40	PLASTIC YIELDING
4	4.40-4.70	PLASTIC YIELDING
5	3.95-4.40	BRITTLE
5	5.25-5.45	PARTIAL SHEAR
6	3.30-3.60	PARTIAL SHEAR
6	5.20-5.40	PLASTIC YIELDING
7	8.85-9.10	PARTIAL SHEAR
7	11.20-11.50	PARTIAL SHEAR
8	2.50-3.00	PARTIAL SHEAR
9	2.70-3.00	PLASTIC YIELDING

Figure 1: K- σ diagram for project #2, sample 4.35-4.60 for brittle failureFigure 2: K- σ diagram for project #3, sample 3.05-3.40 for plastic yieldingFigure 3: K- σ diagram for project #5, sample 5.25-5.45 for partial shear failure

The graphs show remarkable differences in behavior for the three modes but also some similarities.

From the σ - k diagrams the variation of k_{unc} along compression of the sample towards failure is depicted. The range of k values is important for foundation design, but also the interpretation of the curves. The range can be taken from the σ - k diagram. The values of k are undrained values, since the rate of loading in UC tests is very quick. The σ - k looks similar to σ - ϵ diagrams, but gives different information. For the interpretation of the curve the index K_c is introduced and defined as the slope of the σ - k diagram at each loading stage. The index is expressed:

$$K_c = \Delta k / \Delta \sigma \text{ with units } M^{-1} \text{ or per m length} \quad (1)$$

The values of K_c can be negative or positive, depending on the direction of the curve. The first measurement has always positive K_c , but then it changes sign. Measured values for K_c and k_{unc} are shown in Table 3.

Table 3: Measured range of values for k_{unc} and K_c values

Range of K_c before failure (m^{-1})		
Brittle failure	Plastic yielding	Partial shear
-811 to +1000	-7289 to +1000	-1030 to +1000
Range of measured k_{unc} before failure (KN/m^3)		
Brittle failure	Plastic yielding	Partial shear
21793-91393	9069-150595	12211-135101

From the tests conducted it has been found that each mode of failure produces one of the three family curves shown in Figs 1 to 3. The curve always consists of 3 branches, the first has positive K_c , the second negative K_c and the third has again $K_c > 0$. At the point of inflection between branch 2 and branch 3 failures occur. For example failure occurs at $k_{unc} = 32684,23$ in Fig.1 (sample 4.35-4.60-brittle failure). At the very next measurement K_c becomes positive and failure has already occurred. It is evident from all σ - k curves that k_{unc} decreases during loading. This is due to the fact that irrecoverable strains occur in the structure and the soil "spring" loses stiffness which means greater displacements at working loads (for example foundation loads). It is also evident that the value of k_{unc} is not only stress dependent but also depends on the previous measurement or how much compression the soil spring has already undergone.

5 COMPARISON BETWEEN K_{CONS} AND K_{UNC}

The modulus of subgrade reaction k_{cons} can also be measured with oedometer tests (Barounis et. al., 2007). The modulus that can be obtained from consolidation tests on clays is substantially greater than k_{unc} . This is due to the absence of sample confinement in unconfined compression testing but also to the fact that soil does not fail in one dimensional consolidation test.

From the theory of elasticity settlement of foundations is calculated from the following formula rewritten in terms of k_{unc} as:

$$k_{unc} = \sigma / \rho = E_{unc} / (B(1-v^2)I) \quad (2)$$

where $B = 0,08m$ (typical diameter of soil sample in UC tests)
 $I = 0,79$ for circular rigid loaded areas (NAVFAC DM-7.1, 1982)
 $v = \text{Poisson's ratio}$ with values 0,1 to 0,4.

The modulus of subgrade reaction k_{cons} as determined from consolidation tests is:

$$k_{cons} = \sigma / \rho \quad (3)$$

And for continuity it should be

$$k_{cons} = E_{unc} / (0,08 \times 0,79 \times (1-v^2)) = E_{unc} / (0,063(1-v^2)) \quad (4)$$

Or for $v = 0,1$

$$k_{cons} = E_{unc} / 0,062 = 16,1 E_{unc} = 16,1 k_{unc} \quad (5)$$

And for $v = 0,4$

$$k_{cons} = E_{unc} / 0,052 = 19,2 E_{unc} = 19,2 k_{unc} \quad (6)$$

Which means theoretically that $k_{unc} = 5,2\%$ to $6,2\%$ k_{cons} .

This is also in agreement with the findings from Crawford and Burn (1962) who estimated that in situ elasticity modulus was 4 to 13 times greater than that obtained from laboratory unconfined compression tests.

