A probabilistic approach to estimate one-dimensional consolidation settlements

Approche probabilistique pour l'evaluation du tassement du terrain sur un plan uni-dimensionnel

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ABSTRACT

Probabilistic analyses were performed to estimate consolidation settlements using an extensive field and laboratory data set from the Namak New City site in Korea. The results from 70 borings, 106 Dutch-cones and 103 oedometer tests were statistically analyzed. An analytical approximation method was developed and compared to Monte Carlo simulation techniques to estimate distributions of settlement ratios. Among the parameters of interest, the most dominant factor that controlled the magnitude of settlement was the soft clay thickness. For fill heights of 3 to 9 m, average settlement ratios of 11.1 to 21.0 % were estimated from the probabilistic approach. The settlement distributions from the analytical approximation were comparable to that from the Monte Carlo simulation.

RÉSUMÉ

On a effectue des analyses probabilistiques afin d'evaluer la consolidation du tassement du terrain en utilisant les donnees sur place et en laboratoire du site de la nouvelle ville Namak en Corée. Apres 70 forages, 106 tests Dutch-cones et 103 odometres, on a analyse les resultats statistiquement. Une methode d'analyse approximative a alors ete developpee et on l'a comparee aux techniques de simulation Monte-Carlo afin de faire une estimation des rapports du tassement du terrain sur le site. Parmi les differents parametres, le facteur essentiel qui avait le plus d'importance pour l'ampleur du tassement du terrain etait la couche de l'argile souple. Grace a l'etude probabilistique, on a determine des rapports moyens de 11.1 a 21.0 % pour des hauteurs allant de 3 a 9 metres. A partir de cette etude approximative, on a remarque que les repartitions du tassement du terrain sur le site etaient semblables a celles de la simulation Monte-Carlo.

Keywords: Consolidation, Probability, Uncertainty, Settlement, Monte Carlo simulation, Marine clay, Soft clay

1 INTRODUCTION

A probabilistic approach is a systematic method to evaluate the uncertainties in geotechnical engineering. Probability density functions can be used to model the inherent variability of soil and soil properties rather than deterministic parameters. Sources of uncertainty in soil include inherent spatial variability, limited availability of subsurface information and measurement errors (Vanmarcke 1977). Among these, inherent variability of soil is an important source of uncertainty in consolidation analysis. Numerous probabilistic approaches (Chang 1985, Corotis et al. 1975, Freeze 1977, Hong & Shang 1998, Krizek et al. 1977 and Vanmarcke 1977) have been published for geotechnical engineering problems. However, probabilistic approaches on one-dimensional (1-D) consolidation problems based on actual data from a specific site were seldom carried out, partly because of the limited availability of data. Corotis et al. (1975) have collected over 700 consolidation results from Greece and USA. They interpreted the soil properties as random variables regardless of their origins. Geological aspects of a site must be considered and site-specific distribution functions should be used in a well-designed probabilistic analysis.

Currently, a major construction project to develop Namak New City is underway on a reclaimed area, near to the Youngsan Sea Dyke, Jeollanam-do in Korea. The majority of construction work was areal fills for housing, commercial complexes and roads. A large portion of the site is situated on a soft marine clay deposit. Therefore consolidation settlements due to the surcharge loads were one of the critical calculations during the design phase. An extensive site characterization program had been performed including field and laboratory tests. The field tests included borings, Dutch-cones, Piezocones and field vane tests. The laboratory tests included index tests, unconfined compression tests (UC), oedometer tests and triaxial tests (UU and CU).

Unlike many other geotechnical design cases, deterministic soil parameters were selected using the extensive data set. For the settlement calculations, the site was divided into small sub-sectors and the same soil parameters were assigned for each sub-sector. Although statistical analysis results were taken into account for the soil parameter determinations, the settlements at the site were calculated using conventional methods.

