

3-D elasto-plastic model of tunnel performance under shadow of case history

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ABSTRACT

Tunneling in cohesionless soil leads to ground movement. In urban environment, the ground movement may adversely affect surface or subsurface constructions. In the present study, the ground movement under the impact of tunnel construction is studied using a 3-D finite element analysis (FEA). The analysis utilizes an elasto-plastic stress-strain relationship, a yielding function of the Mohr-Coulomb type and a plastic potential function of the Drucker-Prager type. The ground movement due to tunneling is also calculated using the surface displacement equation (SDE) proposed by Peck and Schmidt (1969). For assessing the reliability of the FEA and the SDE, a case history along the Greater Cairo Metro tunnel Line 2 has been considered. A comparison between the field measurements and those obtained by the FEA and the SDE is made. The FEA has been found to give better estimate of the actual surface settlements than the SDE. Both methods of analysis have been extended to other sandy soils. The results show that the surface settlement profiles using the SDE agree reasonably well with those obtained with the FEA for loose to medium sandy soils. However, for dense to very dense sand soils, the results from the SDE do not agree well with those of the FEA. This is due to the SDE does not take into account the difference in soil denseness.

RÉSUMÉ

Construire des tunnels dans une terre lâche cause un mouvement dans le sol. Dans un environnement urbain le mouvement du sol peut affecter les constructions de la surface ou les sous surface. Dans cette étude, le mouvement du sol causé par la construction du tunnel est étudié avec l'analyse du 3-D élément fini. L'analyse emploie une relation élastique-plastique tension-effort, une yield fonction du Mohr-Coulomb type et une fonction potentielle plastique du Drucker-Prager type. Le mouvement du sol causé par la construction des tunnels est calculé avec l'équation du déplacement de la surface (SDE) proposé par Peck et Schmidt (1969). Pour juger la fiabilité du FEA et SDE, un cas historique au long du Grand Caire Metro Souterrain ligne 2 est considéré. Une comparaison entre les mesures aux champs et les résultats obtenus avec la FEA et la SDE est conduite. La méthode FEA a prouvé une meilleure estimation de l'arrangement réel de la surface que la méthode SDE. Les deux méthodes d'études sont utilisées pour autre terre sableux. Les résultats ont prouvé que les profils de l'arrangement de la surface obtenue par la SDE sont en accord raisonnablement bien avec les résultats obtenue par la FEA pour lâche jusqu'au moyen terrain sableux. Pourtant, pour dense jusqu'au extrême dense terrain sableux les résultats de la SDE ne sont pas en bien accord avec les résultats de la FEA. Ceci a lieu parce que la SDE ne compte pas la différence en densité.

Keywords: Tunnels, settlement, numerical modeling and analysis, nonlinear displacement, surface displacement equation, deformations.

1 INTRODUCTION

Geotechnical problems in surface and subsurface environments are expected during the construction of a tunnel in cohesionless soil. This study is carried out to understand the performance of the tunnel system due to tunneling. The tunnel system performance is expressed in terms of the surface settlement caused by the tunnel construction. The study presents the prediction of the surface displacement profile obtained using a 3-D finite element analysis (FEA) and the surface displacement equation (SDE) proposed by Peck and Schmidt (1969). The modeling of such a problem should include the details of tunnel construction phases and the associated changes of stresses around the tunnels. A nonlinear stress-strain constitutive model is adopted for the soil surrounding the tunnel. A yield function of the Mohr-Coulomb type and a plastic potential function of the Drucker-Prager type are employed. In addition, linear elastic behavior is assumed for the tunnel liners.

A case study on the Greater Cairo Metro tunnel Line 2 (Fig. 1) is conducted to assess the accuracy of the FEA. The computed surface settlements as the metro tunnel is constructed are compared with the actual measurements from the field. A good agreement is found.

2 FINITE ELEMENT MODEL

The finite element computer program (COSMOS/M) is used in this study. The finite element model takes into account the effects of the vertical overburden pressure, the lateral earth pressure, the soil parameters, the tunnel liner, and the ground loss. Fig. 1 shows the configuration of the metro tunnel along the regional Line 2 of Cairo metro.

