# Development and preliminary investigation for a resilient modulus prediction model of sub-ballast and subgrade materials

# Le développement et l'enquête préliminaire pour un modèle de prédiction de module élastique de sous-lest et de matériel de sous-qualité

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ABSTRACT

An alternative method for resilient modulus evaluation was developed using nonlinear dynamic stiffness, which can be measured by in-situ and laboratory seismic techniques. The resilient modulus prediction model was verified by comparing the calculated and measured vertical displacements during train passages. The proposed model performs well and maintains its consistency as well because of its dependency on well developed seismic measurement techniques.

RÉSUMÉ

Une method alternative pour l'évaluation de module élastique a été développée en utilisant la raideur dynamique non linéaire, qui peut être mesurée par dans-situ et les techniques sismiques de laboratoire. Le modèle de prediction de module élastique a été vérifié en comparant le calculé et a mesuré des déplacements verticaux pendant les passages de train. Le modèle propose joue bien et maintient sa consistance aussi à cause de sa dépendence aux techniques de mesure sismiques bien développées.

Keywords : railroad trackbed, resilient modulus prediction model, shear modulus, elastic displacement measurement

## 1 INTRODUCTION

In the railroad trackbed design using elastic multilayer model, the stress-dependent resilient modulus ($E_R$) is an important input parameter, which reflects substructure performance under repeated traffic loading. However, the evaluation of resilient modulus using the repeated loading triaxial test has been hindered, from the practical perspective, by high cost of equipment and inconsistency of measured values depending on the testing equipment and laboratory personnel. As an alternative proposition to circumvent the difficulty, a prediction model was developed by combining maximum Young's modulus and its normalized reduction curve for high strain, converted from nonlinear shear modulus using Poisson's ratio. The maximum Young's modulus was modeled by the power of the mean effective principal stress (or the first stress invariant) and the reduction curve was represented by the modified hyperbolic model using the parameters of reference strain and curvature coefficient. The model was explained further in detail in the following section.

The model parameters were evaluated for typical materials of sub-ballast or subgrade such as weathered soil, crushed stone, and crushed rock-soil mixture. To assess the model and its parameters, the elastic response of the test trackbed near PyeongTaek, Korea was calculated using a 3-D elastic computer program (GEOTRACK) and compared with measured elastic vertical displacements during the passages of freight and passenger trains.

## 2 RESILIENT MODULUS PREDICTION MODEL

The model, proposed by May and Witczak (1981) for granular materials, was adopted and adjusted into an easier form to use. The model is in the form of multiplication of maximum Young's modulus ($E_{max}$) and its strain-dependent reduction curve ($f(\varepsilon)$). The maximum Young's modulus is the power function of the mean effective principal stress ($I_1$ in kPa) normalized by the atmospheric pressure ($P_a$ = 101.3 kPa) as Equation 1.

$$E_{\max} = A_E P_a \left(\frac{I_1}{P_a}\right)^{n_E} \tag{1}$$

where $A_E$ and $n_E$ are the dimensionless model parameters. The Young's modulus reduction curve is represented by modified hyperbolic model (Darendeli 2001), which is defined by reference strain ($\varepsilon_r$) and curvature coefficient (a) as Equation 2.

$$f(\varepsilon) = E / E_{\max} = \frac{1}{1 + \left(\frac{\varepsilon}{\varepsilon_r}\right)^a} \tag{2}$$

where $\varepsilon_r$ is the strain, at which E is the half of the maximum value and a governs the curvature of the reduction curve. By combining Equations 1 and 2, the resilient modulus ($E_R$) is finalized as Equation 3.

$$E_R = A_E P_a \left(\frac{I_1}{P_a}\right)^{n_E} \frac{1}{1 + \left(\frac{\varepsilon}{\varepsilon_r}\right)^a} \tag{3}$$

It should be stressed that $I_1$ include both effective overburden stress and dynamic stress caused by traffic load, whereas the axial strain ($\varepsilon$) be calculated from the deviator stress of traffic load only.

