Designing for the effects of tunnelling on buried pipelines

Conception pour les Effets des Travaux en Souterrain sur Canalisations Enterrées

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ABSTRACT

Tunnelling and trenchless technologies can cause additional loading to existing pipelines. This paper highlights key findings of research conducted at the University of Cambridge regarding Greenfield ground movement response and estimating pipeline response in design. Four general load cases and a general procedure for pipeline design is provided, showing the implementation of key observations made during centrifuge testing, numerical modelling and field testing.

RÉSUMÉ

La technologie des travaux en souterrain peut causer une charge supplémentaire aux oléoducs existants. Ce document met en évidence les principaux résultats des recherches menées à l'Université de Cambridge, en ce qui concerne la réponse des mouvements du sol de Greenfield et l'estimation de la réponse de canalisaton dans la conception. Quatre cas de charge générale et une procédure générale pour la conception de canalisation est prévue, avec la mise en œuvre des principales observations faites durant les essais de centrifugeuses, la modélisation numérique et des essais sur le terrain

Keywords : pipelines, tunnelling, shear strain, load cases, design process, interaction, mechanisms, jointed, continuous

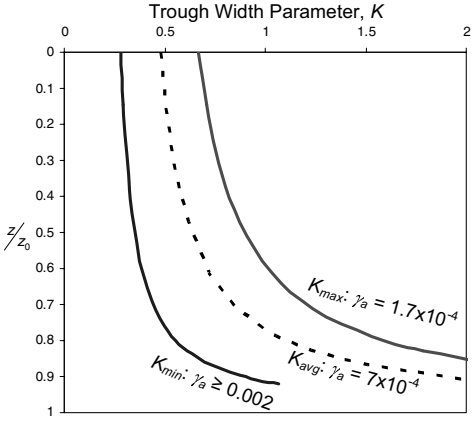
1 INTRODUCTION

Large cities across the world make use of tunnelling as a sustainable solution to traffic congestion. In urban areas, trenchless technologies such as pipe jacking allow the installation of pipelines and small tunnels with minimal surface disruption compared to open cut methods. These solutions are elegant for their respective purposes but potentially provide an additional flexural load component to the design of existing and future services and utilities. General structural design practice for new pipelines generally focuses on the circumference of pipelines, which means that there is a need to extend this design philosophy to include longitudinal bending, taking account of pipe-soil interaction.

This paper highlights key findings of research conducted at the University of Cambridge regarding Greenfield ground movement response and estimating pipeline response in design. A procedure for pipeline design is provided, showing the implementation of key observations made during centrifuge testing, numerical modelling and field testing.

2 GREENFIELD SOIL RESPONSE

It was found that ground movements and associated stress paths observed at positions above a model tunnel during centrifuge testing in sand were similar to observations in practice. Notably, volume losses in the soil increased with distance away from the tunnel and toward the surface. It was also observed that the trough width, i (and hence K, the trough width parameter), is a function of the shear strain state of the soil and can therefore be expected to depend on the tunnelling method. K decreased with increasing $S_{\max}/i^2$ (S_{max} is maximum Greenfield soil settlement) and tunnel depth, z_0, but increased with increased proximity to the tunnel z/z_0. On average, the relationship by Mair *et al*. (1993) describing the change of K with depth also fits centrifuge test results, suggesting that it would also be appropriate for granular material in general. This finding was further supported by plotting results for various published field cases (Vorster, 2005). Relationships for estimating upper and lower approximations of K, were developed based on centrifuge observations and the mentioned field cases. It is shown that the shear strain state of the soil should be taken into account for a more accurate estimate of the lateral extent of surface and subsurface settlement troughs. Using Vorster *et al.*'s (2005) equation to estimate the representative greenfield engineering shear strain, γ_a, an indicative relation of shear strain at a particular depth associated with the maximum, minimum and average values of K (Eq. 4.11) are shown in Figure 1.



$$K_{max} = \frac{0.231 + 0.436\left(1 - z/z_0\right)}{1 - z/z_0}$$

$$K_{avg} = \frac{0.155 + 0.319\left(1 - z/z_0\right)}{1 - z/z_0}, \text{ or Mair et al. (1993)}$$

$$K_{min} = \frac{0.068 + 0.214\left(1 - z/z_0\right)}{1 - z/z_0}$$

Notes:

1. γ_a is the representative greenfield engineering shear strain at a particular depth based on centrifuge test results and applying the estimation of shear strain published by Vorster et al. (2005), while assuming that ground movement vectors are directed to the tunnel axis
2. For K_{min} it should be noted that, after initially changing steadily, changes in K become very subtle after higher volume loss (Vorster, 2005). Beyond this point, changes in γ_a are not represented by changes in K. The magnitude of γ_a where this occurs is ~0.002 engineering shear strain.