The objectives of this paper are to implement a probabilistic settlement estimation utilizing the extensive data set and to evaluate variability of the soil parameters. This site is well suited to a probabilistic approach because of the vast data set available at this site. Furthermore, the soft clay layer in the site is geologically uniform (Won & Chang 2007), thus can be treated as a single layer. The extensive field and laboratory data were statistically analyzed as random. An analytical approximation method for the Terzaghi 1-D consolidation was developed. Settlement ratios from the analytical approximation were compared to Monte Carlo simulation results.

2 SITE CHARACTERIZATION PROGRAM

2.1 *The Namak New City site*

The site of interest is located in the estuary of Youngsan River in the southwestern part of Korea (Figure 1). The area of the site is 9 km^2 (2,224 acres) and its tidal range is about 2.7 m. The Youngsan Sea Dyke, completed in 1981 transformed the marshes or submerged lands to onshore paddy lands. Detailed

Proceedings of the 17th International Conference on Soil Mechanics and Geotechnical Engineering
M. Hamza et al. (Eds.)

doi:10.3233/978-1-60750-031-5-2012

descriptions for the site and clay deposit were reported by Won & Chang (2007).

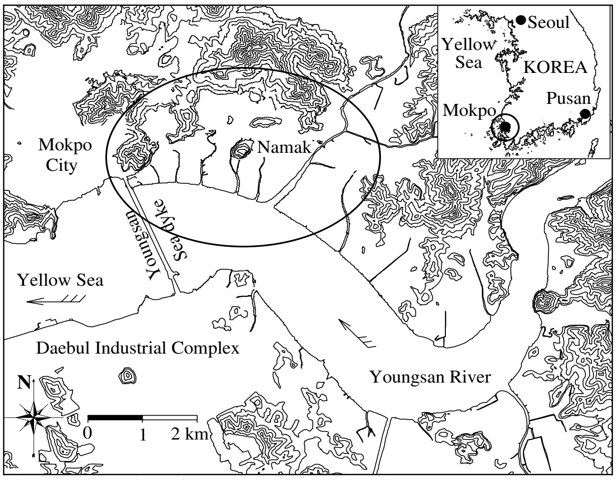

Figure 1. Location of the Namak New City site.

2.2 *Field tests*

The thicknesses of soft clay layer (H) were determined by 70 borings and 106 Dutch-cone tests. Locations of borings and Dutch-cones are shown in Figure 2. The Holocene soft marine clay deposit is underlain by Paleosol or fluvial sand and gravel. Distinctive tip resistance changes from the Dutch-cones were observed from the boundaries between the soft clay layer and sandy layer (or Paleosol). In Figure 2, the contour map of H was superimposed on the geographic shaded map (Won et al. 2008). The grids were generated by the ordinary Krigging method based on the field data. A fixed-piston type sampler (76-mm inner diameter) was used to retrieve undisturbed samples. In most cases, undisturbed samples were retrieved from ½ the height of the soft clay layer from a separate boring.

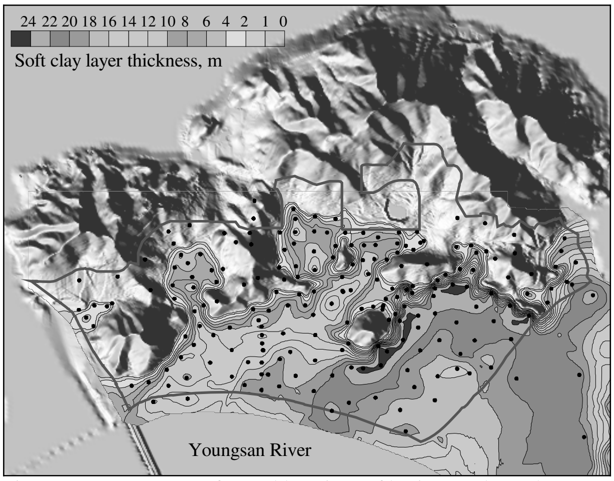

Figure 2. Contour map of H and locations of borings and Dutch-cones.