## 3 EVALUATION OF MODEL PARAMETERS

### 3.1 *Proposed methodology*

The nonlinear shear modulus, measured by laboratory resonant column tests and in situ seismic tests, was utilized by converting shear modulus (G) and shear strain ($\gamma$) to Young's modulus (E) and axial strain ($\varepsilon$), using the relationships of $E = 2G/(1+\nu)$ and $\varepsilon = \gamma/(1+\nu)$, respectively. A series of maximum Young's modulus ($E_{max}$), converted from the corresponding maximum shear modulus ($G_{max}$) measured at various $I_1$s, is normalized and

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plotted in logarithm scale as shown Figure 1. The value of $A_E$ is the value of $E_{max}/P_a$ at which $I_1/P_a$ is equal to one and $n_E$ is the slope of the best-fit straight line of the data. To evaluate model parameters for large strain range of nonlinear modulus ($\varepsilon_r$ and a), the values of normalized Young's modulus ($E/E_{max}$) for each $I_1$ are plotted on semi-logarithm scale as shown in Figure 2. The data is approximated with the best-fit modified hyperbolic curve shown by Equation 2. The reference strain, $\varepsilon_r$ is the value at which $E/E_{max}$ is 0.5 and the curvature coefficient, a is the very trial value which gives the best-fit. The model parameters of typical materials were evaluated in the following sub-sections.

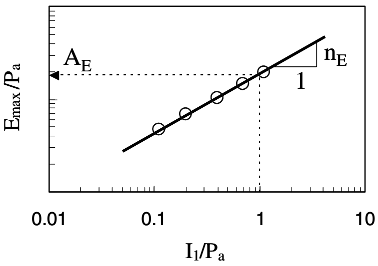


Figure 1. Evaluation of $A_E$ and $n_E$.

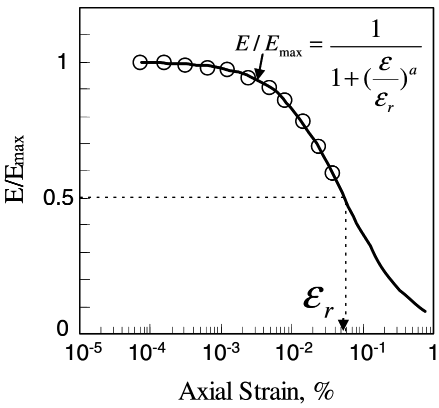


Figure 2. Evaluation of $\varepsilon_r$ and a.

### 3.2 *Weathered granite soil*

Granite residual soil was compacted to 95% of standard proctor maximum dry density ($\gamma_{d,max}$) to use sub-ballast at the test trackbed near PyeongTaek, Korea. The soil was classified SM (by Unified Soil Classification System) with $D_{50}$ (the grain size at which 50 % of soil by weight is finer) of 3.4 mm and coefficient of uniformity, $C_u$ of 22.2. In situ maximum shear modulus was measured by crosshole testing and also a resonant column test was performed on the specimen compacted with the same density and water content as the in situ values. The maximum shear modulus was converted to Young's modulus with the Poisson's ratio (ν) of 0.33, which was determined from crosshole results. The model parameters, $A_E$ and $n_E$ were determined as 2312 and 0.1, respectively from the $E_{max}/P_a$ - $I_1/P_a$ plot as shown in Figure 3.

For the model parameters for large strain, resonant column data were converted, using the same ν of 0.33, to normalized Young's modulus ($E/E_{max}$), which was plotted against axial strain as shown in Figure 4. Each set of data was nicely fitted with each hyperbolic function for $I_1$ of 31, 61 or 123 kPa, respectively. The values of reference strain and curvature coefficient ($\varepsilon_r$ and a), determined from each curve, were plotted against $I_1/P_a$ as shown in Figure 5 and 6, respectively. The linear relationships of the parameters on logarithm scale were approximated with Equation 4 and 5, respectively.

$$\varepsilon_r = 0.05\times(\frac{I_1}{P_a})^{0.23} \tag{4}$$

$$a = 0.9\times(\frac{I_1}{P_a})^{0.084} \tag{5}$$

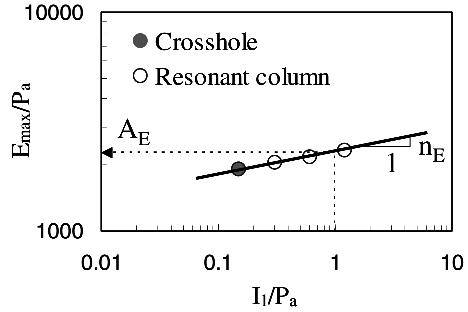


Figure 3. $A_E$ and $n_E$ of weathered soil.