Figure 1. Representative Greenfield Shear Strain associated with K_{max}, K_{min} and K_{avg} (Vorster, 2005)

The changes in trough width with increasing volume loss and the relation between trough width and shear strain also

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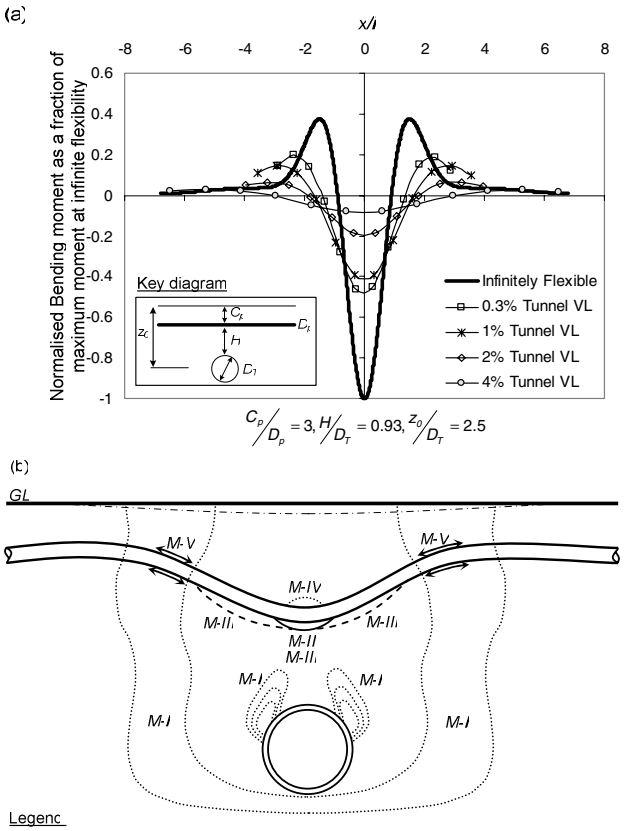
suggests that horizontal ground movements, S_h, are affected by the shear strain state of the soil. The estimation of S_h has a significant impact on pipeline design as it is frequently used in methods such as suggested by Attewell *et al*. (1986) to estimate the effect of pipe-soil interface shear, or for estimating pipe joint pullout / push-in for jointed pipelines as illustrated in regard to centrifuge test results and field trial by Vorster (2005). Current methods of estimating S_h were in the past found to underestimate observations in practice (e.g. Hong and Bae, 1995; Cording, 1991). Data from Mair (1979) on soft clay and Potts (1976) on dense sand revealed that S_h/S_v (with S_v the vertical soil movement) changed both with distance from the tunnel centreline, x/i , and proximity to the tunnel, z/z_0 . In the case of dense sand, however, the influence of z/z_0 was less pronounced and the ratio S_h/S_v remained more-or-less constant with depth, with ground movement vectors directed above the tunnel axis. More data is however required for providing a generic description of the relation between S_v and S_h.

3 CONTINUOUS PIPELINE BEHAVIOUR

Vorster (2005) showed that a pipeline could behave 'flexibly' in one instance, while under another set of circumstances behaving 'stiffer' in relation to the behaviour of the greenfield soil (for instance Figure 2a). This behaviour was shown to be due to global and local (pipe-soil interaction) mechanisms, including Mechanism I (the greenfield state), Mechanism II (gap formation), Mechanism III (positive downdrag failure), Mechanism IV (negative downdrag failure) and Mechanism V (longitudinal interaction); Figure 2b.