2.3 *Laboratory tests*

The undisturbed samples were extruded in the laboratory and physical properties such as water content, specific gravity, Atterberg limits and particle size distribution were measured. Unit weights were measured using 38-mm diameter specimens for UC and UU tests as well as 60-mm diameter oedometer specimens. Consolidation properties were determined from 103 conventional oedometer tests using 60-mm diameter and 20-mm thick undisturbed specimens.

3 DISTRIBUTIONS OF SETTLEMENT RATIO

3.1 *Assumptions*

In the probabilistic analyses, 1-D consolidation condition was assumed because the construction type was mainly areal fill. Ground water levels were assumed to be the same as ground levels (Won & Chang 2007). Effects of ground water level changes and submergence of fill materials due to settlements (i.e. reduction of applied loads) were neglected. Measured wet unit weights were assumed to be saturated ones.

Won & Chang (2007) divided the clay deposit within the site into two layers: the upper apparently overconsolidated layer and the lower normally consolidated layer. However, for simplicity, the soft clay layer was considered as a single normally consolidated layer. It seems that the difference between single layer and multilayer models for probabilistic consolidation analysis is insignificant (Chang 1985).

Effective overburden pressures (P_o) and additional loads (ΔP) are probabilistic parameters rather than constant values (Matyas 1977). However, in this study, constant ΔP (assuming fill material of 17.6 kN/m^3 unit weight) was used for a given fill height. In the later part of this paper, settlement ratios for various ΔP will be presented, considering ΔP as a constant.

During the construction, sand mat layers were installed between the soft clay and fill materials, then drain wells and different types of instrumentation were maintained to drain excess pore water from the ground (Won et al. 2008).

3.2 *Probability density functions*

The soft marine clay layer was classified as CH or CL with clay fractions of 26 to 53 % (Won & Chang 2007). Basic statistical parameters for each soil property are tabulated in Table 1. Initial water content (W_n), specific gravity (G_s), compression index (C_c) and wet unit weight (γ_t) were measured values, whereas initial void ratios (e_o) were calculated ones. The unit weights and void ratios from the UC, UU and consolidation tests for a given undisturbed sample were averaged.

Table 1. Statistical parameters of the soil properties.

	N	Min./ Max.	Mean	Median	Variance	Std. Dev.
W_n (%)	145	34.2 / 76.7	56.1	55.5	68.9	8.30
G_s	145	2.50 / 2.73	2.63	2.64	0.004	0.06
LL (%)	144	31.4 / 68.8	49.5	49.5	56.9	7.55
PI	144	13.7 / 43.0	28.6	28.7	39.7	6.30
C_c	103	0.48 / 1.32	0.73	0.69	0.028	0.17
e_o	103	1.19 / 2.12	1.55	1.53	0.035	0.19
γ_t (kN/m^3)	104	15.1 / 17.4	16.1	16.1	0.21	0.46
H (m)	176	0.2 / 25.0	11.9	11.0	48.43	7.00

Normal and lognormal probability density functions for the soil parameters were compared to each histogram in Figure 3. All the soil parameters, including thicknesses of the soft clay layer were considered as random variables. Although actual soil properties usually follow lognormal distributions, in the past, normal distributions of soil properties were generally accepted (Krizek et al. 1976) due to their simplicity. From the Chi-square tests, lognormal distributions were considered more appropriate models for the parameters involved. Furthermore, the facts that the parameters have strictly positive values and the settlement equation is expressed as a multiplication of the parameters made lognormal distributions more applicable. Moment values of the lognormal distribution functions are listed in Table 2 for the soil parameters involved.

