Application of virtual cohesion concept to stability analysis of reinforced soil massifs

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ABSTRACT
The paper is dedicated to analysis of stability of soil massifs, reinforced with soil nails or geomaterials, based on assumption that the effect of reinforcement is equivalent to additional soil cohesion. If virtual cohesion c^* ensures stability of soil massif, having initial cohesion c, then *cohesion deficit* $\Delta c = c^* - c$ can be compensated by *equivalent reinforcement*. Formulas for reinforcement parameters, equivalent to Δc, are proposed, accounting for the reinforcement strength to rupture and pull-out force for geotextiles plus resistance to bending and shear for soil nails and stiff geogrids. Such approach enables application of all available non-reinforced soil stability analysis methods and computer codes to determine parameters of required reinforcement of soil massifs. This concept is applicable to 1D, 2D and 3D distributions of c^*, c and Δc. Reinforcement parameters can be optimized.

RÉSUMÉ
Stabilite des massifs des sols, armees par cloutage ou par geotextiles, etaites analyzees avec assumption que l'effet de armature est equivalent au cohesion de sol complementaire. Si cohesion virtuelle c^* assure stabilite de massif de sol, ayant cohesion initial c, alors cohesion deficit $\Delta c = c^* - c$ peut etre compensee par reinforcement equivalent. Les formules pour les parameters d'armature, equivalent au Δc, sont proposees, compte tenue de la resistance de reinforcement a rupture et a arrachment pour geotextiles aussi que a flexion et a force tranchant pour clouts et geogrilles rigids. Avec ce principe on peut calculer les parametres d'armature necessaire, profitant des methodes et des codes d'ordinateur existent, developees pout massifs non-armees, Ce principe est applicable au *1D*, *2D* et *3D* distributions de c^*, c et Δc. Les parameters d'armature peut etre optimisees.

Keywords: reinforced earth, soil reinforcement, geomaterials, soil nails, composite, homogenization, virtual cohesion, tension, pullout, bending moments, shear forces, localization.

1. INTRODUCTION

The concept of reinforced soil relates to man-made massifs (H.Vidal's *terre armée*), filled and reinforced layer-by-layer with geotextiles or geogrids *from ground up*, as well as to already existing soil massifs either natural or man-made (slopes, excavation shores), reinforced with soil nails *top down*. Soil nails are similar to soil anchors, differing in that *soil anchors are active* elements, which are mostly prestressed, they transfer soil lateral pressure to the retaining enclosure immediately after their installation, while *soil nails are passive*, activated only by soil deformations after installation. The installation technology is described in detail elsewhere (Skormin et al., 1981), (Moroz A.I. 1987), (Lazarte et al., 2003) .

Numerical analysis of reinforced earth and nailed soil massifs can be performed by the same methods. The analysis can include several ultimate limit states (ULS), as is shown on Fig.1.

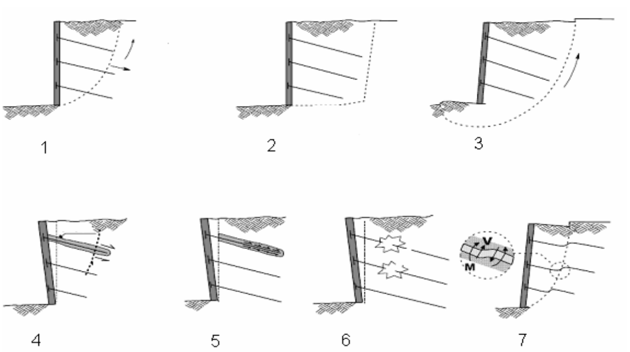

Figure 1. Ultimate limit states (ULSs) of reinforced soil massifs (Lazarte et al, 2003).

Most methods of reinforced soil massifs stability analysis use reinforcement parameters (strength and geometry) as input data. In such analysis reinforcement is replaced by point forces, applied to the solid sliding block. A different approach is proposed in the paper. Instead of reinforced soil massif a virtual non-reinforced soil massif is analyzed. The virtual massif has the same configuration as the real one, but its actual cohesion c is replaced by virtual cohesion c^*. If the massif is stable with virtual cohesion c^* and $c^* > c$ then in order to ensure stability of the real massif the difference (deficit) $\Delta c = c^* - c$ should be compensated by adequate reinforcement. Such approach makes it possible to apply conventional methods of non-reinforced soil massifs stability analysis to the analysis of reinforced massifs. The reinforcement parameters and geometry can be obtained and optimized if Δc is determined.