In general unconfined compression testing gives conservative estimates for elasticity modulus (Bowles, 1997) but also for subgrade reaction modulus.

6 CORRELATION BETWEEN CONSISTENCY OF CLAYS AND K_{UNC}

Based on the measured k_{unc} one could classify the consistency of the clay. A proposal for classification for the consistency of clays is presented in Table 4. Values for k_{unc} , q_u and SPT N are also shown for reference. Further research on specific clays may result in a narrower range of values and a more strict classification scheme.

Consistency	Very soft to soft	Soft to medium stiff	Medium stiff to very stiff	Hard to very hard	Very hard
k_{unc} (MN/m^3)	1,6-19	19-70	50-90	70-130	90-160
q_u (KPa)	0-50	50-100	100-400	400-600	600-700
SPT N	0-4	4-8	8-30	30-50	>50

Table 4: Classification scheme for the consistency of clays based on k_{unc}

7 CALCULATION OF K VALUE FOR A FULL SIZE FOUNDATION FROM K_{UNC}

The usual types of shallow foundation on clays are strip or long footings with $L/B \geq 5$ and raft (or mat) foundations with dimensions $L \times B$.

For a strip or raft foundation, with dimensions $B \times L$, the $k_{B \times L}$ value can be calculated as follows. First the k_{unc} is converted to the equivalent plate subgrade reaction modulus $k_{0,3}$ with the following relationship:

$$k_{0,3} = (8/30) k_{unc} = 0,26 k_{unc} \quad (7)$$

Then is converted to $k_{B \times L}$ as follows:

$$k_{B \times L} = k_{0,3} \times \left(\frac{m + 0,5}{1,5m} \right) \quad (8)$$

where $m = L/B$. (Bowles, 1997)

8 CALCULATED VALUES FOR K_{BXL} FROM K_{UNC}

The proposed k_{BXL} for the foundation of the buildings in greater Larisa city area are shown in Table 5. The foundation types are also shown. The values proposed were adopted for design from the structural designer of the buildings.

Table 5: Proposed K_{BXL} values for foundation design based on k_{unc} values

Type of foundation	L (m)	B (m)	kcons (KN/m ³)	k_{BXL} (KN/m ³)
Strip	10	2	72085	13744
Strip	10	2	49696	9475
Mat	22	15	28997	6739
Strip	10	2	117952	22489
Strip	10	2	61977	11816
Mat	15	10	29294	6770
Mat	20	10	11576	2508
Strip	10	2	102427	19529
Mat	20	15	30666	7308
Mat	20	12	34830	7848
Strip	10	2	108811	20746
mat	20	15	5457	1300

9 CONCLUSIONS

The modulus of subgrade reaction for clays can be estimated economically from unconfined compression tests. The results obtained from such tests are considerably lower than those from consolidation tests. It is proposed to make σ -k graphs which show k during loading of the sample. The main difficulty that arises from the σ -k curves is the selection of k_{unc} for foundation

design. The most suitable values for foundation design are those that are before failure of the sample. The values tend to decrease as the soil compresses and could become smaller and smaller as the test continues. Assuming a linear relation in the initial part of the stress-strain curve, the most suited k_{unc} results for design are those that correspond to the points of the tangent elasticity modulus. The k_{unc} values corresponding to failure or beyond failure are not suitable for foundation design.

A classification scheme for consistency of clays based on the measured k_{unc} is introduced. The values entered in the table should be those that correspond to the elastic part of the stress-strain curve.

REFERENCES

- Barnes, G. 1995. Soil mechanics, principles and practice. Macmillan press Ltd, London.
- Barounis, N. Orr, T.L.L. Barounis, A. and Neratzis, P 2007.. Modulus of subgrade reaction from consolidation tests. Proceedings XIV European Conference on Soil Mechanics and Geotechnical Engineering, Madrid, Spain, 24-27 September, Vol. 2, pp309-316
- Bowles, J. 1997. Foundation analysis and design. McGraw-Hill editions.
- Crawford, C. B. & Burn, N.K. 1962. Settlement studies on the Mt. Sinai Hospital. Engineering Journal of Canada, Ottawa, vol.45, no 12.
- Day, R. 2000. Geotechnical engineer's portable handbook. McGraw-Hill editions, New York.
- Das, B.M. 1990. Principles of Foundation Engineering. PWS Publishing Company, California.
- NAVFAC DM-7.1 1982. Soil mechanics, Design Manual 7.1, Department of the Navy, Naval Facilities Engineering Command, Alexandria, Va, 364pp
- Terzaghi, K. 1943. Theoretical soil mechanics. John Wiley and Sons Inc., New York.
- Whitlow, R. 2001. Basic soil mechanics. Pearson education limited.