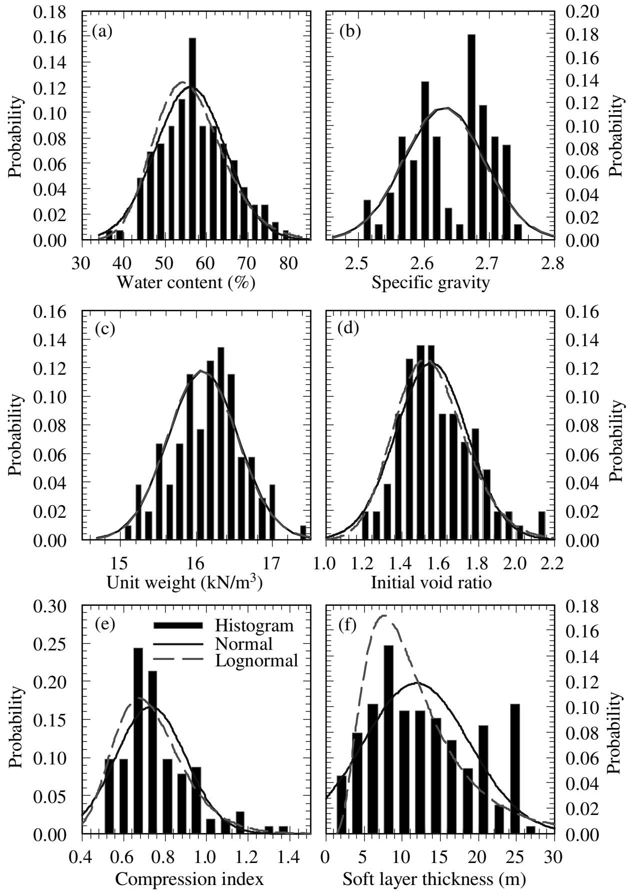

Figure 3. Probability density functions for the soil parameters.

Table 2. Moment values for lognormal distribution functions.

	Mean (μ)	Stdev. (σ)	λ	ζ
C_c	0.73	0.17	-0.3404	0.2265
e_o	1.55	0.19	0.4341	0.1193
γ_t (kN/m^3)	16.1	0.46	2.7779	0.0284
H (m)	11.9	7.00	2.3275	0.5431

($\lambda = \mu(\ln(X)) = \ln(\mu) - 0.5\zeta^2$ and $\zeta^2 = \ln(1+\sigma^2/\mu^2)$)

3.3 *Analytical approximation*

3.3.1 *Effective overburden pressure at mid-height of H*

The lognormal distribution of effective overburden pressure (P_o) at mid-height of soft layer thickness (H) was modeled from the distributions of γ_t and H, assuming normally consolidated homogeneous condition (equation 1).

$$
\begin{aligned}
&P_o = 0.5(\gamma_t - \gamma_w)H = 0.5\gamma_s H \\
&\ln(2P_o) = \ln(\gamma_s) + \ln(H) \\
&\lambda_{2P_o} = \lambda_{\gamma_s} + \lambda_H \quad \zeta_{2P_o}^2 = \zeta_{\gamma_s}^2 + \zeta_H^2 + 2\rho_{\gamma_s,H}\zeta_{\gamma_s}\zeta_H \\
&\lambda_{\gamma_s} = 1.837 \quad \zeta_{\gamma_s} = 0.073 \quad \rho_{\gamma_s,H} = 0.467 \\
&\mu_{P_o} = 38.07 \quad \sigma_{P_o} = 24.10 \quad \lambda_{P_o} = 3.471 \quad \zeta_{P_o} = 0.580
\end{aligned} \tag{1}
$$

To simplify the mathematical calculation, the logarithmic term of P_o and ΔP in the Terzaghi 1-D consolidation equation (equation 3) was transformed as a shifted power function. For an assumed constant ΔP (fill height = 6 m) and the P_o distribution (equation 1), the following approximation (equation 2) was determined with a correlation coefficient, R^2=0.9981 (Figure 4). It should be noted that for a different ΔP condition or a different distribution of H, i.e. P_o , new approximation parameters in equation (2) should be determined.

$$
\log\left(\frac{P_o + \Delta P}{P_o}\right) = 4.316 \times (P_o + 2.926)^{-0.546} \tag{2}
$$

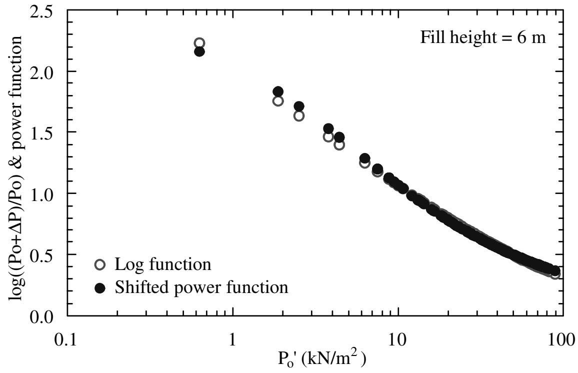

Figure 4. Comparison between log function and power function.