2. REINFORCED SOIL AS COMPOSITE (HOMOGENIZATION)

Reinforced soil is *locally non-homogeneous* due to presence of reinforcement, but it may be *homogenized* and viewed as *homogeneous* composite material (Savitsky, 2000), characterized by virtual cohesion $c^* = c + \Delta c$. After $\Delta c(z)$ is determined, the reinforcement geometry and strength parameters can be determined and optimized, using the calculated deficit Δc.

The application of this concept can be illustrated by the simple case of homogeneous vertical excavation shore with horizontal upper surface to which uniform load q is applied. According to (Lazarte et al., 2006), (Designer handbook, 1985), (Budin, 1982), (Snitko, 1962), (Construction Code, 2004) active pressure on vertical enclosure (facing)

$$\sigma_a = K_a(\gamma z + q) - 2c\sqrt{K_a}\ , \qquad (1)$$

with $K_a = tg^2(\pi/4 - \phi/2)$ as active pressure factor; φ and c as internal friction angle and cohesion; γ as soil weight density.

The facing of any reinforced soil massif shall be non-bearing, therefore, $\sigma_a = 0$. Hence, the shore is unstable at depth z if $c < 0.5\sqrt{K_a}\,(\gamma z + q)$, because then $\sigma_a > 0$. But, if the soil massif had virtual cohesion $c^* = c + \Delta c$ with virtual cohesion deficit $\Delta c > 0.5\sqrt{K_a}\,(\gamma z + q) - c$ then it would be stable. The deficit Δc could be compensated by geotextiles (zero bending

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stiffness), by geogrids and or soil nails (with non-zero bending and shear stiffness).

Active pressure coefficient for non-homogeneous shore is defined by the following equation (Designer handbook, 1985), (Budin, 1982), (Snitko, 1962), (Construction Code SNiP 2.06.07-87, 1987)

$$K_a = \frac{\cos^2(\phi-\beta)}{\left[1+\sqrt{\frac{\sin(\phi+\delta)\cdot\sin(\varphi-\alpha)}{\cos(\delta+\beta)\cdot\cos(\beta-\alpha)}}\right]^2 \cos^2\beta\cdot\cos(\delta+\beta)}, \quad (2)$$

where α is upper surface slope, β is front surface tilt (if $\beta>0$ then the front surface tilts away from the shore).

Equations (1, 2) apply to non-homogeneous soil masiffs as well, then the input values are functions of z (horizontal soil layers) or functions of x and z or functions of x, y and z (non-horizontal soil layers).

3. SOIL-NAILS INTERACTION

Consider a nail (or geogrid) in soil. Its length is $2L$, diameter is d, bending stiffness is D. A slip-line crosses the nail at point x=0 at depth z. The displacements $S=S(x)$ satisfy the following differential equation

$$D\cdot S^{IV}+K\cdot d\cdot z\cdot S=0\,, \quad (3)$$

with boundary conditions $S(0)=0$, $S(\pm L)=\pm S_0$, $S^{II}(\pm L)=S^{III}(\pm L)=0$, where $\pm S$ are displacements of the nail ends, the value of K can be borrowed from the following Table 1, used for analysis of laterally loaded piles (Construction Code 50-102-2003, 2004):

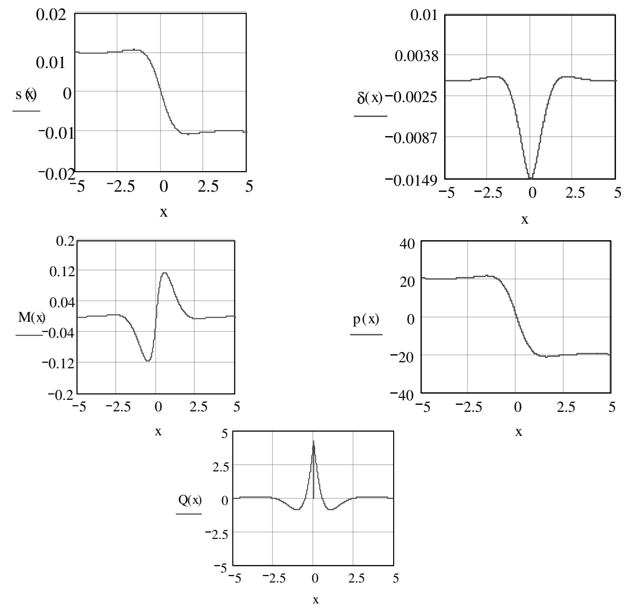

Figure.2. Displacements s(x), inclinations δ(x), bending moments M(x), shear forces Q(x), soil reactions p(x) (tons and meters)

General of solution equation (3) and the derivatives of this solution have the following complex form:

$$\overline{S}(x,k,n) = \sum_{k=0}^{3} A_k r_k^n e^{\cdot r_k x}\,, \quad (4)$$

where r_k is k-th root of the characteristic equation of equation (3); n is the order of x derivative of function S; A_k is constant value obtained by solving a system of four linear equations corresponding to the boundary conditions, listed above.