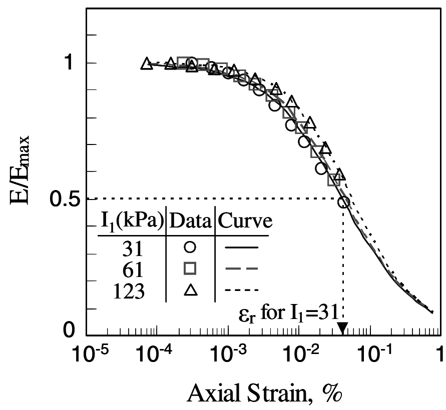


Figure 4. Plot of $E/E_{max}$ vs. ε for weathered soil.

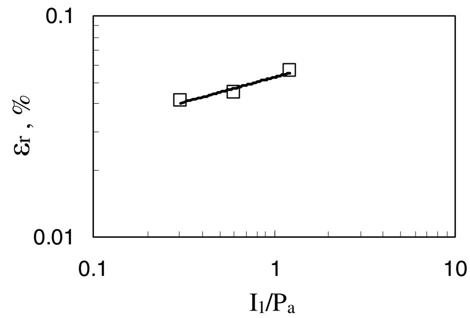


Figure 5. Relationship of $\varepsilon_r$ and $I_1/P_a$ for weathered soil.

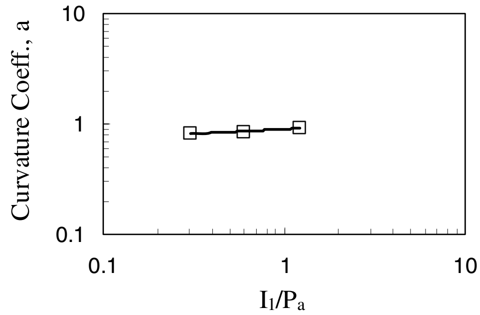


Figure 6. Relationship of a and $I_1/P_a$ for weathered soil.

### 3.3 *Crushed stone*

Crushed stone, engineered to $D_{50}$ of 8mm and $C_u$ of 13.5, was used as sub-ballast at one section of the test trackbed. Maximum shear modulus and Poisson's ratio were measured by crosshole testing. Menq (2003) evaluated the variation of shear modulus of gravel at various gradation curves using large free-free resonant column test. $n_E$ was suggested in terms of maximum shear modulus for various gradations as:

$$n_E = 0.48 \times C_u^{\,0.09}, \quad 1.2 \le C_u \le 50 \tag{6}$$

where $C_u$ is the coefficient of uniformity in the range of 1.2 to 50. Because the $n_E$ value does not change even in terms of the maximum Young's modulus, the Equation 6 can be directly used in the evaluation of $A_E$. The $A_E$ of 18590 was calculated from the crosshole results and the $n_E$ value of 0.61. The reference shear strain ($\gamma_r$), suggested by Menq, was converted to $\varepsilon_r$ by simply dividing with (1+ν) and substituted the value $C_u$ of 13.5. The downsized equation of $\varepsilon_r$ is represented as Equation 7 and the curvature coefficient, suggested as Equation 8, can be used regardless of modulus type, shear or Young's.

$$\varepsilon_r = \frac{0.12}{(1+\nu)} \times C_u^{-0.6} \times \left(\frac{I_1}{P_a}\right)^{0.5 \times C_u^{-0.15}} \tag{7}$$

$$a = 0.86 + 0.1 \times \log\left(\frac{I_1}{P_a}\right) \tag{8}$$

### 3.4 *Crushed rock-soil mixture*

The material is typically used as sub-ballast in mountainous area by engineering cuttings from slope-cut or tunnel excavation. At a high speed railroad site, the gradation was controlled with the maximum grain size of 200 mm, $D_{50}$ of 16 mm and of $C_u$ 37.5. A crosshole test was performed at various depths and the set of maximum Young's modulus was plotted with the $I_1$ value corresponding to each measured depth as shown in Figure 7. The values of $A_E$ and $n_E$ were determined as 13135 and 0.35, respectively. The reference strain was determined as aforementioned as Equation 9 and the curvature coefficient was used directly used as suggested by Menq.