The soil media, the tunnel system, and the interface are simulated using the appropriate finite elements. Solid elements have been used for modeling the soil media and thick shell elements for modeling the tunnel liners. A 3-node triangular thick shell element is used with each node having 6 degrees of freedom (three translations and three rotations). As shown in Fig. 2, the solid element is prismatic in shape and has 6 nodes with each node having 3 degrees of freedom (three translations). The prismatic solid element and the triangular shell element interface are used between the soil media and the tunnel to ensure compatibility conditions at the interface between them.

The 3-D finite element mesh used in the analysis models a soil block with height, width, and length in the x, y, and z directions, respectively, as shown in Fig. 2. The vertical boundaries of the 3-D finite element model are restrained by roller supports to prevent a movement normal to the boundaries. The horizontal plane at the bottom of the mesh represents a

rigid bedrock layer and the movement at this plane is restrained in all three directions. The movement at the upper horizontal plane is free to simulate a free ground surface.

The metro tunnel was constructed in 1996 (El-Nahhass, 1999). Due to the construction of the metro tunnel, the ground surface was subjected to settlement. The lining is composed of 40-cm thick segments. The segments joints are never aligned along the tunnel and the thickness reduction is not as local construction as it is simulated in the model, which is conservative. The computed normal forces and bending moment values must comply with the strength of the 40-cm thick reinforced segments and the 24-cm thick joints between the segments.

3 PROPERTIES OF TUNNEL LINING AND SOIL

Soil displacements would be induced in the urban environment due to tunneling. The ground surface displacement due to the construction of the tunnel is calculated in this study. The final diameter (D) for the metro tunnel is 9.48 m and the excavation diameter of the metro tunnel is 10.28 m. The circular tunnel liner consists of seven segments and one key. The length of the ring is 1.5 m. The characteristics of the tunnels are tabulated in Table 1.

The project area under analysis lies within the alluvial plain, which covers the major area of the low land portion of the Nile valley in Cairo vicinity (Campo and Richards, 1998; El-Nahhass et al., 1994; Ezzeldine, 1999; National Authority for Tunnels, 1993, 1999, 2007). Site investigations along the project alignment have indicated that the soil profile consists of a relatively thin surficial fill layer ranging from two to four metres in thickness. A natural deposit of stiff, overconsolidated silty clay underlies the fill. This deposit includes occasional sand and silt partings of thickness from four to ten metres. Beneath the clay layer, there is thick alluvial sand that extends down to bedrock, which is well below the metro tunnel. The watertable varies between two meters to four meters from the ground surface. The geotechnical parameters are presented in Table 2.

Since soil behavior is generally inelastic, the constitutive relationship adopted in the analysis is an elasto-plastic model. The Mohr-Coulomb criterion is adopted. Excavation of the tunnel is simulated by removing elements from the excavated boundary. The friction angles (ϕ) adopted for the layers have been obtained using laboratory test results from reconstituted samples. The vertical initial drained modulus (E_v) is related to the effective pressure based on Janbu's empirical equation (Janbu, 1963) as given by Eq. 1

$$E_v = m p_a \left(\frac{\sigma_3}{p_a} \right)^n \quad (1)$$

where the modulus number (m) and the exponent number (n) are both pure numbers and p_a is the value of the atmospheric pressure expressed in appropriate units.

The geotechnical parameters of the soils at the site have been presented in National Authority for Tunnels (NAT) documents (National Authority for Tunnels, 1993, 1999, 2007). The soil parameters used for elasto-plastic finite element analysis for different types of the soil are presented in Table 3 (National Authority for Tunnels, 1993, 1999, 2007).

4 VOLUME LOSS IMPACT

The volume loss is considered in this study. The volume loss is the ratio of the difference between the excavated soil volume and the tunnel volume over the excavate soil volume. The

volume loss ranged from 1.5 % to 4.5 % and reached 6 % at some locations (El-Nahhass, 1999). The volume loss of 4 % is adopted in this study using the 3-D finite element analysis, as shown in Fig. 3.

5 3-D FINITE ELEMENT MODEL VERIFICATION

This case studied here is located along the Greater Cairo Metro Line 2, as shown in Fig. 1. The 3-D finite element model is used to predict the performance of the metro tunnel by computing the surface settlement. The computed values are compared with the field measurements so as to understand the behavior of the metro tunnel system. This comparison is used to assess the accuracy of the numerical model, as shown in Fig. 4. The comparison shows that there is good agreement between the computed and measured results.

Based on the good agreement between the computed and measured values, one can proceed to use the 3-D numerical model to explore other aspects of the tunnel system performance under the tunnel construction. In fact, the proposed model can help to predict the ground surface displacement at the different sandy soils.