The solution of equation (3) in real form is, as follows:

$$S(x,k,n) = \mathrm{Re}[\overline{S}(x,k,n)] + \mathrm{Im}[\overline{S}(x,k,n)]\,, \quad (5)$$

Solution (5) was coded in MathCad, and numerical simulation was carried out for a typical 8 cm dia 10 m long concrete soil nail. The calculated results for displacement S_0=1 cm are shown on Fig. 2.

Failure of a nail can be due to tension, pullout, bending or shear that cannot happen simultaneously. The critical value of S_0 corresponds to at least one failure mode among the above four i.e., tension, pullout, bending or shear.

Maximum value of bending moment M is reached at a certain distance x_M, where the first derivative Q = 0. The value of x_M can be determined approximately: $x_M \approx 0.5$ m on Fig. 2.

Numerical simulation showed that the maximum nail inclination before failure is less than 0.05 (less than 3°) i.e., geometrical non-linearity is negligible, and the problem can be viewed as linear. Similar conclusion is made in (Lazarte et al., 2003).

Table 1

Soil around nail and its characteristics	Proportionality coefficient K, MN/m^4	
	Nails top down	Reinfoced earth from ground up
Clays and clay loams ($0.75<I_L<1$)	650-2500	500-2000
Clays and clay loams ($0.5\le I_L\le 1$); silty sands ($0.6\le e\le 0.8$)	2500-5000	2000-4000
Loams ($I_L\le 0.5$); fine sands ($0.6\le e\le 0.75$); medium sands ($0.55\le e\le 0.7$)	5000-8000	4000-6000
Hard clays and clay loams ($I_L<0$); coarse grained sands ($0.55\le e\le 0.7$)	8000-13000	6000-10000
1. Intermediate values of K are determined by interpolation. 2. Values of K for dense sands are 30% higher		

4. TRANSFER FROM COMPOSITE TO REINFORCEMENT (LOCALIZATION)

Localization i.e., conversion of deficit $\Delta c(z)$ into reinforcement parameters is based on the assumption that Coulomb law is applicable to composite materials, hence, $\tau = \sigma\cdot tg\phi + c^* = \sigma\cdot tg\phi + c + \Delta c\cdot$ Deficit Δc can be substituted by flexible geotextiles, resisting to tension and pullout, or to soil nails and geogrids, resisting to tension and pullout as well as to bending and shear.

4.1 *Geotextiles* follow the movements of surrounding soil and resist to tensile and pullout forces, therefore, thanks to their high flexibility, the reinforced soil behaves as isotropic massif. If geotextile layers are continuous in direction, perpendicular to the plane of the drawing, then the required tensile geotextile strength per unit width $T(z)\approx \Delta c(z)h(z)$, where $h(z)$ is distance between geotextile layers at depth z. For regularly spaced reinforcement strips $T(z)\approx\Delta c(z)h(z)B(z)/b(z)$, where $B(z)$ is lateral spacing and $b(z)$ is strip width at depth z. This approach corresponds to ULS 1, 4-7 (Fig. 1). The reinforcement length shall be sufficient to ensure stability, corresponding to ULS 2 and 3. This length may vary versus depth z.

4.2 *Soil nails* (or geogrids) resist to bending and shear beside tension and pullout. If failure occurs along slip-line R (Fig.3) then the deficit Δc can be compensated by nails. Such nails shall have maximum resistances, defined by the following equations:

$$T(z) = \frac{\Delta c(z) \cdot ctg\,[\phi(z)] \cdot b(z) \cdot h(z) \cdot \cos\, B(z)}{\cos[\,A(z) + B(z)]}, \quad (5)$$

$$Q_M(z) = \frac{\Delta c(z) \cdot b(z) \cdot h(z) \cdot \cos\, B(z)}{\sin\,[A(z) + B(z)]}, \quad (6)$$

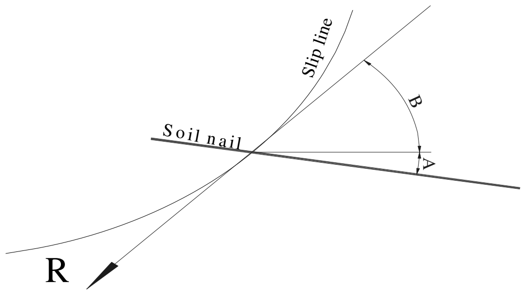

Figure 3. Soil nail, crossed by slip-line

where *T(z)* is maximum resistance to tension or pull-out at depth *z* (ULS 1, 4-6 on Fig. 1);

$Q_M(z)$ is maximum shear resistance, causing nail bending or shear failure at depth *z* (ULS 7 on Fig.1);

b(z)h(z) is area per one reinforcement element at depth *z* in vertical plane, perpendicular to the drawing plane;

A(z) and *B(z)* are angles of mutual position of reinforcement element and the slip-line tangent (Fig.3).