$$\varepsilon_r = 0.01 \times \left(\frac{I_1}{P_a}\right)^{0.29} \tag{9}$$

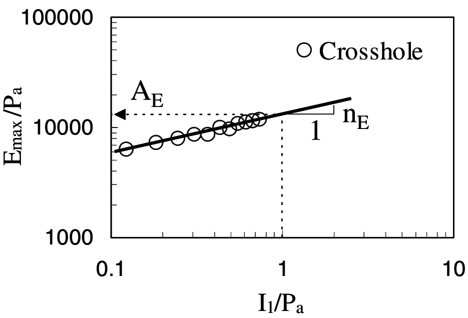


Figure 7. $A_E$ and $n_E$ of crushed rock-soil mixture.

## 4 ASSESMENT OF THE PREDICTION MODEL

To investigate the performance of the power models developed herein, the elastic responses of the test trackbed near PyeongTaek, Korea were evaluated using a 3-D elastic computer program (GEOTRACK) and compared with measured elastic vertical displacements during the passages of passenger and freight trains.

### 4.1 *Calculation of dynamic response*

Two types of trackbed were analyzed. One type (called trackbed A) consists 0.3m thick gravel ballast, 0.8m thick crushed stone as sub-ballast and 2.2 m thick subgrade of weathered soil overlying foundation soil as shown in Figure 8. The other type (called trackbed B) is the same except of sub-ballast, which is the weathered soil compacted to 95% of standard proctor maximum dry density ($\gamma_{d,max}$). The subgrade of both types was the same weathered soil compacted to 90 % of $\gamma_{d,max}$. The proposed resilient modulus model was used for each layer except the gravel ballast, whose model was recommended by the program as Equation 10.

$$E_R = 1569 P_a \left(\frac{\theta}{P_a}\right)^{0.535} \tag{10}$$

where $E_R$ is in MPa and θ is the bulk stress in kPa. The elastic deformations of trackbed A and B were calculated with the wheel load of 60 kN and 134 kN for passenger and freight train, respectively.

The calculated vertical displacements under passenger and freight train loads were plotted for trackbed A and B as shown in Figure 9 and 10, respectively. The displacements in the sub-ballast layer of trackbed A are in the order of 0.2 mm and 0.4 mm under passenger and freight train loads respectively. The elastic deformations are relatively small and do not vary significantly with depth because the high modulus of crushed stone sub-ballast governs the response of trackbed A. In the other hand, the deformations in the compacted soil sub-ballast of trackbed B vary from 0.3mm to 0.2 mm under passenger train load and, from 0.6 mm to 0.4mm under freight train load. The displacements reduce with depth in high rate in the soil sub-ballast layer and were smoothly extended to the soil sub-grade layer because the two layers have the similar modulus.

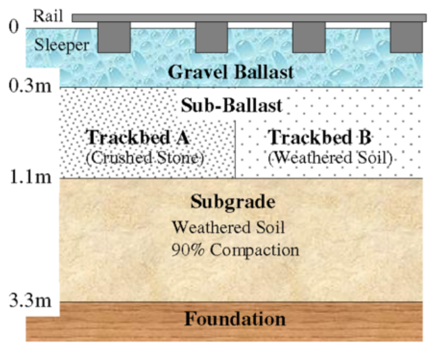


Figure 8. Layer system of trackbed A and B.

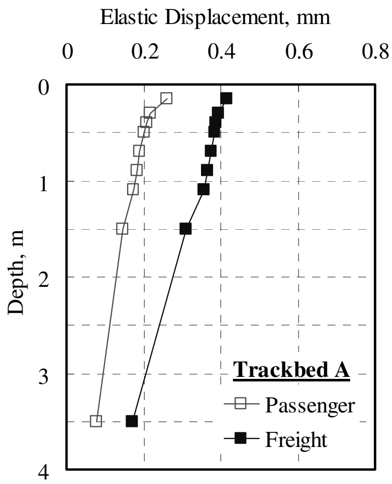


Figure 9. Vertical displacements of trackbed A.