Legend
M-I...M-V = Global and Local Interaction Mechanisms
GL = Ground Level
Tunnel VL = Volume loss applied at tunnel level in the centrifuge mode

Figure 2: (a) Centrifuge observations of increasingly stiff behaviour with increasing volume loss; (b) Mechanisms affecting pipeline behaviour (Vorster, 2005)

An analytical study gave insight into requirements for the allocation of design and data gathering resources, to accurately define the Greenfield condition on the one hand, and performing interaction analysis on the other (Vorster *et al.*, 2005). These requirements relate to the estimation of pipe bending strain, based on relative pipe-soil bending rigidity, R (eq. 1), and were supported by centrifuge test results, where:

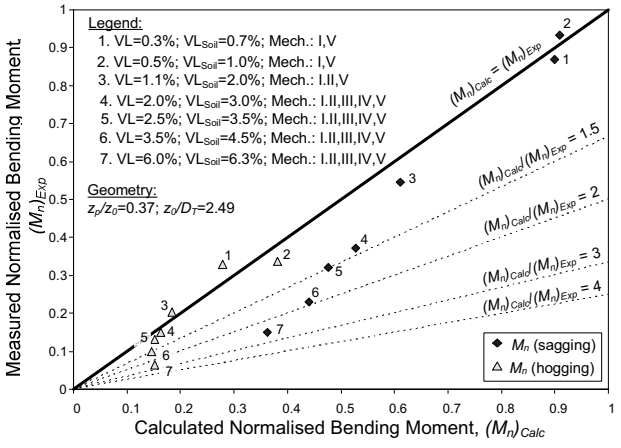
$$R = \left(E_p I_p\right) / \left(E_s r_0 i^3\right) \quad (1)$$

E_pI_p = Pipe bending stiffness [kN.m^2]
E_s = Soil stiffness [kN/m^2]
r_0 = Pipe radius [m]

The main findings are as follows:

- R<~0.1: The pipe-soil system behaves flexibly, requiring detailed information on the curvature of the Greenfield subsoil settlement trough at a level corresponding to pipeline level. Interaction analysis is not important since the pipeline practically follows the curvature of the soil.
- R>~5: The exact curvature of the Greenfield settlement trough is less important since the pipeline provides significant resistance to ground movement. Pipe-soil interaction analysis is required to capture the effect of global and local mechanisms and to avoid being overly conservative in the estimation of strain.
- ~0.1<R<~5: Both good estimation of the Greenfield curvature and interaction analysis are required.

Regardless of R, good information on likely values of S_{max}, i and soil stiffness are required to enable making reasonable estimates of pipeline behaviour. Vorster *et al.*'s (2005) method of estimating engineering shear strain was shown to be effective in accounting for the effect of the Greenfield state on soil stiffness degradation. The method can, however, not take account of the effects of interaction mechanisms as illustrated in Figure 3. Bending moment is overestimated and the magnitude of overestimation increases as ground movement is increased, if interaction mechanisms are omitted in the analysis for cases where a pipeline resists ground movement.



Note:
$(M_n)_{calc}$ = Normalised bending moment calculated using Vorster *et al.*, (2005)
$(M_n)_{exp}$ = Normalised bending moment measured in the centrifuge (Vorster (2005)

Figure 3: Effect of local mechanisms (and hence relative pipeline stiffness) on the prediction of normalised bending moment

In addition to bending, pipelines may also be subjected to pipe-soil interface shear, inducing an axial strain, ε_{sh}. Based on a quasi-analytical study Vorster (2005) showed that:

- If R<1.5, both bending strain, ε_b, and ε_{sh} are required to verify the location of the true maximum tensile strain, while if R>1.5, the location of maximum ε_b

also indicates the location of the combined maximum tensile strain; and

- A conservative estimate of maximum tensile strain would be obtained from estimating only ε_b (in sagging) if R>0.4. This was confirmed during a field study at Chingford, where it was found that this value could be reduced slightly to R>0.3 (Vorster, 2005). If R<0.3, it is likely that the combined tensile strain in hogging would exceed ε_b in sagging. Maximum tensile strain should in that case be estimated using the combined strain condition (ε_b+ε_{sh}).

4 JOINTED PIPELINE BEHAVIOUR

Similar to continuous pipelines, jointed pipelines also respond with rigidity. Apart from the Greenfield state (Mechanism I) and four interaction mechanisms observed to influence continuous pipelines, jointed pipeline behaviour is also influenced by positive and negative locking, positive downdrag in sagging and preferential pipe section movement. These mechanisms were confirmed during centrifuge testing and a field trial (Vorster, 2005).