3.3.2 *Settlement ratio*

Settlement ratios (S/H) of the soft clay layer can be calculated using the following Terzaghi 1-D consolidation equation.

$$
\frac{S}{H} = \frac{C_c}{1+e_o}\log\left(\frac{P_o + \Delta P}{P_o}\right) \tag{3}
$$

After the mathematical transformation in equation (2), equation (3) can be re-written as equation (4).

$$
\frac{S}{H} = \frac{C_c}{1+e_o} \times (4.316) \times (P_o + 2.926)^{-0.546} \tag{4}
$$

For the first-order analytical approximation (Ang & Tang 1990), moment values for lognormal distribution of S/H are given as follow

$$
\begin{aligned}
&\ln(S/H) = \ln(C_c) - \ln(1+e_o) + \ln(4.316) - (0.546)\ln(P_o + 2.926) \\
&\lambda_{S/H} = \lambda_{C_c} - \lambda_{1+e_o} + 1.462 - (0.546)\lambda_{P_o+2.926} = -1.837 \\
&\zeta_{S/H}^2 = \zeta_{C_c}^2 + \zeta_{1+e_o}^2 + (-0.546)^2\zeta_{P_o+2.926}^2 + 2\left[\rho_{C_c,1+e_o}\zeta_{C_c}\zeta_{1+e_o}\right] \\
&\quad = 0.143 \\
&\mu_{S/H} = \exp\left(\lambda + 0.5\zeta^2\right) = 0.171 \\
&\sigma_{S/H} = \sqrt{\mu^2 \times (\exp(\zeta^2) - 1)} = 0.067
\end{aligned} \tag{5}
$$

In equation (5), the correlation coefficient of compression index vs. void ratio ($1+e_o$), i.e. $\rho_{C_c,1+e_o}$ =0.747, whereas ρ_{P_o,e_o} and ρ_{P_o,C_c} were considered to be 0.

From the analytical approximation (equation 5), settlement ratios at the site can be predicted as a lognormal distribution with a mean value of 0.171. It was shown that, among the parameters, the most important and uncertain factor in predicting the magnitude of settlement was soft layer thickness. It follows that spatial variation of soft layer thickness is the governing factor that controls the construction cost. Thus, for an additional site exploration program, more drillings or soundings such as Dutch-cone and Piezocone tests should be emphasized over laboratory tests to acquire more information about the distribution of soft layer thickness.

3.4 *Monte Carlo simulation*

Monte Carlo simulations were carried out using equation (3) to simulate distributions of settlement ratio at the site. Unlike the analytical approximation, the distribution of effective overburden pressure (P_o) was simulated using correlated Monte Carlo techniques directly from the distributions of γ_t and H ($\rho_{\gamma_s,H}$ =0.467). Yet, a constant ΔP was assumed. Distributions of e_o, C_c and P_o were considered as lognormal functions.

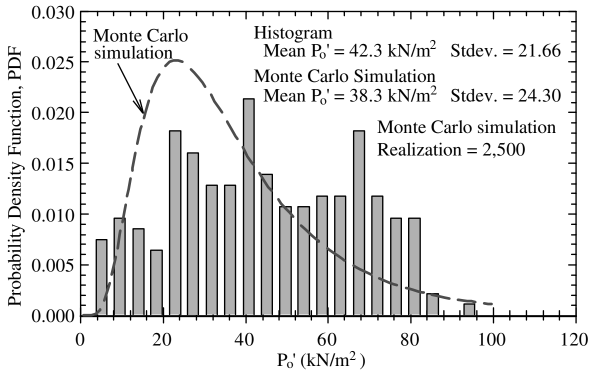

Figure 5. Distributions of the effective overburden pressures.