Minimum allowable nail diameter *d(z)* at depth *z* with nail material unit tensile strength *t*

$$d(z) = 2\sqrt{\frac{T(z)}{\pi \cdot t}}, \quad (7)$$

Minimum allowable nail length *L(z)* outside the virtual failure block at depth *z*

$$L(z) = \frac{T(z)}{\pi\, d\,(\gamma\, z \cdot tg\,\delta + c_0)}, \quad (8)$$

where γ is mean soil weight density above the nail;

q is uniform load, distributed on the soil massif surface;

δ, c_0 are nail-soil friction angle and cohesion.

ULSs 1, 4, 5 may be reached if $L_i > L_e$, and ULSs 6 if $L_i < L_e$ where L_i – is the length of reinforcement inside the potential sliding block.

If $\phi(z) \approx 0$ then it is better to use *dowels* (Lazarte, 2003), perpendicular to the slip-line ($A+B=\pi/2$ on Fig.1) which resist just to bending and shear. Perpendicularity to the slip-line i.e., $cos(A+B)=0$ and $T(z)=\infty$, means that, however great is the dowel tensile strength, its bending and shear resistances alone contribute to the sliding soil block stability.

If a nail/geogrid is located tangentially to the slip-line ($A+B=0$) then its bending and shear resistances do not contribute to the soil block stability i.e., $Q_M(z)=\infty$, and it only depends on tensile and pullout resistances.

The nails shall be checked for bending and shear resistance in compliance with ULS 7 (Fig. 1). This is done, as described in section 2. Firstly, displacement *S* for each *z* is determined for which $Q_{max}=Q(0)=Q_M(z)$, then $M_{max}=max(M(x))$ for this value of *z* and *S*. Then nail diameters d_M and d_Q are determined, ensuring nail strength to bending and shear. The resultant diameter at depth *z* is equal to max (d_M, d_Q).

4.3 *Example*

4.3.1 Consider an excavation pit 10 m deep to be reinforced with concrete nails, injected into pre-bored holes and reinforced with steel bars. The soil is homogeneous clay loam, whose parameters c=30 kPa , φ=19°, γ=17 kN/m³. Uniform load q=50 kPa is distributed over the top surface. The nails are spaced at 1.5 m horizontally and at 1 m vertically.

According to equation (1) soil cohesion deficit $\Delta c(z)>0$ occurs only below depth *z*

$$z = \frac{1}{\gamma}\left(\frac{2c}{\sqrt{K_a}} - q\right) \quad (9)$$

Using equation (3), obtain the required nail tension strength at depth 10 m: *T*(10)= 218.7 kN, If nail reinforcement tension strength *t*=375 MPa (steel A3) then its diameter shall be *d*=0.01 m, as per equation (4).

Assign the outside diameter of concrete nails equal to *D*=8*d*~0.08 m to provide optimal reinforcement. Then the lowermost nail length *L*=8.0 m, as per equation (5).

The above analysis pertains to ULSs 4-6 (Fig. 1).

In order to verify ULS 7 the potential slip-line shall be found, this can be done by any available method. Also a conservative approach may be applied: assume that nail axis is normal to slip-line at any point i.e., A+B=90° (Fig. 3). Equation (4) gives shear force Q_M =6 N equivalent to deficit Δc. Q_M value is very small i.e., each nail can evidently bear it. It all means that ULSs 4-6 are predominant, while ULS 7 may be neglected.

ULSs 2 and 3 relate to the failure of soil massif with no nails. ULS 1 is practically the same as ULSs 2 and 3, because of one or two nails may be neglected to stay on conservative side.

5. TECHNOLOGICAL STABILITY OF NAILED SOIL MASSIF

The top-down technology of soil nailing makes it necessary to separately consider stability of the lowermost portion of the nailed massif. This portion stays unprotected, after it is excavated, before the nails and the facing are installed. Although such situation lasts just for a short period of time, before the nails and the facing are put in place, it could be a source of local soil failure, followed by progressive failure of the whole nailed soil massif. In order to define such event ULS 8 shall be introduced in addition to ULS 1-7 on Fig.1.