From a design perspective, maximum joint rotation occurs in the joint closest to the tunnel centreline and will be a maximum if the joint is located on the tunnel centreline. However, joint rotation in hogging may be of similar magnitude (albeit slightly lower), which means that, although the central joint would be most critical, the joints in hogging should at least be considered from a monitoring point of view (Vorster, 2005).

Where joint axial and/or rotational stiffness are negligible across all joints in the affected area and $L_j/i < 2$ (L_j is joint spacing), pipe strains are not likely to be critical if R≥0.3 (as found by the combination of centrifuge test results and a field study; Vorster 2005). If a pipeline is rigidly jointed, with $(M_\theta D_p)/(E_p I_p)$ and $(E_j A_j)/(E_p A_p)$ greater than or equal to unity, however, the pipeline should be considered a continuous pipeline (M_θ is joint rotational stiffness [kN.m/rad]; E_jA_j is joint axial stiffness [kN]; E_pA_p is pipe section axial stiffness). In-between these limits the importance of tensile strain might be more prominent (e.g. continuous/jointed pipelines where the pipe is partly continuous and partly jointed or if joint stiffness is not negligible, but the pipeline is not continuous). In such cases the combination of axial tensile resistance in hogging and axial compressive resistance in sagging and the possibility of exceeding bending resistance in joints should be investigated. Individual pipe section curvature would also have to be taken into account. Vorster (2005) gives a detailed description.

Joint pullout was generally found to be less critical than joint rotation. In combination with a joint located on the tunnel centreline, however, its relevance as a design parameter and quantity to be monitored is of similar importance to joint rotation. As such it should be considered during design and monitoring. Joint pullout occurs in joints subjected to ground movement in hogging. The quantification of joint pullout is highly dependent on the general direction of ground movement vectors, which is likely to be different for soils subjected to different magnitudes of shear strain. Its magnitude also depends on pipe section length, joint position, joint stiffness and joint type, joint condition, the likelihood and magnitude of pipe-soil interface shear, and the general degradation of stiffness of the soil around the pipeline.

5 IMPLICATIONS FOR PIPELINE DESIGN

The objective of pipeline design is to take account of the most critical scenarios that increase tensile strain and to ensure that acceptable limits for a specific pipeline are not exceeded (both during and after construction). Pipeline design should take into account whether pipeline behaviour is continuous, perfectly jointed (negligible joint rotational and axial resistance) or in-between these bounds. An assessment of pipe-soil interaction should be made, including the potential formation of pipe-soil interaction mechanisms which may affect relative pipe-soil rigidity and subsequent loading of the pipe.

5.1 *Defining Behavioural Limits*

Specific limits depend on a variety of issues such as pipe material, joint configuration, loading history and likely loading imposed on a pipeline in future (e.g. tunnel-induced ground movement). In addition to circumferential limits, standard pipe design should therefore include allowable tensile and compressive strain limits of pipe material in a longitudinal direction, type of joint restraint, joint filler material (if applicable), and allowable joint movements (incorporating rotation and axial push-in and pullout of a specific type of restraint), taking account of initial conditions if possible. These limits also depend on the application of the pipeline in question (e.g. gas or water).

5.2 *Components of Strain for Design*

Pipe strain due to tunnelling has been shown to comprise a combination of bending strain, ε_b, pipe soil interface shear, ε_{sh}, and additional strain components due to joint behaviour, C_j. Although it would be convenient from a design point of view to ignore ε_{sh} and C_j, the effect of ground movement parallel to the pipeline may be significant where local joint conditions impact on pipeline behaviour. This is the case where some joints allow axial movement, effectively reducing the relative axial and bending pipe-soil rigidity of the pipe, while other joints have the capacity to resist tensile strain. Under these circumstances, increases in axial tensile strain due to the effect of ground movement parallel to the pipeline may be estimated by the magnitude of ε_{sh} possible at the location of axial movement. Where joint tensile capacity is exceeded, the effect of ε_{sh} in design can be neglected altogether since the lack of a tensile component of ε_{sh} in hogging, increases compressive strain in sagging, and is therefore not conservative for design. Vorster (2005) gives examples of the impact of joint conditions on strain.