In Figure 5, calculated P_o ($=0.5\gamma_s H$) from the data set were plotted as a histogram. The simulated distribution of P_o was somewhat different from the histogram, because of the discrepancies in the distributions of γ_t and H (Figure 3). It appeared that the errors from the two parameters propagated on the P_o distribution. The distribution of P_o from the analytical approximation (equation 1) was almost identical to that from the Monte Carlo simulation.

In the Monte Carlo simulations, correlation coefficients ρ_{C_c,e_o} =0.734 and ρ_{H,P_o} =0.985, whereas ρ_{P_o,e_o} and ρ_{P_o,C_c} were considered to be 0, because there are little physical relationships between the parameters. In the Monte Carlo simulation, the realization number for each condition was 5,000, the percentage of error in the mean value was 0.0016 % and the percentage of error in variance was 3.92 %, with 95 % confidence bounds.

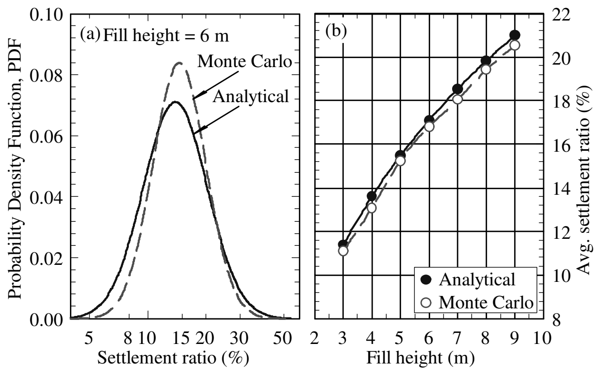

Figure 6. (a) Settlement ratio distributions for fill height of 6 m and (b) average settlement ratios for different fill heights.

In Figure 6 (a), for fill height of 6 m, the Monte Carlo simulation result seemed to have a narrow distribution compared to that of the analytical approximation. The wider distributions from the analytical approximation may be attributed to the error from the mathematical transformation in equation (2). In Figure 6 (b) settlement ratios from both analyses for different fill heights were presented. Different approximation parameters in equation (2) were used for the different fill height conditions. In Table 3, as an example for the case of 6 m fill, distribution parameters from the analytical approximation and Monte Carlo simulation were compared. It was shown that the results from the analytical approximation were comparable to that from the Monte Carlo simulation.

Table 3. Results of analytical and simulation methods for 6 m fill.

	Analytical method	Monte Carlo simulation
Mean	0.1714	0.1681
Median	0.1596	0.1600
Std. Deviation	0.0672	0.0538
c.o.v.	0.3922	0.3203
λ	-1.8354	-1.8324
ζ	0.3783	0.3124

Both from the analytical approximation and Monte Carlo simulation technique, settlement ratios at the site ranged between 11.1 and 21.0 % for fill heights of 3 to 9 m (52.8 to 158.4 kN/m^2). Settlements can also be estimated by the presented probabilistic approach (both from analytical and simulation methods). Following this probabilistic approach one can systematically evaluate the uncertainties in the soil parameters for a settlement prediction. Furthermore, based on a distribution of settlement ratio for a given average fill height, a reasonable amount of borrowing materials and construction cost can be estimated.

4 CONCLUSIONS

Distributions of consolidation settlement in the Namak New City site were estimated using the analytical approximation and Monte Carlo simulation techniques. Distributions of each soil parameter were constructed using the field and laboratory test results. Using the Terzaghi 1-D consolidation settlement equation, a first-order analytical approximation solution was derived. The distribution of soft layer thickness appeared to be the dominant factor in the settlement estimations. Based on the probabilistic approach, average settlement ratios of 11.1 to 21.0 % were estimated for fill heights of 3 to 9 m. The distributions of settlement ratio from the analytical approximation were comparable to that from the Monte Carlo simulation.

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