6 SURFACE DISPLACEMENT DUE TO TUNNELING

The ground surface displacement due to tunneling is also calculated using the surface displacement equation proposed by Peck and Schmidt (1969). The surface displacements for different sandy soils computed by the proposed finite element model and the surface displacement equation are studied here. The calculated results using the proposed finite element model and the surface displacement equation are also compared with those obtained by the field measurements.

The surface displacement trough can be approximated by the normal probability curve as written in Equation (2) (Peck and Schmidt, 1969).

$$S = S_{\max} \exp \left(\frac{-x^2}{2i^2} \right) \quad (2)$$

where S is the surface displacement; S_{\max} is the maximum surface settlement at the point above the tunnel centreline; x is the distance from the tunnel centreline in transverse direction. Based on the case study involving medium sand, S_{\max} was recorded in the field. For loose sand, dense sand, and very dense sand, S_{\max} was estimated using the 3-D finite element analysis.

The width parameter (i) is the horizontal distance from the tunnel centreline inflexion point of the curve. O'Reilly and New (1982) proposed the empirical relationship as presented in Equation (3).

$$i = 0.28 Z + 0.1 \text{ for cohesionless soils} \quad (3)$$

where; Z is the depth of the tunnel axis below ground level; both i and Z are in meters.

The surface displacement profile above a tunnel with diameter 9.48 meters are calculated and plotted in Fig. 5 to Fig. 8 using the surface displacement equation.

The finite element analysis is also conducted to determine the surface displacements due to tunneling in different sandy soils. The average values of the different sandy soil parameters adopted in the finite element analysis are summarized in Table 3 (Duncan et al., 1980). Based on the finite element analysis, the surface displacements along the centerline of the metro tunnel for different sandy soil types are presented in Fig. 5 to Fig. 8.

The surface displacement profiles obtained by the finite element analysis are used to examine those obtained by the surface displacement equation. Fig. 5 shows the comparison between the results obtained by the finite element analysis with those obtained by the surface displacement equation for loose sand. The comparison indicates that the surface displacement profile computed by the finite element analysis has the same

trend as the surface displacement profile calculated by the surface displacement equation. It is also observed that the surface displacements calculated by finite element analysis are larger than those calculated by surface displacement equation in the region of zero m to 20 m from the centreline of the tunnel. However beyond 20 m from the centreline of the tunnel, the surface displacements calculated by finite element analysis is the same as those calculated by surface displacement equation. Generally, the results obtained by the finite element analysis agree well with those obtained by the surface displacement equation.

Fig. 6 shows the comparison of the calculated maximum surface settlements obtained by the finite element model, the surface displacement equation, and the field measurements at the Greater Cairo Metro Line 2, which is in medium sand. The comparison shows that the surface displacement profiles calculated by the surface displacement equation and the finite element analysis are in reasonable agreement with those obtained by the field measurements around the centreline of tunnel. The comparison also shows that the surface displacements obtained by the finite element analysis are higher than those calculated by the surface displacement equation, and closer to the measured settlements, in the region of 5 m to 20 m from the centreline of the tunnel. However beyond 20 m from the centreline of the tunnel, the surface displacements calculated by finite element analysis is the same as those calculated by surface displacement equation. Generally, the surface displacements calculated by the finite element analysis agree well with those calculated by the surface displacement equation.

Fig. 7 shows the comparison between the results obtained by the finite element model and those obtained by the surface displacement equation in dense sand. The surface settlement profile calculated by the finite element analysis has a different shape than the one obtained by the surface displacement equation. Generally, the surface displacements obtained by the finite element analysis are larger than those calculated by the surface displacement equation for dense sand.

In the case of very dense sand, Fig. 8 shows the comparison between the surface displacement profiles obtained by the finite element analysis with those obtained by the surface displacement equation. The surface settlement profile calculated by the finite element analysis again differs from that obtained by the surface displacement equation. Generally, the surface displacements obtained by the finite element analysis are higher than those calculated by the surface displacement equation. The difference between the two sets of computed settlements is greater in the very dense sand than in the dense sand.