For continuous pipelines with low relative pipe-soil bending and/or axial rigidity and associated low D_p/i-ratio, the distribution of ε_{sh} should be taken into account since the pipeline would be expected to closely follow ground movements (Attewell *et al.*, 1986). In reality the importance of ε_{sh} depends on the likelihood of the occurrence of local mechanisms or other factors which increase soil strains at the pipe-soil interface, or decrease the pipe-soil shear transfer area, to an extent where the transfer of ε_{sh} to the pipe becomes negligible. A conclusive criterion as to when ε_{sh} becomes insignificant for continuous pipelines is therefore illusive. Nonetheless, based on Vorster's (2005) centrifuge test results (indicating the formation of local mechanisms), the Chingford field case described by Vorster (2005), where R > 0.27, and Attewell *et al.*'s (1986) estimation of axial strain, a conservative bound for the relevance of ε_{sh} in the design of continuous pipelines surrounded by granular material, may be defined. If D_p/i≥0.2 (i.e. the ground movement disturbance is very local), or R>0.3 the influence of ε_{sh} is likely to be small. In reality, R might even be smaller if the transfer of ε_{sh} to the pipe is affected.

5.3 *Critical Design Load Cases*

Based on observations made in this study, four design load cases for continuous, continuous/jointed and perfectly jointed pipelines (with negligible axial and rotational joint resistance) are defined in the Appendix to account for longitudinal bending effects in pipeline design. Note that these cases do not account for circumferential design, which is normally accounted for in

standard design procedures. It should also be noted that additional strain caused by external factors other than tunnelling are not shown and should also be taken into account as appropriate.

Where Load Cases A, B-1 and B-2 are concerned for use with continuous/jointed pipelines, the following should be noted:

- Load Case A estimates a maximum design strain, ε_d, if the pipeline remains intact for the duration of ground movement;
- Load Case B-1 estimates a maximum for ε_d if joint tensile stiffness is high in hogging and joint compression is viable in any of the joints in sagging; and
- Load Case B-2 estimates a maximum for ε_d when joint compressive stiffness in sagging is high and tensile capacity in hogging is low.

The effect of some interaction mechanisms when included in design is generally regarded as favourable for improving the estimated strain state of a pipeline since they reduce the estimated bending or axial loading imposed on a pipeline (Vorster *et al.*, 2005). As such it may be argued that it is conservative to ignore local pipe-soil interaction effects in design unless they can be reasonably estimated. However, gap formation may not have the same effect. If gap formation is possible the pipeline may in the long-term be subjected to additional flexural strains over and above those caused by tunnelling alone. This is due to the combination of loss of bedding support and continuation of external loading (e.g. traffic) after tunnelling construction ends (Vorster *et al.*, 2005b) and has to be taken into account (Rajani and Tesfamariam, 2004).

5.4 *A General Design Process*

A process of applying the load cases described for continuous, continuous/jointed and perfectly jointed pipelines is proposed in the Table 1 of the Appendix. The process aims to allow users to start from a simple, conservative approach requiring only limited data. The user could then develop the design by utilising the benefits of implementing interaction mechanisms in terms of strain reduction, through implementation into interaction design to improve the design estimate as more sophisticated data becomes available, or financial constraints require in-depth evaluation of the problem. The general design philosophy implemented entails: (1) Defining the Greenfield soil behaviour; (2) Anticipating pipeline response (whether the pipeline is expected to behave in a continuous, continuous / jointed, or perfectly jointed manner); (3) Defining allowable limits; (4) Conducting a preliminary assessment whereby the pipeline profile is fitted to the estimated Greenfield ground movement and serviceability limits are tested. (5) If limits are exceeded for continuous or continuous / jointed pipelines, re-evaluate pipe-soil bending and axial rigidity and the localisation of loading, D_p/i, to establish whether the focus should be on improving Greenfield ground movement data, interaction analysis or both.