Consider equation (1), in which presence of soil nails is taken into account by assuming that the nailed massif has virtual cohesion *c**, whose value is sufficient to ensure the massif stability after completion of nailing operations. However, the above-mentioned non-reinforced segment, having yet no facing, shall be characterized by initial cohesion *c*, for which the deficit $\Delta c=0$, and this means that Coulomb plastic condition could be achieved in at least one point below the lowermost nail i.e., soil collapse is possible. The same situation is also possible at all preceding stages of excavation and nailing, but the lowermost stage is the most vulnerable.

In practice such failure does not happen, because nailing operations are strictly supervised, and the height of the unprotected segment is evaluated by practical methods on site. Also the lowermost section could be excavated horizontally by short intervals, i.e. with berms so that 2D plane strain condition would never happen. In order illustrate the above situation in a FEM (PLAXIS) simulation was carried out. The results are shown on Figs. 4. 5.

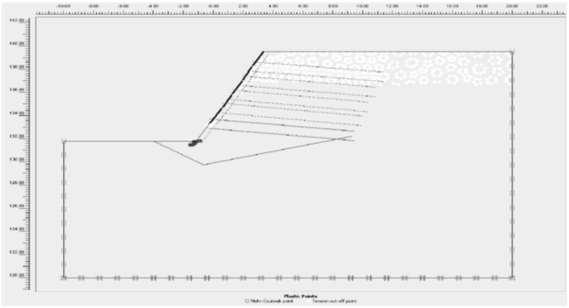

Figure 4. Plastic zone at the slope toe, before the lowermost nail is installed.

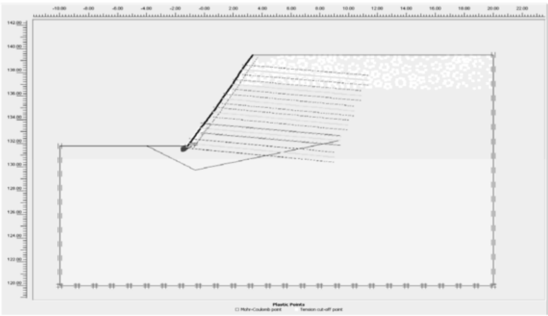

Figure 5. Plastic zone at the slope toe, after the lowermost nail is installed.

Figs. 4 and 5 might not be exactly realistic, because, in spite of its universal capacities, FEM, implemented in PLAXIS, might not reflect the real situation. But such results certainly demonstrate that ULS 8 shall be analyzed.

6. OPTIMIZATION OF SOIL REINFORCEMENT

There are the following possibilities to optimize soil reinforcement, using virtual cohesion concept.
1. It is not advisable to design reinforced soil massifs of uniform strength, in which all ULSs, shown on Fig. 1 could be reached simultaneously. E.g. in the case of seismic or other short-term actions it might be better to overdesign reinforcement so that beside global failure (ULS 2, 3) only ULSs 4, 5 would prevail (could be reached first) with the reinforcement staying intact, thus shortly preventing total collapse of the massif, although its shape could change under seismic forces.
2. Reinforcement is easily optimized if virtual cohesion concept were applied. Virtual cohesion distribution could be optimized by trial and error method, and then reinforcement strength and geometry could be optimized at each point separately.
3. Optimization of soil reinforcement geometry and strength with virtual cohesion already determined
its derivative by nail inclination angle is equal to zero.

7. CONCLUSIONS

1. A practical approach is proposed for reinforced soil massif stability analysis, based on simulation of reinforced soil as *homogenized composite,* characterized by *virtual cohesion* equal to the sum of actual cohesion plus certain *deficit*, simulating the required reinforcement.. The virtual cohesion is determined so that the reinforced massif is stable as regards all potential virtual ULSs then the deficit is found as difference of virtual and actual cohesion. *Localization* procedure converts the deficit into strength and geometry parameters of required reinforcement.
2. The virtual cohesion concept was applied to ULS analysis of excavated pit shore, based on well-known active pressure formulas.
3. Intermediate techological stages of soil nailing shall be analyzed for intermediate ULSs at each stage of operations, when the lowermost section of the massif is not strengthened by nails and protected by facing. Such ULSs can generate progressive failure and shall be taken into account in addition to ULS 1-7, related to stability of completed nailed massif with facing.
4. The virtual cohesion concept can be used for ULS analysis of soil massifs of arbitrary shape, based on critical slip-line concept. Equations were elaborated for converting cohesion deficit into parameters of reinforcement. This enables application of available computer codes of slope stability analysis to nailed slope analysis.
5. The virtual cohesion concept makes it possible to optimize reinforcement parameters and geometry.

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