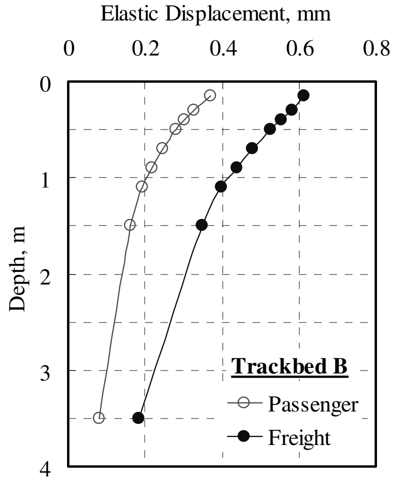


Figure 10. Vertical displacements of trackbed B.

### 4.2 *Field measurement of vertical displacements*

Cased boreholes were installed at trackbed A and B for crosshole seismic tests. For each trackbed, two 4.5Hz vertical geophones were installed inside the borehole using packers at the depth of 0.4m and 1.1 m below the ballast surface. The particle velocities were measured during passages of passenger and freight trains and converted to vertical displacements by integration. The typical vertical displacement time history, measured at the crushed stone sub-ballast (trackbed A) during the passage of freight train, is shown in Figure 11.

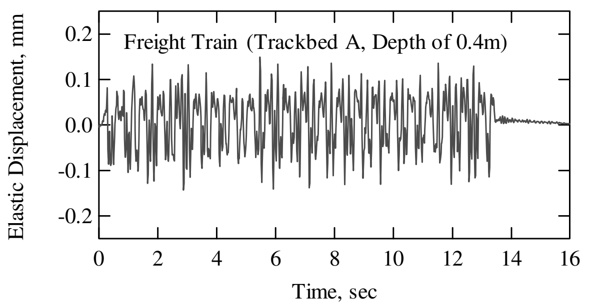


Figure 11. Typical displacement time history.

### 4.3 *Comparison of calculated and measured displacements*

The calculated vertical displacements under passenger and freight train loads (60 kN and 134 kN, respectively) were plotted as shown in Figure 12 and 13, respectively. The calculated vertical displacements agree well with measured values with the reasonable margin and are in the same decreasing patterns in both trackbeds as well. The proposed prediction model is thus concluded to work properly and to be very useful in engineering practice of high caliber.

## 5 CONCLUSIONS

A resilient modulus prediction model was developed by combining maximum Young's modulus and its normalized reduction curve for high strain, converted from nonlinear shear modulus using Poisson's ratio. The maximum Young's modulus was modeled by the power of the mean effective principal stress and the reduction curve was represented by the modified hyperbolic model. The prediction model was verified by comparing the calculated and measured vertical displacements of the test trackbed. Conclusions are:

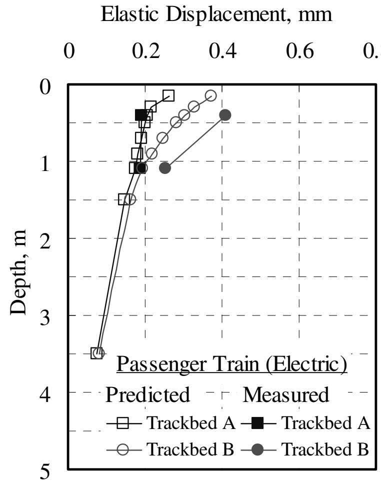


Figure 12. Comparison of displacements under passenger train.

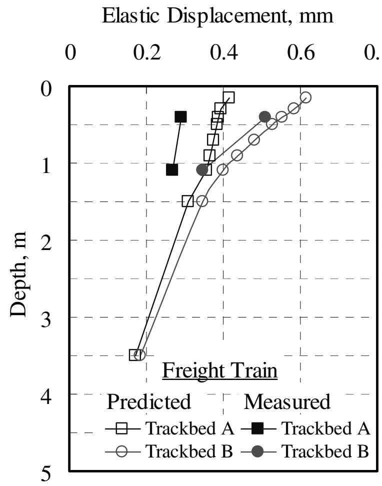


Figure 13. Comparison of displacements under freight train.

(1) The prediction model performs well with the typical sub-ballast and subgrade materials.
(2) The proposed methodology is theoretically sound and maintains its consistency because of its dependency on well developed seismic measurement techniques.

## ACKNOWLEDGEMENT

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