For continuous pipelines, the emphasis is on predicting pipe strain. Use Vorster *et al.* (2005) or Attewell *et al.* (1986) to improve predictions of bending and pipe-soil interface shear. If required, improve the analyses with more sophisticated methods and the inclusion of mechanisms as per guidance given by Vorster (2005). For continuous/jointed pipelines, the emphasis is both strain and joint movement based (rotation and axial movement). The pipeline is first assumed continuous and then gradually 'decomposed' into continuous portions coupled with joints with increasing ground movement (based on joint axial and rotational capacities) until, ultimately, it resembles a perfectly jointed pipeline (if ground movement is sufficient). To improve analyses, mechanisms and parametric effects as described by Vorster (2005) should be introduced into the predictive model. For perfectly jointed pipelines (negligible axial or rotational resistance), the emphasis is on joint movement, unless $L_j/i \geq 2$ (resembling a continuous/jointed pipeline), or $L_j/i < 2$ and R<0.3, when bending strain of individual pipe sections / continuous pipeline portions should be taken into account. If after initial considerations, joint movement or strain limits are exceeded, Greenfield ground movement data should be improved. If possible, more sophisticated analysis taking account of interaction mechanisms and parametric effects described by Vorster (2005) should be introduced. Finally, the design is concluded by defining parameters for pipeline monitoring (if applicable).

6 CONCLUSIONS

The following conclusions are presented:

- Tunnelling and trenchless technologies can cause additional loading to existing pipelines.
- Relationships for estimating upper and lower approximations of K are given based on centrifuge data and published field cases. It is shown that the shear strain state of the soil needs to be defined for more accurate estimation of the ground settlement trough at a particular depth; significant for pipeline design.
- Depending on the applicable global and local interaction mechanisms developed, a pipe may react 'flexibly' (following the ground profile) or 'stiff' (resisting the ground profile). The applicable value of R determines the accuracy of S_{max}, i, Es and curvature of the ground settlement trough required to be able to make reasonable estimates of pipe-soil interaction and induced pipe strain for continuous, continuous/jointed and jointed pipelines.
- Depending on R (continuous pipelines), D_p/i, axial stiffness (including joint effects), joint rotational stiffness, L_j (jointed pipelines) and the occurrence of interaction mechanisms, either axial or bending strain or both and joint rotation and pullout are required in design.
- Based on the study four general load cases and a design process are provided to estimate the behaviour of pipelines subjected to tunnel-induced ground movement.

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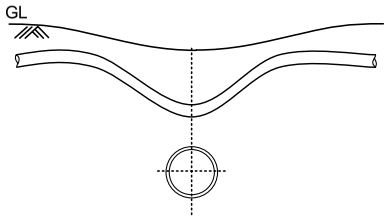
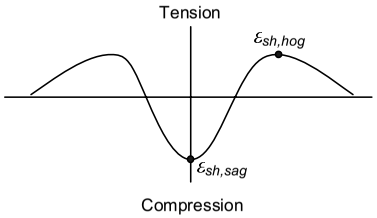
Appendix: Design Load Cases and General Design Process

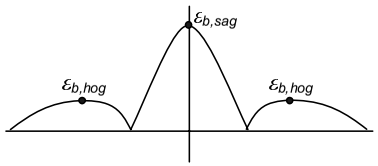
Case A
Continuous Pipeline Behaviour



Axial Strain: Pipe-Soil Interface Shear



Tensile Bending Strain



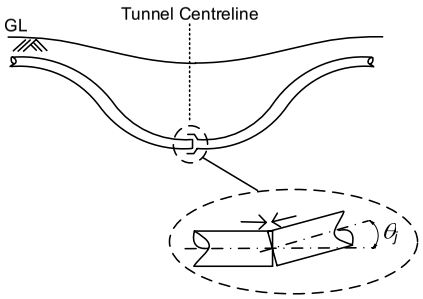
Continuous Pipelines:
Design strain, ε_d, highest of:

$$\underbrace{\varepsilon_d = \varepsilon_{b,sag}}_{\text{Likely if } R\geq 0.3,\ D_p/l\geq 0.2} \quad \text{or} \quad \varepsilon_d = \varepsilon_{b,hog} + \varepsilon_{sh,hog}$$

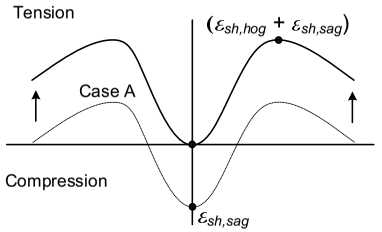
Continuous/Jointed Pipelines:

If $\varepsilon_d > \varepsilon_{joint}$, ***then*** $\varepsilon_d = \varepsilon_{joint}$

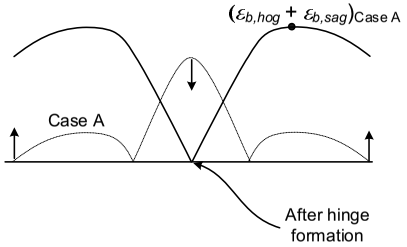
Case B-1:
Continuous / Jointed Pipelines
Maximum Hogging Strain



Axial Strain: Relative Joint Compression



Tensile Bending Strain: Hinge Formation (Sagging Region)

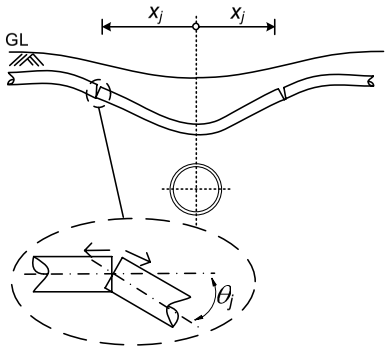


$$\varepsilon_d = \underbrace{\varepsilon_{b,hog} + \varepsilon_{sh,hog}}_{\textit{Case A}} + \underbrace{\varepsilon_{sh,sag}}_{\textit{Joint Compression}} + \underbrace{\varepsilon_{b,sag}}_{\textit{Hinge Formation}}$$

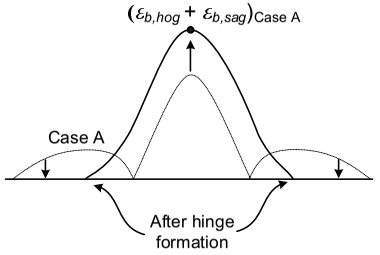
BUT

If $\varepsilon_d > \varepsilon_{joint}$, ***then*** $\varepsilon_d = \varepsilon_{joint}$

Case B-2:
Continuous / Jointed Pipelines
Maximum Sagging Strain



Tensile Bending Strain: Hinge Formation

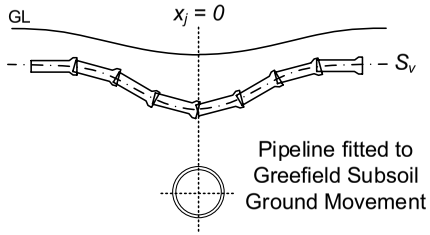


$$\varepsilon_d = \underbrace{\varepsilon_{b,sag} + \varepsilon_{b,hog}}_{\textit{Case A}}$$

BUT

If $\varepsilon_d > \varepsilon_{joint}$, ***then*** $\varepsilon_d = \varepsilon_{joint}$

Case C:
Perfectly Jointed Pipelines (i.e. negligible joint rotational or axial resistance)



Maximum Joint Rotation, $\theta_{j,max}$:
$x_j = 0$

Maximum Joint Pullout:
a. Critical joint is located closest to maximum subsoil curvature in hogging

b. For $x_j/l \geq 2$ the assumption for the direction of ground movement vectors (if not taken into account with sophisticated FE analysis) should consider directions above the tunnel axis

Tensile Strain:
Not considered for design if $L_j/i<2$, $R\geq 0.3$ and joint rotational and axial stiffness are negligible. If not, apply Load Cases A and B.

General Notes for use of Load Cases A to C:

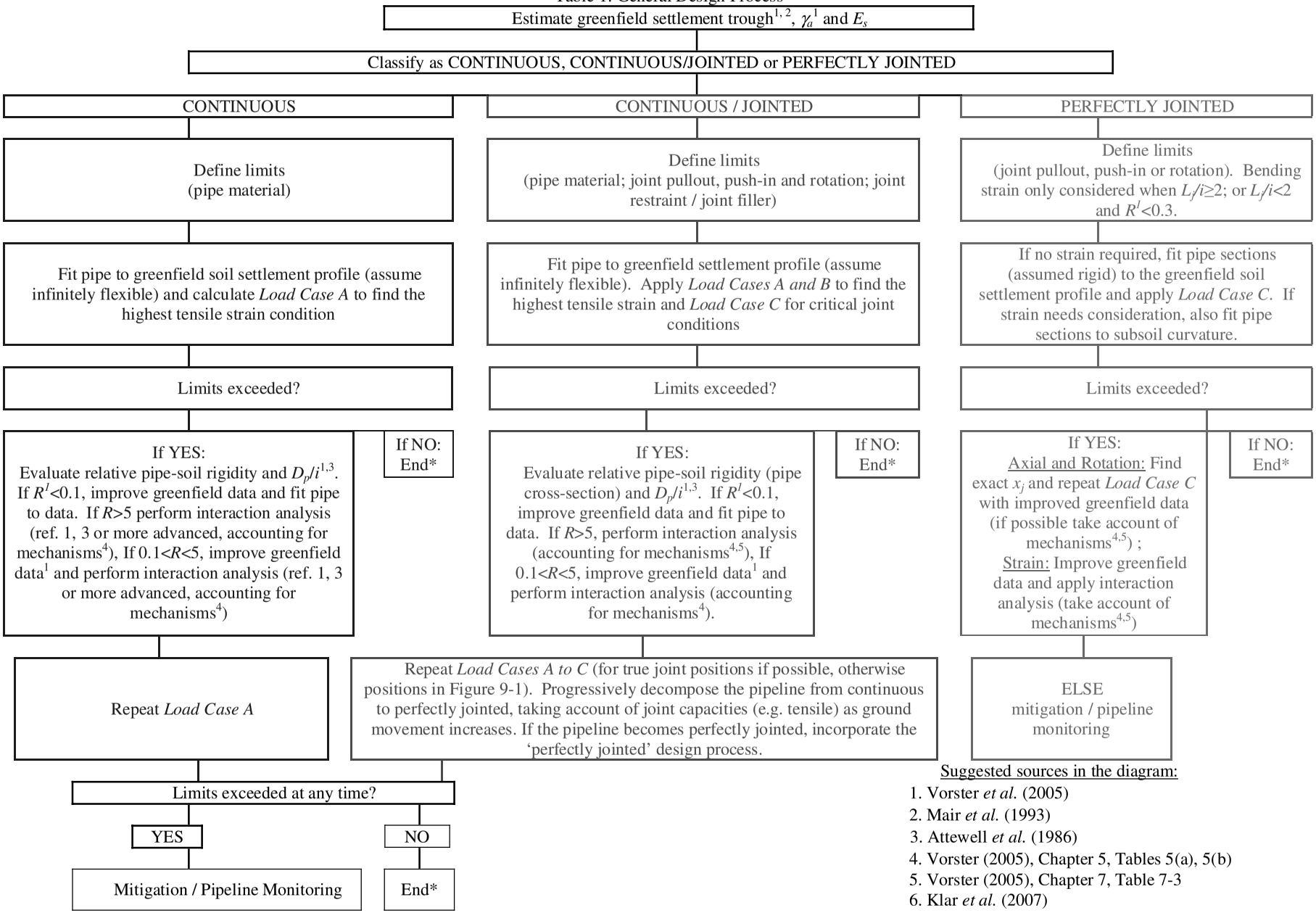
1. The diagrams depict specific pipeline conditions and associated strain diagrams.

2. The exact location of hinge-formation (Cases B-1 and B-2) will depend on the actual joint location, x_j. The cases shown are the most critical locations.

3. Reference to R is according to the method of Vorster et al. (2005) and should be based on the cross-section of the continuous pipeline portion.

4. Calculate $\varepsilon_{b,sag}$, $\varepsilon_{b,hog}$, $\varepsilon_{sh,sag}$ and $\varepsilon_{sh,hog}$ from Case A.

Table 1: General Design Process



Suggested sources in the diagram:

1. Vorster *et al.* (2005)
2. Mair *et al.* (1993)
3. Attewell *et al.* (1986)
4. Vorster (2005), Chapter 5, Tables 5(a), 5(b)
5. Vorster (2005), Chapter 7, Table 7-3
6. Klar *et al.* (2007)

**** Consider effect of gap formation (loss of bedding support) on long-term behaviour, e.g. traffic loading, etc.***