The difference between the two sets of computed settlements lies in the use of the width parameter, i , in Equations (3). This equation is for use with cohesionless soils but takes no account of the different geotechnical parameters associated with different denseness of cohesionless soil. The proposed finite elements model takes into account the effects of the characteristics of the cohesionless soils of different denseness. Therefore the finite element analysis gives a better estimation of the surface settlement as demonstrated with the comparison of the measured settlements in medium dense soils involved in the case study. For the dense and very dense sandy soils, ignoring the appropriate soil characteristics in the surface displacement equation (Eq. 2) probably leads to larger error and farther deviation from the actual values.

7 CONCLUSIONS

A 3-D nonlinear finite element analysis has been used to study the ground surface displacements caused by tunneling. The analysis takes into account the changes in soil stress, the non-linear behavior of the soil, the different soil types, and the construction progress. The following conclusions can be drawn

regarding the performance of the tunnel under the effects of different factors.

- The 3-D finite element model can be successfully used to analyze and estimate the performance of the tunnel system with the construction of the tunnel as demonstrated by the case study of the Greater Cairo Metro tunnel Line 2.
- The surface settlement profile computed using the surface displacement equation by Peck and Schmidt (1969) is in reasonable agreement with the surface settlement profile computed using finite element analysis in loose to medium sandy soils.
- Based on the case study involving medium sand, the 3-D finite element analysis gives a better estimate of the actual settlements than the surface settlement equation.
- The surface settlement profile computed using the surface displacement equation does not agree well with the surface settlement profile computed using the 3-D finite element model in dense to very dense sandy soils.
- For different soil types, the surface settlements calculated by the finite element analysis are larger than those computed by the surface displacement equation.

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Table 1. Characteristics of the road tunnel liner

ν	E_b (t/m ²)	t cm	f_c (t/m ²)
0.2	2.1×10^6	40	4000

Note: ν is Poisson's ratio of tunnel liner, E_b is the elastic modulus of the tunnel lining, t is the thickness of tunnel lining, and f_c is the compressive strength of concrete.

Table 2. Geotechnical parameters

Soil parameter	Fill	Silty clay (drain condition)	Sand
γ_b (t/m^3)	1.8	1.9	2.0
k_o	0.58	0.8	0.37
ν_s	0.4	0.35	0.30
ϕ (Degree)	25	26	40
C (t/m^2)	1.0	0	0
Depth (m)	0.0 to 4.0	4.0 to 10.0	10.0 to end

Note: γ_b is bulk density, k_o is coefficient of lateral earth pressure, ν_s is Poisson's ratio, ϕ is the angle of internal friction for the soil, and C is cohesion.

Table 3. Soil Parameters

Material	m	n	C_u (kPa)	C (kPa)	ϕ_u	ϕ	ν_u	ν
Fill	300	0.74	50	10	20	25	0.4	0.4
Silty Clay	350	0.60	75	0	0	26	0.45	0.35
Sand	400-600	0.5-0.6	0	0	-	40	-	0.3
Loose sand	350	0.5	0	0	-	31	-	0.3
Medium sand	500	0.5	0	0	-	35	-	0.3
Dense Sand	800	0.5	0	0	-	39	-	0.3
Very dense sand	1100	0.5	0	0	-	43	-	0.3

Note: C_u is the undrained cohesion, C the effective cohesion (drained), ϕ_u is angle of internal friction in terms of total stress (for unsaturated fill $\phi_u = 20^\circ$), ϕ is the effective angle of internal friction (drained), ν_u is the undrained Poisson's ratio, and ν is the drained Poisson's ratio.

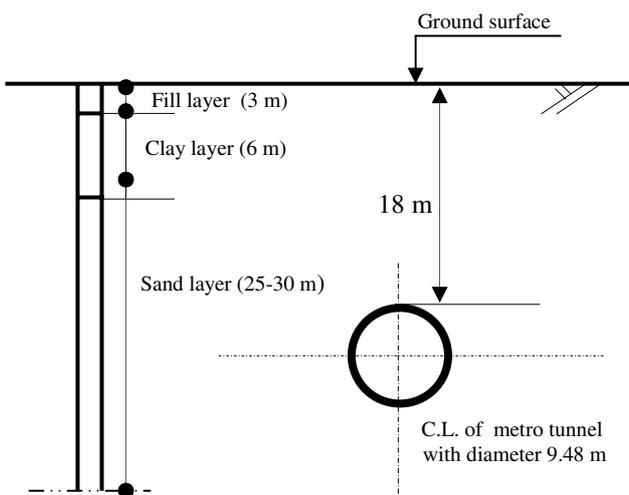


Fig. 1: Cross section along the Greater Cairo Metro tunnel Line 2 (Case study)

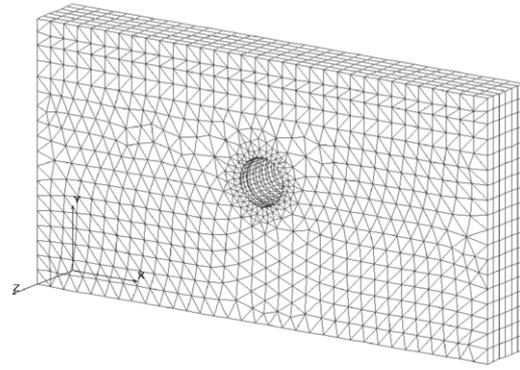


Fig. 2: 3-D finite element model of metro tunnel (case history)

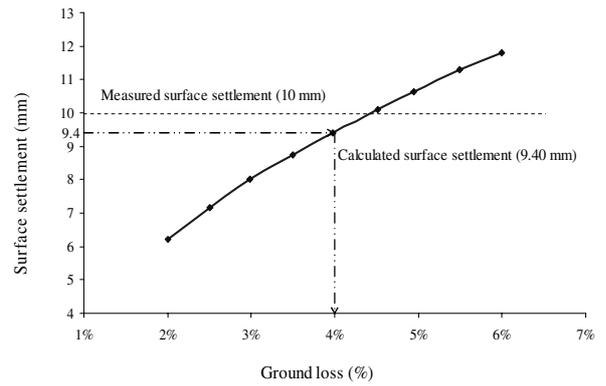


Fig. 3: Calculated settlement at ground surface with different ground loss using 3-D finite element analysis

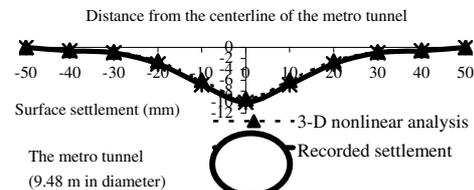


Fig. 4: Vertical displacement of soil at the ground surface of the Greater Cairo metro tunnel construction (case history)

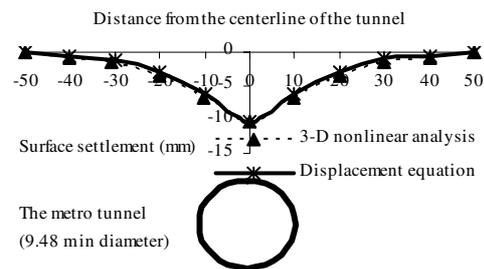


Fig. 5: Comparison between calculated surface settlements obtained by finite element model and surface displacement equation (tunnel located in loose sand)

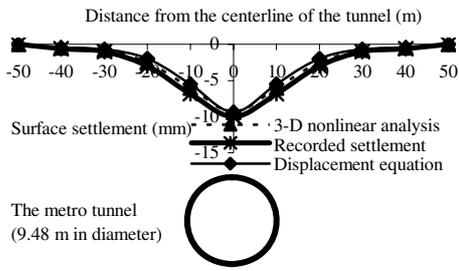


Fig. 6: Comparison among surface settlement obtained by finite model, surface displacement equation, and recorded measurements (tunnel located in medium sand)

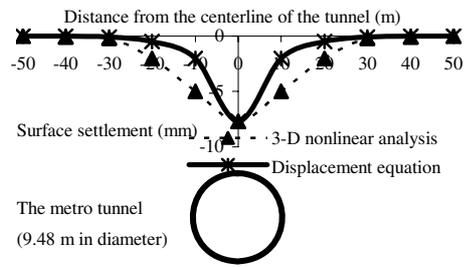


Fig. 8: Comparison between calculated surface settlements obtained by finite element model and surface displacement equation (tunnel located in very dense sand)

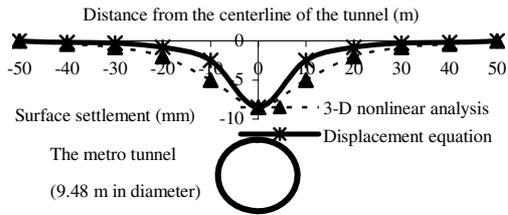


Fig. 7: Comparison between calculated surface settlements obtained by finite element model and surface displacement equation (tunnel located in dense sand)